13. Areas Related to Circles

Exercise 13

1. Question

The circumference of a circle exceeds its diameter by 16.8 cm. Find the circumference of the circle.

Answer

We know that, circumference of a circle, C = 2πr

and diameter, D = 2r

According to the question,

C = D + 16.8

$\Rightarrow 2\pi r = 2r + 16.8$

$\Rightarrow 2\pi r - 2r = 16.8$

$\Rightarrow 2r(\pi - 1) = 16.8$

$$\Rightarrow r\left(\frac{22}{7} - 1\right) = \frac{16.8}{2} \left[\because \pi = \frac{22}{7}\right]$$

$$\Rightarrow r\left(\frac{22-7}{7}\right) = 8.4$$

$\Rightarrow 15r = 8.4 \times 7$

$\Rightarrow r = 3.92$ cm

∴ Circumference of circle = 2πr

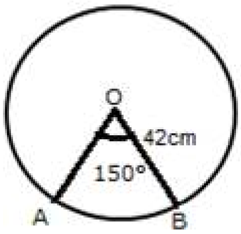
$$= 2 \times \frac{22}{7} \times 3.92$$

=24.64cm

2. Question

A sector is cut from a circle of radius 42cm. The central angle of the sector is 150°. Find the length of the arc.

Answer



Given: Radius of a circle, r = 42cm

Central Angle of the sector, θ = 150°

To find: Length of the arc i.e. AB

Now,

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$

$\Rightarrow$ Length of an arc of a sector of angle $\theta = \frac{150}{360} \times 2 \times \frac{22}{7} \times 42$

$\left[\because \pi = \frac{22}{7}\right]$

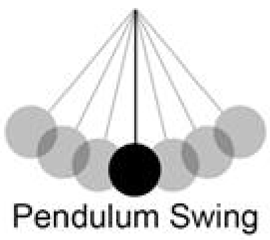
$\Rightarrow$ Length of an arc of a sector of angle $\theta = \frac{5}{12} \times 2 \times 22 \times 6$

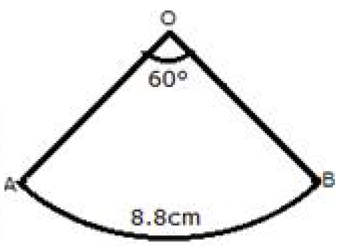
$\Rightarrow$ Length of an arc of a sector of angle θ = 5×22 =110 cm

3. Question

A pendulum swings through an angle 60° and describes an arc 8.8 cm in length. Find the length of the pendulum $\left[\text{Use } \pi = \frac{22}{7}\right]$

Answer





Given: Length of the arc = 8.8 cm

Central Angle of the sector, θ = 60°

To find: Radius of a circle

Now,

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$

$\Rightarrow\ 8.8 = \frac{60}{360} \times 2 \times \frac{22}{7} \times r \left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow\ 8.8 = \frac{1}{6} \times 2 \times \frac{22}{7} \times r$

$\Rightarrow\ 8.8 = \frac{1}{3} \times \frac{22}{7} \times r$

$\Rightarrow\ r = \frac{8.8 \times 21}{22}$

⇒ r = 8.4cm

4. Question

A wire made of silver is looped in the form of circular ear ring of radius 5.6 cm. It is rebent into a square form. Determine the length of the side of the square.

Answer

Given: Radius of the circle = 5.6cm

So, the circumference of the circle = 2πr

$= 2 \times \frac{22}{7} \times 5.6$

= 35.2cm

Now,

The perimeter of square = Circumference of a circle

= 35.2cm

Therefore,

Side of a square $= \frac{\text{The perimeter of a square}}{4}$

$= \frac{35.2}{4}$

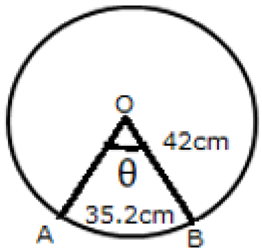
= 8.8cm

Hence, the side of a square = 8.8cm

5. Question

An arc of a circle of radius 42cm has a length 35.2 cm. Find the angle subtended by the arc at the centre of the circle.

Answer



Given: Length of the arc = 35.2 cm

Radius of a circle, r = 42cm

To find: Central Angle of the sector

Now,

Length of an arc of a sector of angle θ= $\frac{\theta}{360} \times 2\pi r$

$$\Rightarrow\ 35.2 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 42$$

$$\left[\because \pi = \frac{22}{7}\right]$$

$$\Rightarrow\ 35.2 = \frac{\theta}{360} \times 2 \times 22 \times 6$$

$$\Rightarrow\ \theta = \frac{35.2 \times 360}{12 \times 22}$$

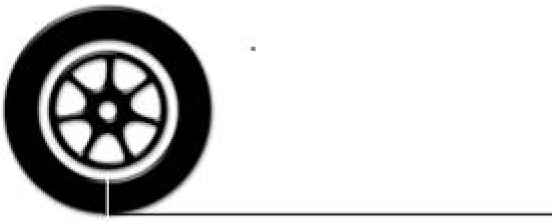
⇒ θ = 48°

Hence, the angle subtended by the arc at the centre of the circle is 48°

6. Question

A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 minutes when the car is moving at a speed of 80 km per hour?

Answer



Circumference $= 2\pi r$

Diameter of a car wheel = 80cm

∴ radius = r $= \frac{80}{2} = 40\text{cm}$

Distance covered in 1 revolution = Circumference of wheel

= $2\pi r$

= $2 \times \pi \times 40$

= 80π cm

Now, we find total distance covered

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Here, Speed = 80 km/hr

Time = 10 minutes

$$= \frac{10}{60}\text{ hour} = \frac{1}{6}\text{hour}$$

Putting a value in the formula, we get

$$80 = \frac{\text{Distance}}{\frac{1}{6}}$$

$$\Rightarrow \text{ Distance} = \frac{80}{6}$$

⇒ Distance = 13.33 km

= 13.33×1000 m

= 1333.3 m

= 1333.3×100 cm

= 1333333.3 cm

Now,

$$\text{Number of revolutions} = \frac{\text{Total distance}}{\text{Distance covered in 1 revolution}}$$

$$= \frac{1333333.3}{80\pi}$$

$$= \frac{1333333.3 \times 7}{80 \times 22}$$

= 5303.02

= 5303 (approx.)

Hence, number of revolutions = 5303

7. Question

Rajeev walks around a circular park of area 88704 sq. m How long will he take to walk 10 rounds at the speed of 4.5 km per hour?

Answer

Area of circular park = 88704 m^2

and Area of circle = πr^2 = 88704

$$\Rightarrow \frac{22}{7} r^2 = 88704$$

$$\Rightarrow r^2 = \frac{88704 \times 7}{22}$$

⇒ r^2 = 28224

⇒ r = $\sqrt{28224}$

⇒ r = 168m

[taking positive root, because radius can't be negative]

Perimeter of circle = 2πr

$$= 2 \times \frac{22}{7} \times 168$$

= 1056 m

= 1.056 km

So, total distance after 10 rounds = 1.056 × 10 = 10.56km

Now, we know that

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Here, Speed = 4.5 km/hr

Distance = 10.56 km

Putting value in formula, we get

$$4.5 = \frac{10.56}{\text{Time}}$$

$$\Rightarrow \text{Time} = \frac{10.56}{4.5}$$

= 2.34 hour

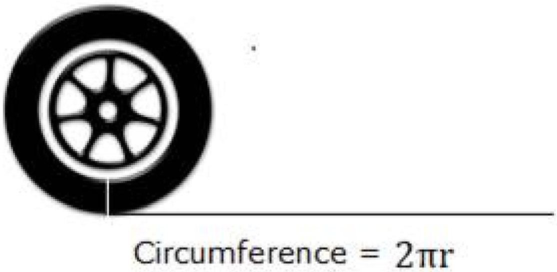
= 2 hours 20 minutes 24 seconds

Hence, Rajeev will take 2 hours 20 minutes and 24 seconds to walk 10 rounds.

8. Question

The diameter of the wheels of a bus is 140cm. How many revolutions per minute must a wheel make to move at a speed of 66km per hour?

Answer



Diameter of a bus wheel = 140cm

∴ radius = $r = \frac{140}{2} = 70cm$

Distance covered in 1 revolution = Circumference of wheel

= 2πr

= 2 × π × 70

= 140π cm

Now, we find total distance covered

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Here, Speed = 66 km/hr

Time = 1 minutes

$$= \frac{1}{60}\text{ hour} = \frac{1}{60}\text{hour}$$

Putting a value in the formula, we get

$$66 = \frac{\text{Distance}}{\frac{1}{60}}$$

$$\Rightarrow \text{ Distance} = \frac{66}{60}$$

⇒ Distance = 1.1 km

= 1.1 × 1000 m

= 1100 m

= 1100 × 100 cm

= 110000 cm

Now,

$$\text{Number of revolutions} = \frac{\text{Total distance}}{\text{Distance covered in 1 revolution}}$$

$$= \frac{110000}{140\pi}$$

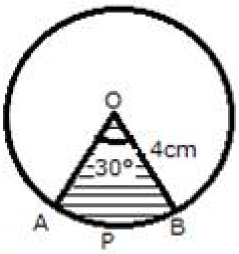
$$= \frac{110000 \times 7}{140 \times 22}$$

Number of revolutions = 250

9. Question

Find the area of the sector of a circle with radius 4cm and angle 30°. Also, find the area of the corresponding major sector [use π=3.14].

Answer



Given: Radius of circle = 4cm

And Central angle, θ = 30°

To find: Area of the sector

Now, Area of the sector of angle θ $= \frac{\theta}{360} \times \pi r^2$

⇒ Area of the sector of angle θ$= \frac{30}{360} \times 3.14 \times (4)^2$ [∵ π = 3.14]

⇒ Area of the sector of angle θ$= \frac{1}{12} \times 3.14 \times 4 \times 4$

$= \frac{3.14 \times 4}{3}$

$=4.19\text{cm}^2$

Now, we have to find the area of major sector (unshaded region)

= Area of circle – Area of sector OAPBO

$= \pi r^2 - 4.19$

$= \{3.14 \times (4)^2\} - 4.19$

$= \{3.14 \times 16\} - 4.19$

$= 50.24 - 4.19$

$= 46.05 \text{ cm}^2$

Hence, the area of sector = 4.19cm^2 and area of the major sector = 46.05cm^2

10. Question

Find the area of a quadrant of a circle whose circumference is 22 cm.

Answer

Given: Circumference of a circle = 22cm

⇒ $2\pi r = 22$

$$\Rightarrow\ 2 \times \frac{22}{7} \times r = 22$$

⇒ 2r = 7

$$\Rightarrow\ r = \frac{7}{2}\ \text{cm}$$

Now, we have to find the area of Quadrant

$$\text{Area of Quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{11 \times 7}{8}$$

= $9.625cm^2$

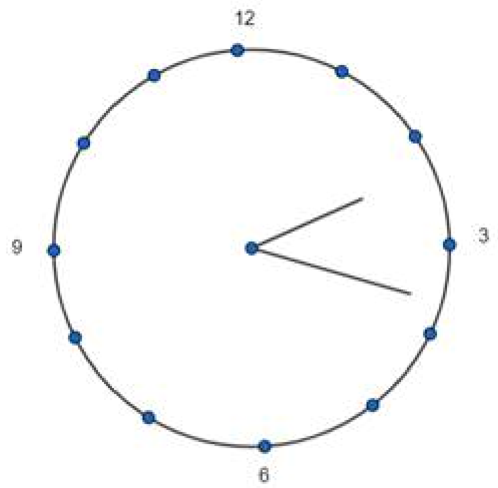
11. Question

The minute hand of a clock is 12cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Answer

Given: Length of minute hand = 12cm

Let us look at a clock:



So, minute hand covers a total of 60 minutes in one round of the clock.

So, 60 minutes = 360°

$$1\ \text{minute} = \frac{360°}{60} = 6°$$

35 minutes = $(35 \times 6)^\circ = 210^\circ$

So in 35 minutes, minute hand subtends an angle of 210°.

Now Area of segment $= \frac{\theta}{360^\circ}\pi r^2$

Where θ is the angle subtended.

Therefore, the area covered by minute hand$= \frac{210^\circ}{360^\circ}\pi r^2$

The radius of the circle = Length of the minute hand = 12cm

The area covered by minute hand $= \frac{7}{12} \times \frac{22}{7} \times 12^2$

The area covered by minute hand = $(22 \times 12)\ cm^2$

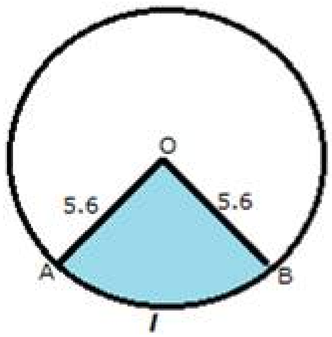
Area = $264\ cm^2$

Hence, The area covered by minute hand in 35 minutes is $264 cm^2$.

12. Question

The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. Find the area of the sector.

Answer



Let OAB be a the given sector with perimeter 27.2cm

Let arc AB = l

Perimeter of sector OAB = 27.2cm

⇒ OA + AB + OB = 27.2

⇒ 5.6 + l + 5.6 = 27.2

⇒ l = 27.2 – 11.2

⇒ l = 16cm

Now, we know that

$$\text{Length of the arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow\ 16 = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow\ \frac{16}{2\pi r} = \frac{\theta}{360} \ldots\text{(i)}$$

Area of sector OAB $= \frac{\theta}{360} \times \pi r^2$

$= \frac{16}{2\pi r} \times \pi r^2$ [from (i)]

= 8r

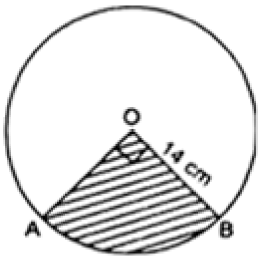
= 8 × 5.6

= 44.8cm^2

13. Question

A chord of a circle of radius 14cm makes a right angle at the centre. Find the areas of the minor and the major segments of the circle.

Answer



Given: Radius of circle = 14cm

and ∠AOB = 90°

$$\therefore \text{Area of minor sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{4} \times 22 \times 2 \times 14$$

=154cm^2

and Area of major sector = Area of circle - Area of minor sector

$= \pi r^2 - 154$

$= \frac{22}{7} \times 14 \times 14 - 154$

$= 22 \times 2 \times 14 - 154$

$= 462 cm^2$

14. Question

The area of a circle is 78.5 sq.cm. Calculate the circumference of the circle [Taken π=3.14].

Answer

Given: Area of circle = $78.5 cm^2$

$\pi r^2 = 78.5$

$\Rightarrow 3.14 r^2 = 78.5$

$\Rightarrow r^2 = \frac{78.5}{3.14}$

$\Rightarrow r^2 = 25$

$\Rightarrow r = \pm 5$

$\Rightarrow r = 5cm$

[taking positive root, because radius can't be negative]

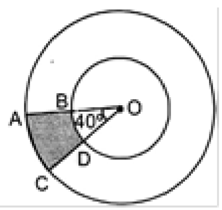
Now, circumference of circle = $2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.4cm$

15. Question

Find the area of the shaded region in the given figure if radii of the two concentric circles with centre O are 7cm and 14 cm respectively and ∠AOC=40°.



Answer

Given: Radius of the small circle, OB = 7cm

Radius of second circle, OA = 14cm

and ∠AOC=40°

∴Area of minor sector OBD = $\frac{\theta}{360} \times \pi r^2$

$= \frac{40}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{1}{9} \times 22 \times 7$

= 17.11 cm^2

∴Area of minor sector OAC = $\frac{\theta}{360} \times \pi r^2$

$= \frac{40}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{1}{9} \times 22 \times 2 \times 14$

= 68.4 cm^2

Area of the shaded region = Area of sector OAC

– Area of sector OBD

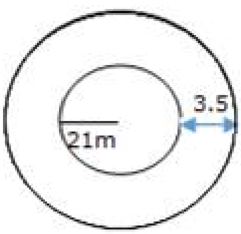
= 68.4 – 17.1

= 51.3 cm^2

16. Question

A circular park, 42m is a diameter, has a path 3.5 wide running round it on the outside. Find the cost of gravelling the path at Rs. 4 per m^2

Answer



The diameter of a circular park = 42m

⇒ the radius of a circular park, $r = \frac{42}{2} = 21m$

Width of path = 3.5m

⇒ radius of the park with path, R = 21 + 3.5 = 24.5m

∴ Area of path = Area of outer circle – Area of the inner circle

$= \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$= \pi (R - r)(R + r)$

$$= \frac{22}{7}(24.5 - 21)(24.5 + 21)$$

$$= \frac{22}{7}(3.5)(45.5)$$

$= 22 \times 0.5 \times 45.5$

$= 500.5\ m^2$

Now, the cost of gravelling the path of $1m^2$ = Rs 4

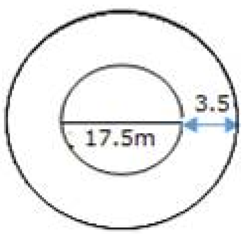
the cost of gravelling the path of $500.5m^2$ = Rs 4×500.5

= Rs 2002

17. Question

The diameter of a circular pond is 17.5m. It is surrounded by a path of width 3.5 m. Find the area of the path.

Answer



The diameter of a circular park = 17.5m

⇒ the radius of a circular park, $r = \frac{17.5}{2}$

Width of path = 3.5m

⇒ radius of the park with path, $R = \frac{17.5}{2} + 3.5 = \frac{17.5+7}{2} = 12.25m$

∴ Area of path = Area of outer circle – Area of inner circle

$= \pi R^2 - \pi r^2$

$= \pi (R^2 - r^2)$

$= \pi (R - r)(R + r)$

$= \frac{22}{7}(\frac{24.5}{2} - \frac{17.5}{2})(\frac{24.5}{2} + \frac{17.5}{2})$

$= \frac{22}{7}(\frac{24.5 - 17.5}{2})(\frac{24.5 + 17.5}{2})$

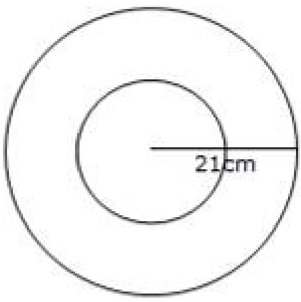
$= \frac{22}{7} \times \frac{7}{2} \times \frac{42}{2}$

= 231 m^2

18. Question

The area enclosed between two concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, find the radius of the inner circle.

Answer



Given: Let the radius of the inner circle = r

Area enclosed between two concentric circles = 770cm^2

and Radius of the outer circle, R = 21cm

∴ The area enclosed between two concentric circles

= Area of the Outer circle – Area of the inner circle

$770 = \pi R^2 - \pi r^2$

$770 = \pi(21^2 - r^2)$

$770 = \frac{22}{7}(441 - r^2)$

$\frac{770 \times 7}{22} = 441 - r^2$

$245 = 441 - r^2$

$\Rightarrow r^2 = 441 - 245$

$\Rightarrow r^2 = 196$

$\Rightarrow r = \sqrt{196}$

$\Rightarrow r = \pm 14$

$\Rightarrow r = 14$cm [taking positive square root, because radius can't be negative]

Hence, the radius of the inner circle is 14cm.

19. Question

The difference between circumference and diameter of circular plot is 105m. Find the area of the circular plot.

Answer

Given: Difference between circumference and diameter of circular plot = 105m

We know that,

The circumference of a circle = $2\pi r$

and diameter of circle = $2r$

According to the question

$2\pi r - 2r = 105$

$\Rightarrow 2r(\pi - 1) = 105$

$\Rightarrow r = \frac{105}{2(\pi - 1)}$

Now, Area of circular path = πr^2

$= \pi \left(\frac{105}{2(\pi - 1)}\right)^2$

$$= \frac{22 \times 105 \times 105}{7 \times 4(\frac{22}{7} - 1)^2}$$

$$= \frac{11 \times 15 \times 105}{2 \times \left(\frac{22-7}{7}\right)^2}$$

$$= \frac{11 \times 15 \times 105 \times 49}{2 \times 15 \times 15}$$

$$= \frac{11 \times 7 \times 49}{2}$$

$= 1886.5m^2$

20. Question

The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs. 0.50 per m^2. Find the cost of ploughing the field $\left[\text{Take } \pi = \frac{22}{7}\right]$

Answer

Total cost of fencing a circular field = Rs 5280

Cost of fencing per meter = Rs 24

So,

$$\text{The total length of the field} = \frac{\text{The total cost of fencing}}{\text{cost of fencing per meter}}$$

$$= \frac{5280}{24}$$

= 220m

Here, the total length of the field would be the circumference of the circular field.

∴ Total length = Circumference of circular field

⇒ 220 = 2πr

$$\Rightarrow \frac{220}{2\pi} = r$$

$\Rightarrow \; r = \dfrac{110}{\pi}$

Now, Area of the field = πr^2

$= \pi \left(\dfrac{110}{\pi}\right)^2$

$= \dfrac{110 \times 110 \times 7}{22}$

= $3850m^2$

Now,

Cost of ploughing $1m^2$ of the field = Rs 0.50

Cost of ploughing $3850m^2$ of the field = Rs 0.50 × 3850

= Rs 1925

Hence, the cost of ploughing the field is Rs 1925

21. Question

A field is in the form of a circle. The cost of ploughing the field at Rs. 1.50 per m^2 is Rs. 5775. Find the cost of fencing the field at Rs. 8.50 per metre.

Answer

Cost of ploughing $1m^2$ of the field = Rs1.50

The total cost of ploughing the field = Rs 5775

So,

$$\text{Area of the field} = \frac{\text{The total cost of ploughing the field}}{\text{Cost of ploughing per square. meter}}$$

$= \dfrac{5775}{1.50}$

$=3850m^2$

Given: A field is in the form of a circle.

Let the radius of the field = r

∴ Area of the field = πr^2

$\Rightarrow 3850 = \pi r^2$

$\Rightarrow\ 3850 = \frac{22}{7}r^2$

$\Rightarrow\ r^2 = \frac{3850 \times 7}{22}$

$\Rightarrow r^2 = 1225$

$\Rightarrow r = \sqrt{1225}$

$\Rightarrow r = \sqrt{(5\times5\times7\times7)}$

$\Rightarrow r = 35m$

Now,

Circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times 35$

= 2 × 22 × 5

= 220 m

Cost of fencing 1m of the field = Rs 8.50

Cost of fencing 220 of the field = Rs 8.50 × 220

= Rs 1870

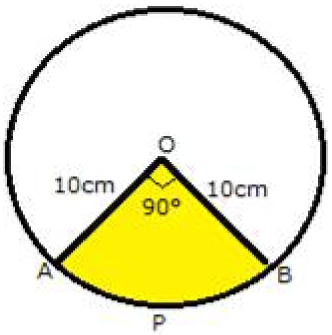
22. Question

A chord of a circle of radius 10cm subtends a right angle at the centre. Use π=3.14 and find:

(i) area of the minor sector

(ii)area of the major sector

Answer



Given: Radius of the circle = OA = OB = 10 cm and θ = 90°

(i) Area of the minor sector

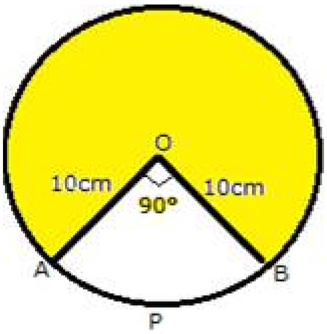
"Area of minor sector " $= \frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times (10^2)$

$= \frac{1}{4} \times 3.14 \times 100$

= 3.14 × 25

= $78.5 cm^2$



(ii) Area of major sector

Area of major sector = Area of circle - Area of minor sector = $\pi r^2 - 78.5$

= $3.14 \times (10)^2 - 78.5$

= 314 – 78.5

= $235.5 cm^2$

23. Question

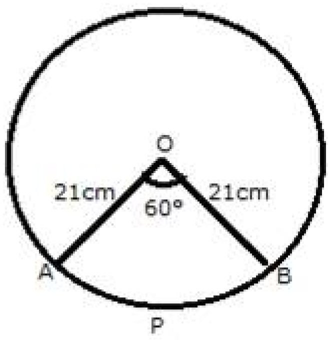
In a circle of radius 21 cm, and arc subtends an angle of 60° at the centre. Find

(i) length of arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord of the arc.

Answer



Given: Radius of the circle = OA =OB = 21cm

and θ = 60°

(i) Length of the arc

$$\text{Length of the arc APB} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21$$

$$= \frac{1}{6} \times 2 \times 22 \times 3$$

= 22cm

(ii) Area of the sector formed by this arc

$$\text{Area of minor sector OAPB} = \frac{\theta}{360} \times \pi r^2$$

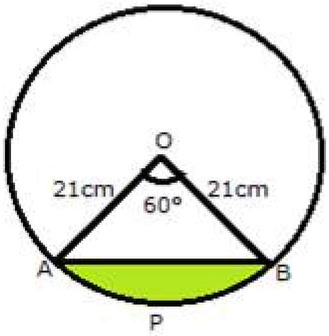
$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times 22 \times 3 \times 21$$

= 11 × 21

= 231 cm^2

(iii) area of the segment formed by the corresponding chord of the arc



In ΔOAB,

∠OAB = ∠OBA (As OA = OB)

∠OAB + ∠AOB + ∠OBA = 180°

2∠OAB + 60° = 180°

∠OAB = 60°

∴ ΔOAB is an equilateral triangle.

$$\text{Area of } \Delta\text{OAB} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (21)^2$$

$$= \frac{441\sqrt{3}}{4}\,\text{cm}^2$$

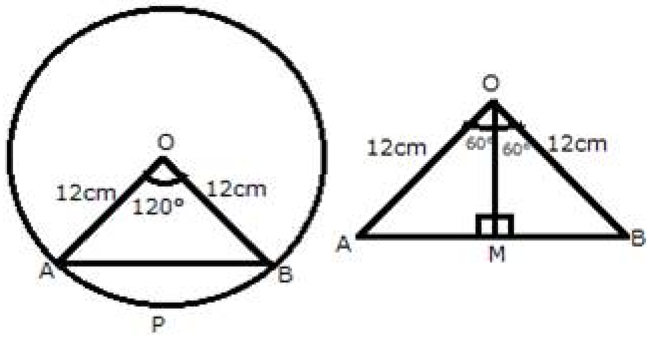
Area of segment APB = Area of sector OAPB – Area of ΔOAB

$$= \left(231 - \frac{441\sqrt{3}}{4}\right)\text{cm}^2$$

24. Question

A chord of a circle of radius 12cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle [Use π=3.14,√3=1.73]

Answer



Given: Radius of the circle = OA = OB = 12cm

and θ = 120°

To find: Area of the corresponding segment of the circle

i.e. Area of segment APB = Area of sector OAPB – Area of ΔAOB

So, firstly we find the Area of sector OAPB

$$\text{Area of minor sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{120}{360} \times 3.14 \times (12)^2$$

$$= \frac{1}{3} \times 3.14 \times 12 \times 12$$

= 150.72cm^2

Now, we have to find the area of ΔAOB

We draw OM ⊥ AB

∴∠OMB = ∠OMA = 90°

In ΔOMA and ΔOMB

∠OMA = ∠OMB [both 90°]

OA = OB [both radius]

OM = OM [common]

∴ OMA ≅ ΔOMB [by RHS congruency]

⇒ ∠AOM = ∠BOM [CPCT]

$$\therefore \angle AOM = \angle BOM = \frac{1}{2}\angle BOA$$

$= \frac{1}{2} \times 120 = 60°$

∴In right triangle OMA, we have

$\sin 60° = \frac{AM}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12}$

⇒ AM = $6\sqrt{3}$ cm

⇒ 2AM =$12\sqrt{3}$ cm

⇒ AB =$12\sqrt{3}$ cm

and

$\cos 60° = \frac{OM}{OA}$

$\Rightarrow \frac{1}{2} = \frac{OM}{12}$

⇒ OM = 6cm

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 12\sqrt{3} \times 6$

= $36\sqrt{3}$

= 36 × 1.73

= 62.28 cm^2

Area of segment APB = Area of sector OAPB – Area of ΔAOB

= (150.72 – 62.28)

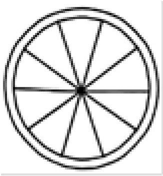
= 88.44 cm^2

25. Question

A brooch is made with silver wire in the form of a circle with diameter 35mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the figure. Find:

(i) the total length of the silver wire required.

(ii) the area of each sector of the brooch



Answer

Here, brooch is made up of silver wire in the form of a circle.

The diameter of the brooch = 35mm

"⇒ The radius of the brooch, r" $= \frac{35}{2}$ mm

Since the wire is used in making 5 diameters and circle

So,

The total length of the silver wire required = length of wire in circle

+ wire used in 5 diameters

= 2πr + 5 × 2r

= 2r(π + 5)

$$= 2 \times \frac{35}{2}\left(\frac{22}{7} + 5\right)$$

$$= 35\left(\frac{22 + 35}{7}\right)$$

= 5×57

= 285 mm

Now, Area of each sector of the brooch

$$= \frac{1}{10} \times \Pi r^2$$

$$= \frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= \frac{385}{4}$$

∴Area of each sector= $\frac{385}{4}mm^2$

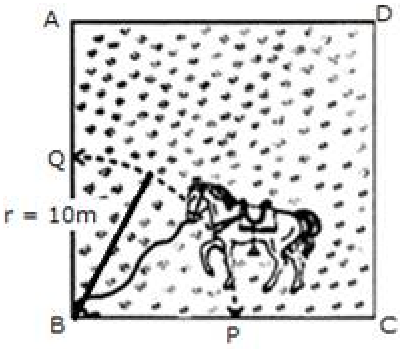
26. Question

A horse is tied to a pole at one corner of a square grass field of side 15 m using 10 m long rope. Find

(i) the area of that part of the field in which the horse can graze.

(ii) the decrease in the grazing area if the rope was 5 m long instead of 10m. [Use π=3.14]

Answer



(i)

Let ABCD be square field

and length of rope, r = 10m

We need to find the area of the field which horse can graze, i.e. the area of sector QBP

As we know that in a square all angles are of 90°

Hence, ∠QBP = 90°

Area of sector QBP = $\frac{\theta}{360} \times \pi r^2$

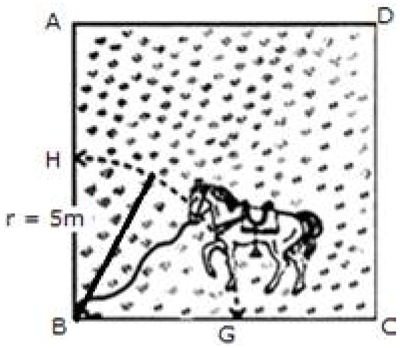
$= \frac{90}{360} \times 3.14 \times 10 \times 10$

$= \frac{1}{4} \times 314$

= 78.5 m^2

Hence, the area of the field which horse can graze = 78.5 m^2

(ii) the decrease in the grazing area if the rope was 5 m long instead of 10m



Length of rope is decreased to 5m

Area grazed by a horse now = Area of sector HBG

Area of sector HBG = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times 5 \times 5$

$= \frac{1}{4} \times 3.14 \times 25$

= 19.625 m^2

So, the decrease in the grazing area

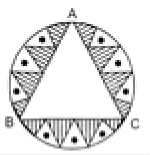
= Area of sector QBP – Area of sector HBG

= 78.5 – 19.625

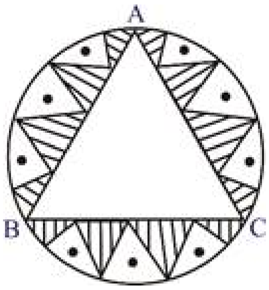
= 58.875 m^2

27. Question

In a circular table cover of radius 32cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the given figure. Find the area of the design (shaded region)



Answer



Given: Radius of circle = 32cm

Area of design = Area of circle – Area of ΔABC

Firstly, we find the area of a circle

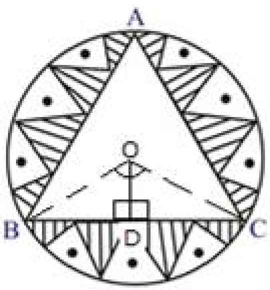
Area of circle = πr^2

$= \frac{22}{7} \times (32)^2$

$= \frac{22}{7} \times 32 \times 32$

$= \frac{22528}{7}$ cm^2 ...(a)

Now, we will find the area of equilateral ΔABC



Construction:

Draw OD ⊥ BC

In ΔBOD and ΔCOD

OB = OC (radii)

OD = OD (common)

∠ODB = ∠ODC (90°)

∴ ΔBOD ≅ ΔCOD [by RHS congruency]

⇒ BD = DC [by CPCT]

or BC = 2BD ...(i)

and, $\angle BOD = \angle COD = \frac{1}{2}\angle BOC = \frac{120}{2} = 60°$

Now, In ΔBOD, we have

$$\sin 60° = \frac{BD}{OB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BD}{32}$$

⇒ BD = $16\sqrt{3}$ cm

From (i), BC = 2BD ⇒ BC = $32\sqrt{3}$ cm

Now, Area of equilateral ΔABC

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2$$

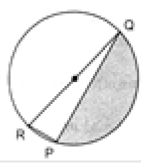
= $768\sqrt{3}$ cm^2 ...(b)

Therefore, Area of design = Area of circle – Area of ΔABC

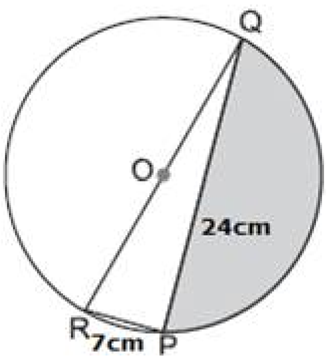
$$= \frac{22528}{7} - 768\sqrt{3}\ \text{cm}^2 \text{ [from (a) and (b)]}$$

28. Question

Find the area of the shaded region in the figure, if PQ=24 cm, PR = 7cm and O is the centre of the circle.



Answer



Given: PQ = 24cm and PR = 7cm

Since QR is a diameter, it forms a semicircle

We know that angle in a semicircle is a right angle.

Hence, $\angle RPQ = 90°$

Hence, ΔRPQ is a right triangle

In ΔRPQ, by Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(QR)^2 = (PQ)^2 + (PR)^2$

$\Rightarrow (QR)^2 = (24)^2 + (7)^2$

$\Rightarrow (QR)^2 = 576 + 49$

$\Rightarrow (QR)^2 = 625$

$\Rightarrow (QR)^2 = (25)^2$

$\Rightarrow QR = 25cm$

$\therefore$ Diameter, QR = 25cm

$$\Rightarrow \text{ Radius} = \frac{25}{2}$$

So,

$$\text{Area of semicircle} = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$$

$$= \frac{11 \times 25 \times 25}{28}$$

$$= \frac{6875}{28}\text{cm}^2$$

Now, Area of ΔPQR

$$\text{Area of } \Delta PQR = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times PQ \times PR$$

$$= \frac{1}{2} \times 24 \times 7$$

= 12 × 7

= $84cm^2$

Area of shaded region = Area of semicircle – Area of ΔPQR

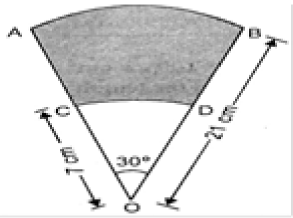
$$= \frac{6875}{28} - 84$$

$$= \frac{6875 - 2352}{28}$$

$$= \frac{4523}{28} cm^2$$

29. Question

AB and CD are arcs of two concentric circles of radii 21 cm and 7 cm respectively and centre O. If ∠AOB =30°, find the area of the shaded region.



Answer

Area of the shaded region

= Area of sector AOB – Area of sector COD

Area of sector AOB

Here, radius = 21cm and θ = 30°

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{30}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{12} \times 22 \times 3 \times 21$$

$= \frac{231}{2} \text{cm}^2$

Area of sector COD

Here, radius = 7cm and θ = 30°

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{1}{12} \times 22 \times 21$

$= \frac{77}{6} \text{cm}^2$

Now, shaded region = Area of sector AOB – Area of sector COD

$= \frac{231}{2} - \frac{77}{6}$

$= \frac{693 - 77}{6}$

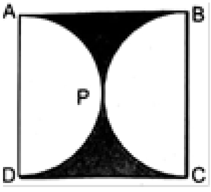
$= \frac{616}{6}$

$= \frac{308}{3} \text{cm}^2$

Hence, area of shaded region is $\frac{308}{3} \text{cm}^2$

30. Question

In the given figure ABCD is a square whose each side is 14cm. APD and BPC are semicircles. Find the area of the shaded region.



Answer

Area of shaded region = Area of square ABCD

– Area of semicircle APD

– Area of semicircle BPC

Area of square ABCD

Given: Side of square = 14cm

Area of square = Side × Side

= 14 × 14

= 196 cm^2

Area of semicircle APD

Diameter = AD = 14cm

So,

$$\text{Radius} = \frac{14}{2} = 7\text{cm}$$

$$\text{Area of semicircle APD} = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

= 11 × 7

= 77cm^2

Similarly, Area of semicircle BPC = 77cm^2

Area of shaded region = Area of square ABCD

– Area of semicircle APD

– Area of semicircle BPC

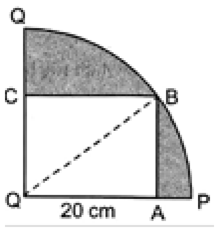
= 196 – 77 -77

= 42cm^2

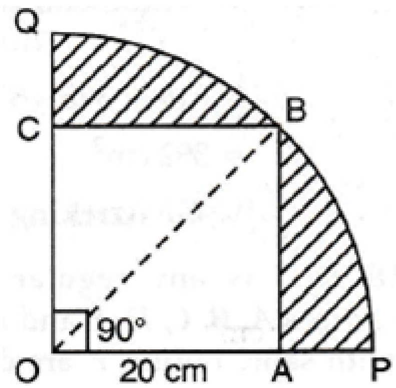
Hence, area of shaded region is 42cm^2

31. Question

In the given figure a square OABC is inscribed in a quadrant OPBQ. If OA =20cm, find the area of the shaded region.



Answer



Area of shaded region = Area of quadrant OBPQ

– Area of square OABC

Area of square OABC

Given: Side of square = 20cm

Area of square = Side × Side

= 20 × 20

= 400 cm^2

Area of quadrant

We need to find the radius

Joining OB

Also, all angles of a square are 90°

∴∠BAO = 90°

Hence, ΔOBA is a right triangle

In ΔOBA, by Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(OB)^2 = (AB)^2 + (OA)^2$

$\Rightarrow (OB)^2 = (20)^2 + (20)^2$

$\Rightarrow (OB)^2 = 400 + 400$

$\Rightarrow (OB)^2 = 800$

$\Rightarrow OB = \sqrt{(10\times10\times2\times2\times2)}$

$\Rightarrow OB = 20\sqrt{2}cm$

$$\text{Area of quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times 3.14 \times (20\sqrt{2})^2$$

$$= 3.14 \times 5 \times 20 \times 2$$

$= 628\ cm^2$

Area of shaded region = Area of quadrant OBPQ

– Area of square OABC

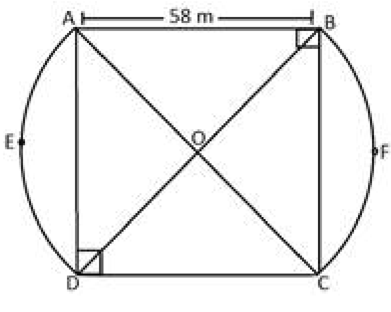
= 628 – 400

$= 228cm^2$

32. Question

It is proposed to add to a square lawn with the side 58m, two circular ends (the centre of each circle being the point of intersection of the diagonals of the square.) Find the area of the whole lawn [Take π=3.14]

Answer



ABCD is a square lawn of side 58m. AED and BFC are two circular ends.

Now, diagonal of the lawn = $\sqrt{(58)^2 + (58)^2} = 58\sqrt{2}m$

It is given that diagonal of square = Diameter of circle

∴The radius of a circle having a centre at the point of intersection of diagonal

$= \frac{58\sqrt{2}}{2} = 29\sqrt{2}\text{m}$

It is given that square ABCD is inscribed by the circle with centre O.

∴Area of 4 segments = Area of circle – Area of square

$= \pi r^2 - (\text{side})^2$

$= \frac{22}{7} \times 29\sqrt{2} \times 29\sqrt{2} - 58 \times 58$

$= \frac{22}{7} \times 29 \times 29 \times 2 - 29 \times 29 \times 4$

$= 29 \times 29 \times 4\left(\frac{11}{7} - 1\right)$

$= 29 \times 29 \times 4 \times \left(\frac{4}{7}\right)$

$\Rightarrow$ Area of two segments $= \frac{1}{2} \times 29 \times 29 \times 4 \times \left(\frac{4}{7}\right)\text{m}^2$

$= \frac{29 \times 29 \times 8}{7}$

$= 961.14\text{m}^2$

Area of whole lawn = Area of circle – Area of two segments

$= \frac{22}{7} \times 29\sqrt{2} \times 29\sqrt{2} - \frac{29 \times 29 \times 8}{7}$

$=5286.28 - 961.14$

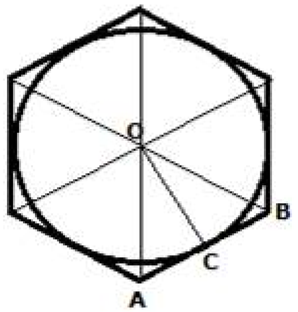
$= 4325.14\ \text{m}^2$

33. Question

Find the difference between the area of a regular hexagonal plot each of whose side is 72 m and the area of the circular swimming tank inscribed in it.

$\left(\text{Take } \pi = \frac{22}{7}\right)$

Answer



The side of hexagonal plot = 72m

Area of equilateral triangle OAB $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 72 \times 72$

$= 1296\sqrt{3}\text{cm}^2$

∴ Area of hexagonal plot = 6 × Area of triangle OAB

$= 6 \times 1296\sqrt{3}$

= 7776(1.732)

$=13468.032\text{m}^2$

In ΔOCA, by Pythagoras theorem

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(OA)^2 = (OC)^2 + (AC)^2$

$\Rightarrow (72)^2 = (OC)^2 + (36)^2$

$\Rightarrow (OC)^2 = 5184 - 1296$

$\Rightarrow (OC)^2 = 3888$

$\Rightarrow r^2 = 3888$

∴Area of inscribed circular swimming tank $= \pi r^2$

$= \frac{22}{7} \times 3888$

$= 12219.429\text{m}^2$

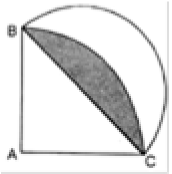
∴ Required difference = 13468.032 – 12219.429

$= 1248.603\text{m}^2$

Hence, the difference between the area of a regular hexagonal plot and the area of the circular swimming tank inscribed in it is 1248.60.3m^2

34. Question

In the figure, ABC is a quadrant of a circle of radius 14cm, and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



Answer

Area of shaded region = Area of segment with chord BC

Now, AC = BA = 14cm

$\therefore$ BC = $\sqrt{(14)^2 + (14)^2} = 14\sqrt{2}$ cm

$\therefore$ Area of semicircle on BC $= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 14\sqrt{2} \times 14\sqrt{2}$

= 154cm^2

Area of ΔABC $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 14 \times 14$

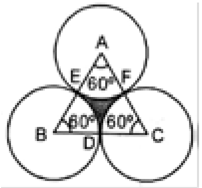
= 98

So, Area of shaded region = 154 – 98

= 56cm^2

35. Question

The area of an equilateral triangle is $100\sqrt{3}$cm^2. Taking each vertex as centre, a circle is described with a radius equal to half the length of the side of the triangle, as shown in the figure. Find the area of that part of the triangle which is not included in the circles [Take π=3.14 and $\sqrt{3}$=1.732]



Answer

Area of the shaded region

= Area of an equilateral triangle – Area of 3 sectors

Given: Area of equilateral ΔABC = $100\sqrt{3}cm^2$

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 100\sqrt{3}$$

$\Rightarrow a^2 = 400$

$\Rightarrow a = 20cm$

It is given that radius is equal to half the length of the side

i.e. $r = \frac{a}{2} = \frac{20}{2} = 10cm$

Now,

$$\text{Area of 3 sectors} = 3\times \frac{\theta}{360} \times \pi r^2$$

$$\text{Area of 3 sectors} = 3 \times \frac{60}{360} \times \frac{22}{7} \times 10 \times 10$$

$$= \frac{11}{7} \times 100$$

$= 157.14cm^2$

Hence, the area of the shaded region

= Area of ΔABC – Area of 3 sectors

$= 100\sqrt{3} - 157.14$

$= 100\times1.732 - 157.14$

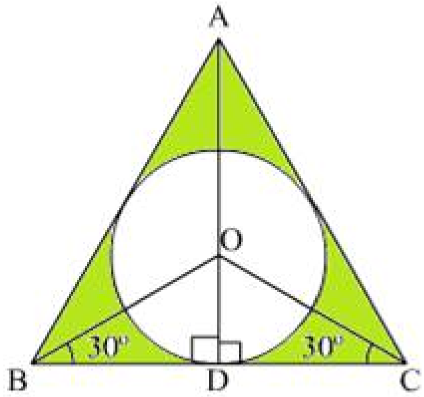
$=173.2 - 157.14$

$= 16.06\ cm^2$

36. Question

In an equilateral triangle of side 12cm, a circle is inscribed touching its sides. Find the area of the portion of the triangle not included in the circle. [Take √3=1.73 and π=3.14]

Answer



Area of shaded region = Area of ΔABC – Area of circle

Given side of triangle = 12cm

$\therefore$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 12 \times 12$

= $36\sqrt{3}\text{cm}^2$

Now, we have to find the area of a circle. For that we need a radius.

Draw AD ⊥ BC

So, In BDO

$\tan 30° = \frac{OD}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{r}{6}$ [$\because$ AD $\perp$ BC $\Rightarrow$ BD $= \frac{1}{2}$BC $= 6$cm]

$\Rightarrow$ r = $2\sqrt{3}$cm

Now, Area of circle = πr^2

= $3.14 \times (2\sqrt{3})^2$

= 37.68cm^2

Area of shaded region = Area of ΔABC – Area of circle

$= 36\sqrt{3} - 37.68$

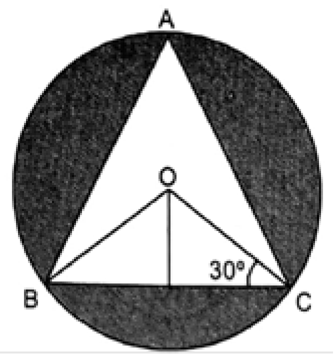
$= 36(1.73) - 37.68$

$= 24.6cm^2$

Hence, the area of the portion of the triangle not included in the circle is $24.6cm^2$

37. Question

In a circular table-cover of radius 16cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the area of the design (shaded region in the figure).



Answer

Given: Radius of circle = 16cm

Area of shaded region = Area of circle – Area of ΔABC

Firstly, we find the area of a circle

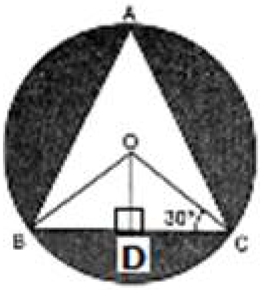
Area of circle = πr^2

$= \frac{22}{7} \times (16)^2$

$= \frac{22}{7} \times 16 \times 16$

$= \frac{5632}{7}$ cm^2 …(a)

Now, we will find the area of equilateral ΔABC



Construction:

Draw OD ⊥ BC

In ΔBOD and ΔCOD

OB = OC (radii)

OD = OD (common)

∠ODB = ∠ODC (90°)

∴ ΔBOD ≅ ΔCOD [by RHS congruency]

⇒ BD = DC [by CPCT]

or BC = 2BD ...(i)

and, $\angle BOD = \angle COD = \frac{1}{2}\angle BOC = \frac{120}{2} = 60°$

Now, In ΔBOD, we have

$$\sin 60° = \frac{BD}{OB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BD}{16}$$

⇒ BD = 8√3 cm

From (i), BC = 2BD ⇒ BC = 16√3 cm

Now, Area of equilateral ΔABC

$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (16\sqrt{3})^2$$

= $192\sqrt{3}$ cm^2 ...(b)

Therefore, Area of design = Area of circle – Area of ΔABC

$= \frac{5632}{7} - 192\sqrt{3}\ \text{cm}^2$ [from (a) and (b)]

= 804.57 – 332.544

= 472.03cm^2