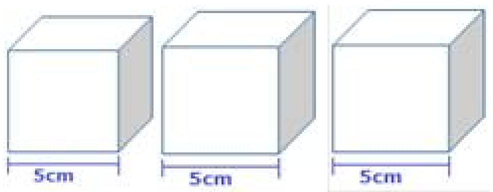
14. Surface Areas and Volumes

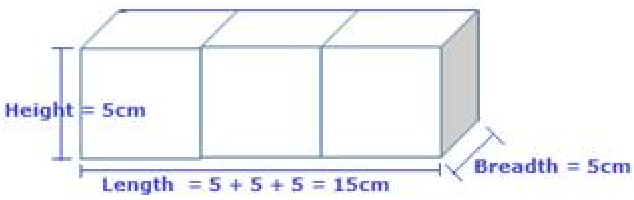
Exercise 14.1

1. Question

Three cubes each of side 5 cm are joined end to end to form a cuboid. Find the surface area of the resulting cuboid.

Answer





Length of each side of cube = 5cm

According to question,

Three cubes are joined end to end to form a cuboid

So, the length of the resulting cuboid = 5 + 5 + 5 = 15cm

But, the breadth and height will remain the same

So, Breadth = 5cm

and height = 5cm

Surface Area of the Cuboid

= 2(Length × Breadth + Breadth × Height + Height × Length)

= 2(lb + bh + hl)

= 2(15×5 + 5×5 + 5×15)

= 2(75 + 25 + 75)

= 2(175)

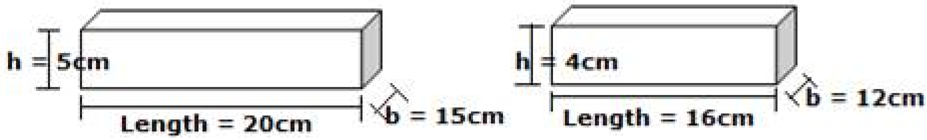
= $350cm^2$

Hence, the surface area of cuboid is $350cm^2$.

2. Question

Cardboard boxes of two different sizes are made. The bigger has dimensions 20cm, 15cm and 5cm and the smaller dimensions 16cm, 12cm and 4cm. 5% of the total surface area is required extra for all overlaps. If the cost of the cardboard is Rs. 20 for one square metre, find the cost of the cardboard for supplying 200 boxes of each kind.

Answer



For bigger box:

Length of the bigger box = 20cm

Breadth of the bigger box = 15cm

Height of the bigger box = 5cm

So, Total Surface Area of bigger box = 2(lb + bh + hl)

= 2(20×15 + 15×5 + 5×20)

= 2(300 + 75 + 100)

= 2(475)

= $950cm^2$

For Smaller box:

Length of the smaller box = 16cm

Breadth of the smaller box = 12cm

Height of the smaller box = 4cm

So, Total Surface Area of bigger box = 2(lb + bh + hl)

= 2(16×12 + 12×4 + 4×16)

= 2(192 + 48 + 64)

= 2(304)

$= 608cm^2$

Total surface area of 200 boxes of each type = 200(950 +608)

= 200 × 1558

$= 311600cm^2$

Extra Area Required = 5% of (950 + 608)

$$= \frac{5}{100} \times 1558 \times 200$$

$= 15580cm^2$

So, Total cardboard Required = 311600 + 15580

$= 327180cm^2$

$$= \frac{327180}{1000} = 327.18 \text{ m}^2$$

Cost of Cardboard for $1m^2$ = Rs 20

Cost of Cardboard for 327.18 m^2 = 20 × 327.18

= Rs 654.36

Hence, the cost of the cardboard for supplying 200 boxes of each kind is Rs 654.36.

3. Question

The length of cold storage is double its breadth. Its height is 3 metres. The area of its four walls (including doors) is 108 m^2. Find its volume.

Answer

Given: Area = $108m^2$, Height = 3m

and Length of a cold storage is double its breadth

So, Let breadth of a cold storage = x

∴ length of the cold storage = 2x

Area of four walls = $108m^2$

∴ 2(l + b)× h = 108

⇒ 2 (x + 2x)×3 = 108

$\Rightarrow 6(3x) = 108$

$$\Rightarrow x = \frac{108}{18}$$

$\Rightarrow x = 6$

∴ Breadth of a cold storage = 6m

and Length of a cold storage = 2×6 = 12m

Hence, Volume of the cold storage = l × b × h

= 12 × 6 × 3

= $216m^3$

4. Question

Find:

(i) the lateral surface,

(ii) the whole surface, and

(iii) the volume of a right circular cylinder whose height is 13.5 cm and radius of the base 7cm

Answer

Given: Radius of base, r = 7cm

Height , h = 13.5cm

(i) Lateral Surface Area of right circular cylinder = 2πrh

$$= 2 \times \frac{22}{7} \times 7 \times 13.5$$

= 2×22×13.5

= $594cm^2$

(ii) Total Surface Area of cylinder = $2\pi rh + 2\pi r^2$

= 2πr(h + r)

$$= 2 \times \frac{22}{7} \times 7(13.5 + 7)$$

= 2×22×20.5

$= 902cm^2$

(iii) Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 13.5$$

= 22×7×13.5

$= 2079cm^3$

5. Question

The radius and height of a right circular cone are in the ratio of 5:12. If its volume is 314 cm^3, find its slant height.

[Take π=3.14]

Answer

Given: Volume of a right circular cone = $314cm^3$

and r : h = 5 : 12

Let r = 5x and h = 12x

The volume of a right circular cone $= \frac{1}{3}\pi r^2 h$

$$\Rightarrow 314 = \frac{1}{3} \times 3.14 \times (5x)^2 \times 12x$$

$$\Rightarrow \frac{314 \times 3}{3.14} = 25x^2 \times 12x$$

$$\Rightarrow \frac{314 \times 3 \times 100}{314 \times 25 \times 12} = x^3$$

$\Rightarrow x^3 = 1$

Hence, x = 1

∴ r = 5x = 5×1 = 5cm

and h = 12x = 12×1 = 12cm

Now, we have to find the slant height, l

Slant height, l = $\sqrt{(r^2 + h^2)}$

$= \sqrt{\{(5)^2 + (12)^2\}}$

$= \sqrt{25+144}$

$= \sqrt{169}$

$= \pm 13$

= 13cm

[taking positive root, because slant height can't be negative]

∴ Slant Height = 13cm

6. Question

A cylinder, whose height is two-thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Answer

Let height of a cylinder = h

and diameter = d

According to question,

$$h = \frac{2}{3}d$$

$$\Rightarrow h = \frac{2}{3} \times 2r$$

$$\Rightarrow h = \frac{4}{3}r$$

Given: Radius of Sphere, R = 4cm

Volume of Cylinder = Volume of Sphere (Given)

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow r^2 \times \frac{4}{3}r = \frac{4}{3}R^3$$

$\Rightarrow r^3 = R^3$

$\Rightarrow r^3 = 4^3$

$\Rightarrow r = 4cm$

Hence, Radius of base of the cylinder is 4cm.

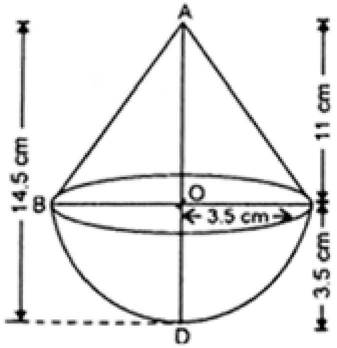
Exercise 14.2

1. Question

A toy is in the form a cone mounted on a hemisphere of diameter 7 cm. The total height of the toy is 14.5 cm. Find the volume of the toy.

$\left[\text{Take } \pi = \frac{22}{7}\right]$

Answer



Given: Diameter of a hemisphere = 7cm

So, Radius $= \frac{7}{2} = 3.5\text{cm}$

Height of the cone, h = 143-.5 – 3.5 = 11cm

The volume of a toy = Volume of cone + Volume of a hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5(11 + 2 \times 3.5)$$

$$= \frac{1}{3} \times 22 \times 0.5 \times 3.5 \times 18$$

$= 231\text{cm}^3$

2. Question

A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 2.1 cm, and the height of the cone is 4 cm. The solid is place in a cylindrical tub full of water in such a way that the whole solid is submerged in water left in the tub.

Answer

Original volume of water in the cylindrical tub

= Volume of Cylinder

= $\pi r^2 h$

$= \frac{22}{7} \times 5^2 \times 9.8$

= 22×25×1.4

= 770cm^3

Given that Radius of hemisphere, R = 2.1cm

and height of cone, h = 4cm

Volume of a solid = Volume of cone + Volume of hemisphere

$= \frac{1}{3}\pi R^2 h + \frac{2}{3}\pi R^3$

$= \frac{1}{3}\pi R^2 (h + 2R)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1(4 + 2 \times 2.1)$

$= \frac{1}{3} \times 22 \times 0.3 \times 2.1 \times 8.2$

= 37.884cm^3

∴ Volume of water displaced (removed) = 37.884cm^3

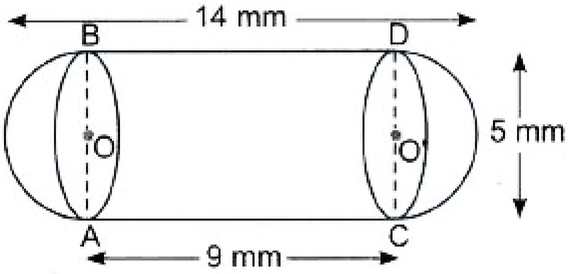
Hence, the required volume of the water left in the cylindrical tub = 770 – 37.884

= 732.116 cm^3

3. Question

A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 5 mm. Find its surface area.

Answer



Given:

Diameter of cylinder = 5mm

The radius of cylinder$= \frac{\text{Diameter}}{2} = \frac{5}{2}$mm

Height of cylinder = 14 – 5 = 9mm

Here, Diameter of hemisphere =. 5mm

So, Radius of Hemisphere $= \frac{\text{Diameter}}{2} = \frac{5}{2}$ mm

The total area of the capsule = CSA of cylinder + CSA of 2 hemispheres

= $2\pi rh + 2\times 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 9 + 4 \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}$

$= \frac{22}{7}(45 + 25)$

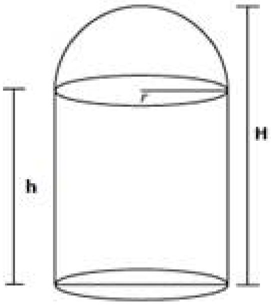
=220 mm^2

4. Question

A room in the form of a cylinder, surmounted by a hemispherical vaulted dome, contains $41\frac{19}{21}$ m^3 of air and the internal diameter of the building is equal to the height of the crown of the vault above the floor. Find the height

$\left[\text{Take } \pi = \frac{22}{7}\right]$

Answer



Let r be the radius of hemisphere and cylinder

and height of the cylinder = h

Given: Volume of air $= \frac{880}{21} m^3$

∴ Diameter of the building = 2r

Height of the building (H) = diameter of the building

∴ Height of the cylinder + Radius of hemispherical dome = 2r

⇒ h + r = 2r

⇒ h = 2r – r

⇒ h = r

Volume of air inside the building = Volume of hemispherical portion

+ Volume of cylindrical portion

$$41\frac{19}{21} = \frac{2}{3}\pi r^3 + \pi r^2 h$$

$$41\frac{19}{21} = \frac{2}{3}\pi r^3 + \pi r^2(r)$$

$$\frac{880}{21} = \frac{2}{3}\pi r^3 + \pi r^3$$

$$\frac{880}{21} = \pi r^3\left(\frac{2}{3} + 1\right)$$

$$\frac{880}{21} = \frac{22}{7} r^3 \times \frac{5}{3}$$

$$\frac{880}{22 \times 5} = r^3$$

⇒ $r^3 = 8$

⇒ r = 2

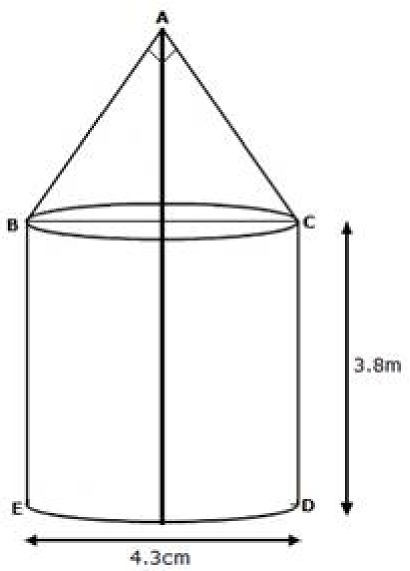
⇒ Height of building = 2r = 2×2 = 4m

Hence, the total height of the building is 4m.

5. Question

The interior of a building is in the form of a cylinder of diameter 4.3 m and height 3.8 m surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building. [Take π=3.14]

Answer



The diameter of a cylindrical portion BCDE of building = 4.3m

∴ The radius of a cylindrical portion= $\frac{4.3}{2} = 2.15$ m

Height = 3.8m

Lateral Surface Area of Cylindrical Portion BCDE = 2πrh

$$= 2 \times \frac{22}{7} \times 2.15 \times 3.8$$

= 51.3543 m^2

Let AB be the slant height of the conical portion of the building = l = AB = AC

Now, the Lateral surface of conical portion = πrl

$$= \frac{22}{7} \times 2.15 \times 3.04$$

= 20.5417 m^2

So,

The total surface area of the building = Surface area of cylindrical portion + Surface area of the conical portion

= 51.3543 + 20.5417

= 71.8960

= 71.90 m^2 (approx.)

Now, In right ΔBAC,

$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = l^2 + l^2$

$\Rightarrow (4.3)^2 = 2l^2$

$\Rightarrow 18.49 = 2l^2$

$\Rightarrow \ l^2 = \frac{18.49}{2}$

$\Rightarrow l^2 = 9.245$

$\Rightarrow l = 3.04$ m

Here, r is the radius of the cone and l is the slant height of the cone

$\Rightarrow l^2 = h^2 + r^2$

$\Rightarrow 9.245 = h^2 + (2.15)^2$

$\Rightarrow 9.245 = h^2 + 4.6225$

$\Rightarrow h^2 = 9.245 - 4.6225$

$\Rightarrow h^2 = 4.6225$

$\Rightarrow h = 2.15$ m

Now, Volume of the conical portion $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (2.15)^2 \times 2.15$

= 10.4116 m^3

Volume of cylindrical portion BCDE = $\pi r^2 h$

$$= \frac{22}{7} \times (2.15)^2 \times 3.8$$

= 55.2059 m^3

So,

The total volume of the building = Volume of the cylindrical portion

+ Volume of the conical portion

= 55.2059 + 10.4116

= 65.6175 m^3

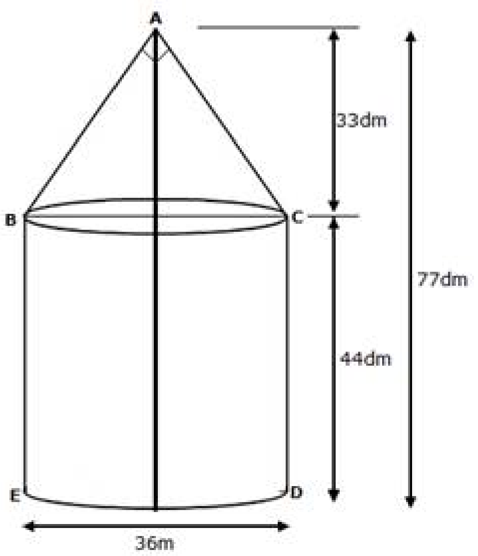
= 65.62 m^3 (approx.)

6. Question

A tent of height 77 dm is in the form of a right circular cylinder of diameter 36m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs 3.50 per m^2.

$$\left[\text{Take } \pi = \frac{22}{7}\right]$$

Answer



Given: Height of the tent = 77dm

$$= \frac{77}{10} = 7.7 \text{ m}$$

The height of the conical portion = 44dm

$= \frac{44}{10} = 4.4$ m

∴ The height of the cylindrical portion

= Height of the tent – Height of the conical portion

= 7.7 – 4.4

= 3.3m

Given Diameter of the cylinder, d = 36m

$\therefore$ Radius $= \frac{36}{2} = 18$m

CSA of cylindrical portion = 2πrh

$= 2 \times \frac{22}{7} \times 18 \times 4.4$

= 497.828

= 497.83 m^2 (approx.)

Firstly, we have to find the slant height (l) of the conical portion

$\Rightarrow l^2 = h^2 + r^2$

$\Rightarrow l^2 = (3.3)^2 + (18)^2$

$\Rightarrow l^2 = 10.89 + 324$

$\Rightarrow l^2 = 334.89$

$\Rightarrow l = \sqrt{334.89}$

$\Rightarrow$ l = 18.3m

∴ CSA of the conical portion = πrl

$= \frac{22}{7} \times 18 \times 18.3$

= 1035.257

= 1035.26 m^2 (approx.)

So,

Total Surface Area of the tent = Surface area of conical portion + surface Area of cylindrical portion

= 1035.26 + 497.83

= 1533.09 m^2

∴ Canvas required to make the tent = 1533.09 m^2

Cost of 1m^2 canvas = Rs 3.50

Cost of 1533.09m^2 canvas = Rs 3.50 × 1533.09

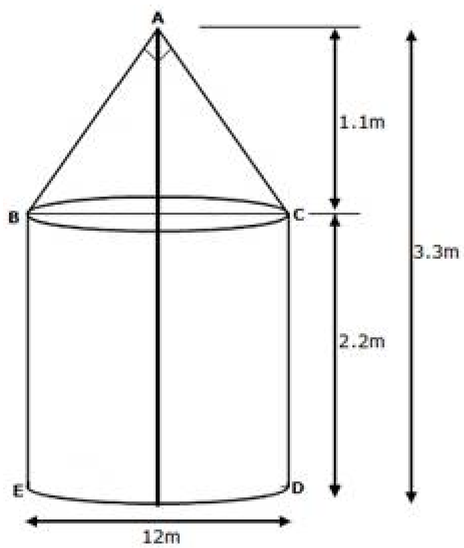
= Rs 5365.815

= Rs 5365.82

7. Question

A tent of height 3.3 m is in the form of a right circular cylinder of diameter 12m and height 2.2m, surmounted by a right circular cone of the same diameter. Find the cost of the canvas of the tent at the rate of Rs. 500 per m^2.

Answer



Given: Height of the tent = 3.3 m

The height of the cylindrical portion = 2.2 m

∴ The height of the conical portion

= Height of the tent – Height of the cylindrical portion

= 3.3 – 2.2

= 1.1m

Given Diameter of the cylinder, d = 12m

$\therefore$ Radius $= \frac{12}{2} = 6\text{m}$

CSA of cylindrical portion = $2\pi rh$

$= 2 \times \frac{22}{7} \times 6 \times 2.2$

$= 82.971\ m^2$

Firstly, we have to find the slant height (l) of the conical portion

$\Rightarrow l^2 = h^2 + r^2$

$\Rightarrow l^2 = (1.1)^2 + (6)^2$

$\Rightarrow l^2 = 1.21 + 36$

$\Rightarrow l^2 = 37.21$

$\Rightarrow l = \sqrt{37.21}$

$\Rightarrow l = 6.1\text{m}$

$\therefore$ CSA of the conical portion = πrl

$= \frac{22}{7} \times 6 \times 6.1$

$= 115.029\ m^2$

So,

Total Surface Area of the tent = Surface area of conical portion + surface Area of cylindrical portion

= 115.029 + 82.971

$= 198\ m^2$

$\therefore$ Canvas required to make the tent = $198\ m^2$

Cost of $1m^2$ canvas = Rs 500

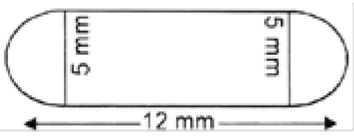
Cost of $198\ m^2$ canvas = Rs 500 × 198

= Rs 99000

8. Question

A medicine capsule as shown in the given figure is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 12 mm, and the diameter of the capsule is 5 mm. Find its surface area.

Answer



Given:

Diameter of cylinder = 5mm

Radius of cylinder $= \frac{\text{Diameter}}{2} = \frac{5}{2}\text{mm}$

Height of cylinder = 12 – 5 = 7mm

Here, Diameter of hemisphere = 5mm

So,Radius of Hemisphere = $\frac{\text{Diameter}}{2} = \frac{5}{2}\text{mm}$

Total area of the capsule = CSA of cylinder + CSA of 2 hemispheres

= $2\pi rh + 2\times 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{5}{2} \times 7 + 4 \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}$

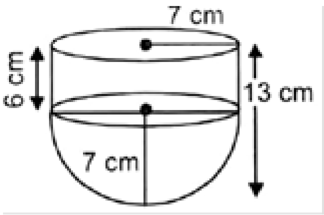
$= \frac{22}{7}(35 + 25)$

$= \frac{22}{7} \times 60$

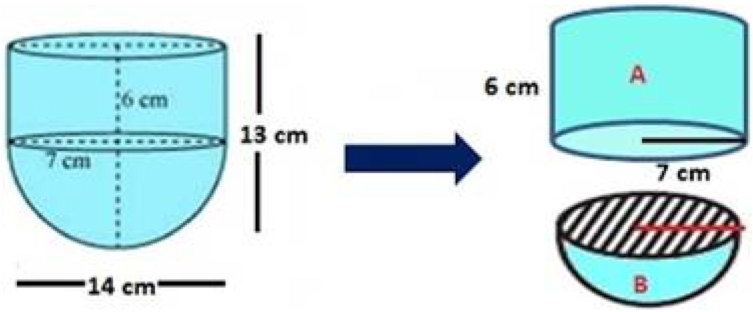
$= 188\frac{4}{7}\ \text{sq. mm}$

9. Question

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm, and the total height of the vessel is 13cm. Find the inner surface area of the vessel.



Answer



Given: Diameter of the hemisphere = 14cm

∴ Radius of the hemisphere= $\frac{14}{2} = 7\text{cm}$

Curved Surface Area of Hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7$$

= 308cm^2

Now,

Radius of cylinder = radius of hemisphere = 7cm

Height of cylinder = Total height – Radius of hemisphere

= 13 – 7

= 6cm

So, CSA of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 6$$

= 264cm^2

The inner Surface area of the vessel

= CSA of hemisphere + CSA of cylinder

= 308 + 264

$= 572\text{cm}^2$

Hence, Inner surface area of vessel is 572cm^2

10. Question

From a circular cylinder of base diameter 10cm and height 12cm, a conical cavity with the same base and height is carved out. Find the volume of the remaining solid.

Answer

Given that Diameter of circular cylinder = 10cm

∴ Radius of the cylinder$= \frac{10}{2} = 5\text{cm}$

and Height of the cylinder = 12cm

So,

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 5 \times 5 \times 12$$

$= 942.857\ \text{cm}^3$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$$

$= 314.285\ \text{cm}^3$

Remaining Volume = Volume of the cylinder – Volume of cone

= 942.857 – 314.285

$= 628.572\ \text{cm}^3$

11. Question

An ice-cream cone consists of a right circular cone of height 14cm and diameter of the circular top is 5 cm. It has hemisphere on the top with the same diameter as of circular top. Find the volume of ice-cream in the cone.

Answer

Given: Height of the cone = 14cm

and diameter of the circular top = 5cm

$\therefore$ Radius $= \frac{5}{2}$cm

Now,

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 14$

= 91.6666cm^3

Now,

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^3$

= 32.738 cm^3

So,

The volume of ice cream in the cone

= Volume of cone + Volume of a hemisphere

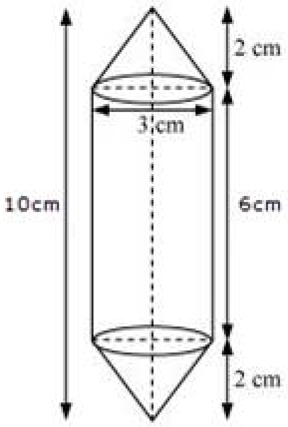
= 91.6666 + 32.738

= 124.404cm^3

12. Question

A student was asked to make a model in his workshop, which shaped like a cylinder with two cones attached at its two ends, using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 10 cm. If each cone has a height of 2cm, find the volume of air contained in the model. (Consider the outer and inner dimensions of the model to be nearly the same).

Answer



Given that diameter of cylinder = 3cm

So, Radius $= \frac{3}{2} = 1.5\text{cm}$

Given: Height of cone = 2cm

and

Height of the cylinder + height of cone + height of cone = 10cm

Height of the cylinder + 2 + 2 = 10

Height of the cylinder = 6cm

So,

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 6$$

= 42.428 cm^3

= 42.43 cm^3 (approx.)

Volume of 1st cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 2$$

= 4.71 cm^3

Volume of 1^{st} cone = Volume of 2^{nd} cone = 4.71cm^3

So, Total volume of the model

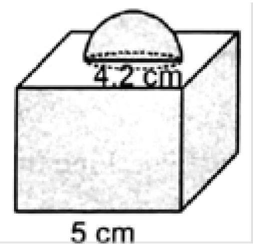
= Volume of cylinder + volume of 1^{st} cone + volume of 2^{nd} cone

= 42.43 + 4.71 + 4.71

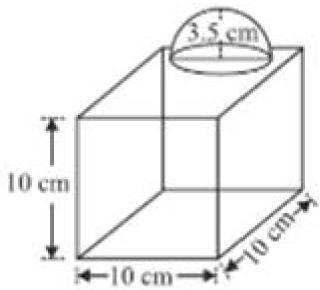
= $51.85cm^3$

13. Question

A decorative block as shown in the given figure is made of a cube and a hemisphere. The edge of the cube is 10cm, and the radius of the hemisphere attached on the top is 3.5. Find the cost of painting the block at the rate of 50 paise per sq.cm.



Answer



Given that side of a cube = 10cm

Total Surface Area of cube = $6(\text{side})^2$

= 6 × 10 × 10

= $600cm^2$

Now, Radius of hemisphere = 3.5cm

Curved Surface Area of hemisphere = $2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5$

= $77cm^2$

Base area of hemisphere = Area of circle = πr^2

$= \frac{22}{7} \times 3.5 \times 3.5$

= $38.5cm^2$

Therefore,

Total Surface Area of block = Total Surface area of a cube + CSA of the hemisphere - Base area of a hemisphere

= 600 + 77 – 38.5

= 638.5cm^2

Cost of painting the block of 1cm^2 = Rs 0.50

Cost of painting the block of 638.5cm^2 = Rs 0.50 × 638.5

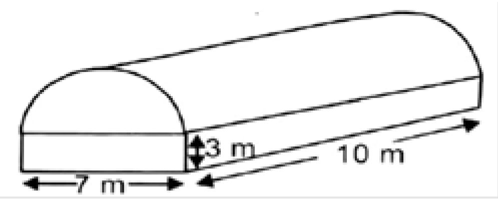
= Rs 319.25

14. Question

A godown building is in the form as shown in the adjoining figure. The vertical cross-section parallel to the width side of the building is a rectangle of size 7 m x 3 m mounted by a semicircle of radius 3.5 m. The inner measurement of the cuboidal portion are 10 m x 7 m x 3 m. Find the

(i) the volume of the godown, and

(ii) the total internal surface area excluding the floor.



Answer

(i) Volume of godown = Volume of cuboid + $\frac{1}{2}$ volume of cylinder

Volume of cuboid = l × b × h = 10 × 7 × 3 = 210 m^3

Volume of cylinder = $\pi r^2 h = \frac{22}{7} \times (3.5)^2 \times 10$ = 385 m^3

∴ Volume of godown= $210 + \frac{1}{2} \times 384.65$

= 210 + 192.5

= 402.5 cm^3

(ii) Internal Surface Area of a godown excluding the floor

$= \text{CSA of cuboid} + \frac{1}{2}$ the total surface area of the cylinder

$= 2h(l + b) + \frac{1}{2} \times 2\pi r(r + h)$

$= 2 \times 3(10 + 7) + \frac{22}{7} \times 3.5(3.5 + 10)$

= 6 × 17 + 22 × 0.5 × 13.5

= 102 + 148.5

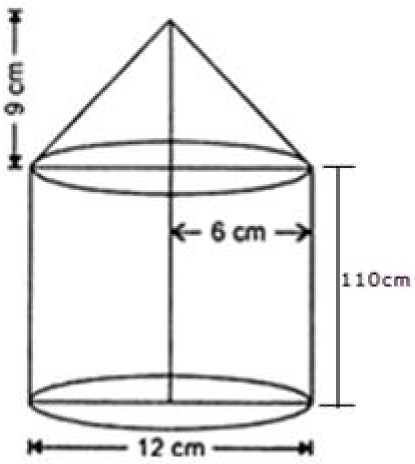
= 250.5 cm^2

Hence, internal surface area of godown excluding floor is 250.5cm^2

15. Question

A solid iron pole is having a cylindrical portion 110 cm high and of base diameter, 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that mass of 1 cm^3 of iron is 8 g.

Answer



Given: Diameter of cone = 12cm

So, Radius of cone = 6cm

and Height of cone = 9cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 9$

= 339.428 cm^3

Radius of the cylinder, R = Radius of cone = 6cm

Height of the cylinder, H = 110cm

Volume of cylinder = $\pi R^2 H$

$$= \frac{22}{7} \times 6 \times 6 \times 110$$

= $12445.714 cm^3$

Hence, the volume of the pole

= Volume of conical part + Volume of the cylindrical part

= 339.43 + 12445.714

= $12785.142\ cm^3$

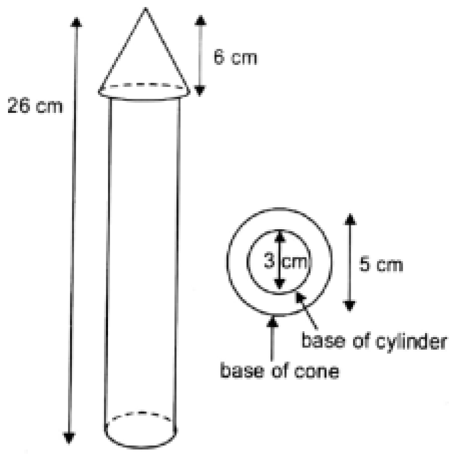
Required mass of pole = 8 × 12785.142

= 102281.13 gm

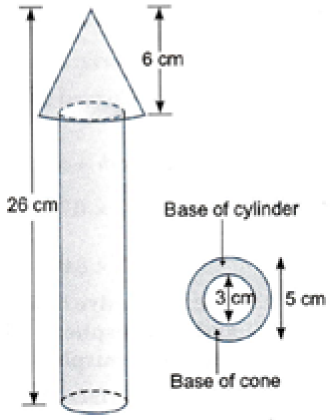
= 102.281kg

16. Question

A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in the given figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6cm. The base of the conical portion has a diameter of 5cm, while the base diameter of the cylindrical portion is 3cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. [Take π=3.14]



Answer



The area to be painted orange

= CSA of cone + Base area of Cone – Base area of the cylinder

Curved Surface Area of Cone

Given that diameter of conical portion = 5cm

The radius of conical portion= $\frac{5}{2} = 2.5\text{cm}$

Height of the conical part = h = 6cm

We need to find the 'l' first

We know that

$l^2 = h^2 + r^2$

$\Rightarrow l^2 = 6^2 + (2.5)^2$

$\Rightarrow l^2 = 36 + (6.25)$

$\Rightarrow l^2 = 42.25$

$\Rightarrow l = \sqrt{42.25}$

$\Rightarrow l = 6.5\text{cm}$

So, CSA of conical portion = πrl

$=3.14 \times 2.5 \times 6.5$

$= 51.025 \text{ cm}^2$

Base area of the cone = πr^2

$=3.14 \times 2.5 \times 2.5$

$=19.625 \text{ cm}^2$

Diameter of the cylinder = 3cm

So, Radius of the cylinder = 1.5cm

Base area of the cylinder = $\pi(r')^2$

$=3.14\times1.5\times1.5$

$=7.065\ cm^2$

So, the area to be painted orange = 51.025 + 19.625 – 7.065

$= 63.585\ cm^2$

Now, the area to be painted yellow

= CSA of the cylinder + Area of one bottom base of the cylinder

$= 2\pi r'h' + \pi(r')^2$

$= 2 \times 3.14 \times 1.5 \times 20 + 7.065$

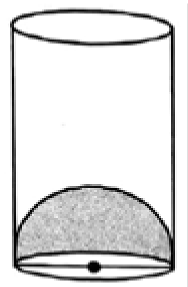
$= 188.4 + 7.065$

$= 195.465\ cm^2$

17. Question

The inner diameter of glass is 7cm, and it has a raise portion in the bottom in the shape of a hemisphere as shown in the figure. If the height of the glass is 16 cm, find the apparent capacity and the actual capacity of the glass.

$\left[\text{Take } \pi = \frac{22}{7}\right]$



Answer

Given the inner diameter of the glass = 7cm

So, the radius of the glass = $r = \frac{7}{2} = 3.5\text{cm}$

Height of the glass = 16cm

The volume of the cylindrical glass = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 16$$

= 616cm^3

Now, radius of the hemisphere = Radius of the cylinder =r = 3.5cm

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$

= 89.83 cm^3

Now,

Apparent capacity of the glass = Volume of cylinder = 616cm^3

The actual capacity of the glass

= Total volume of cylinder – Volume of hemisphere

= 616 – 89.83

= 526.17cm^3

Hence,

Apparent Capacity of the glass = 616cm^3

and actual capacity of the glass = 526.17cm^3

Exercise 14.3

1. Question

Two cylindrical vessels are filled with oil. The radius of one vessel is 15 cm and its height is 25 cm. The radius and height of the other vessel are 10cm and 18 cm respectively. Find the radius of a cylindrical vessel 30 cm in height, which will just contain the oil of two given vessels.

Answer

Radius of 1st cylindrical vessel = 15cm

and height = 25cm

Radius of 2nd cylindrical vessel = 10cm and height = 18cm

So, Volume of 1^{st} cylindrical vessel = $\pi r^2 h$

= $\pi \times (15)^2 \times 25$

= 5625π cm^3

Volume of 2^{nd} cylindrical vessel = $\pi(r')^2 h'$

= $\pi \times (10)^2 \times 18$

= 1800π cm^3

Height of the third vessel = 30cm

and let its radius be R

So,

Volume of third cylindrical vessel = $\pi R^2 H$

= $\pi R^2 \times 30$

= $30\pi R^2$

Volume of 1^{st} cylindrical vessel + Vol. of 2^{nd} Cylindrical vessel

= Volume of the third cylindrical Vessel

$\Rightarrow 5625\pi + 1800\pi = 30\pi R^2$

$\Rightarrow 7425\pi = 30\pi R^2$

$\Rightarrow \frac{7425}{30} = R^2$

$\Rightarrow R^2 = 247.5$

$\Rightarrow R = 15.73$cm

Hence, radius of the required cylinder is 15.73cm

2. Question

A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube (Assume that there is no loss of metal during melting)

Answer

Let the edge of the third smaller cube be x cm.

Three small cubes are formed by melting the cube of edge 12 cm.

Edges of two small cubes are 6 cm and 8 cm.

Now, volume of a cube = $(\text{side})^3$

Volume of the big cube = sum of the volumes of the three small cubes

$\Rightarrow (12)^3 = (6)^3 + (8)^3 + (x)$

$\Rightarrow 1728 = 216 + 512 + x^3$

$\Rightarrow 1728 = 728 + x^3$

$\Rightarrow x^3 = 1728 - 728$

$\Rightarrow x^3 = 1000$

$\Rightarrow x = 10$ cm

Therefore, the edge of the third cube is 10 cm

3. Question

A hemisphere of lead of radius 8 cm is cast into a right circular cone of base radius 6 cm. Determine the height of the cone, correct to two places of decimals.

Answer

Radius of hemisphere = 8cm

and Radius of right circular cone = 6cm

According to question,

Volume of the hemisphere = Volume of cone

$$\Rightarrow \frac{2}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$$

$$\Rightarrow \frac{2}{3} \times \pi \times 8 \times 8 \times 8 = \frac{1}{3} \times \pi \times 6 \times 6 \times h$$

$$\Rightarrow \frac{1024}{36} = h$$

$\Rightarrow h = 28.44$cm

4. Question

A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of small balls thus obtained.

Answer

Given that Radius of bigger sphere (R) = 3cm

and diameter of smaller spherical balls = 0.6cm

So, radius of small spherical balls (r) = 0.3cm

Let the number of small balls = n

According to the question,

n × volume of small balls = Volume of bigger sphere

$\Rightarrow n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow nr^3 = R^3$

$\Rightarrow n = \frac{R^3}{r^3}$

$\Rightarrow n = \left(\frac{R}{r}\right)^3$

$\Rightarrow n = \left(\frac{3}{0.3}\right)^3$

$\Rightarrow n = (10)^3$

$\Rightarrow n = 1000$

Hence, the number of small balls = 1000

5. Question

A solid metal cone with radius of base 12 cm and height 24 cm is melted to form solid spherical balls of diameter 6 cm each. Find the number of balls thus formed.

Answer

Given that Radius of cone = 12cm

Height of the cone = 24cm

So,

Volume of cone $= \frac{1}{3}\pi R^2 h$

$= \frac{1}{3}\pi \times 12 \times 12 \times 24$

= 1152π cm^3

It is also given that diameter of each spherical balls = 6cm

so, Radius of each ball = 3cm

Volume of each ball $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3 \times 3 \times 3$

= 36π cm^3

Total number of balls formed by melting the cone

$= \frac{\text{Volume of cone}}{\text{Volume of a ball}}$

$= \frac{1152 \times \pi}{36 \times \pi}$

= 32

Hence, 32 balls are formed by melting the cone.

6. Question

A solid metallic sphere of diameter 21 cm is melted and recast into a number of smaller cones, each of diameter 3.5 cm and height 3cm. Find the number of cones so formed.

Answer

Let the number of cones formed be n

Diameter of a metallic sphere = 21cm

So, Radius $= \frac{21}{2}$cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$

= 11 × 21 × 21

= $4851 cm^3$

Now, Diameter of cone = 3.5cm

So, Radius $= \frac{3.5}{2} = \frac{7}{4}$ cm

and height = 3cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{4}\right)^2 \times 3$

$= \frac{22\times7}{4\times4} cm^3$

According to question,

n × volume of cone = volume of sphere

$\Rightarrow\ n \times \frac{22 \times 7}{4 \times 4} = 4851$

$\Rightarrow\ n = \frac{4851 \times 16}{22 \times 7}$

$\Rightarrow$ n = 504

Hence, the number of cones formed = 504

7. Question

Spherical ball of diameter 21cm is melted and recasted into cubes, each of side 1 cm. Find the number of cubes thus formed. [Use π=22/7]

Answer

Diameter of sphere = 21 cm

So, radius of the sphere $= \frac{21}{2}$ cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$

= 11 × 21 × 21

= 4851cm^3

Volume of cube = a^3 = 1^3

Let number of cubes formed be n, then

Volume of sphere = n × (volume of cube)

⇒ 4851 =n×1

⇒ n = 4851

Hence, number of cubes is 4851.

8. Question

The internal and external diameters of a hollow hemispherical shell are 6cm and 10cm respectively. It is melted and recast into a solid cone of base diameter 14cm. Find the height of the cone so formed.

Answer

Given:

Internal Diameter of a hollow hemispherical shell = 6cm

So, internal radius, (r) = 3cm

External Diameter of a hollow hemispherical shell = 10cm

So, external radius (R) = 5cm

Volume of material in the shell $= \frac{2}{3}\pi[R^3 - r^3]$

$= \frac{2}{3}\pi[5^3 - 3^3]$

$= \frac{2}{3} \times \frac{22}{7} \times 98$

Given that diameter of cone =14cm

So, radius = 7cm

Volume of cone $= \frac{1}{3}\pi(r')^2h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2h$

According to the question.

Volume of a shell = Volume of a cone

$\frac{2}{3} \times \frac{22}{7} \times 98 = \frac{1}{3} \times \frac{22}{7} \times (7)^2h$

$\Rightarrow \frac{2\times98}{7\times7} = h$

$\Rightarrow$ h = 4cm

9. Question

A solid sphere of radius 6cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5cm and its height is 32 cm, find the uniform thickness of the cylinder.

Answer

Let inner radius of the cylinder be x cm

Given that radius of sphere = 6cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi(r')^3$

$= \frac{4}{3} \times \pi \times (6)^3$

$= 4 \times \pi \times 2 \times 36 \text{ cm}^3$

It is also given that external radius of base of cylinder (R) = 5cm

and height = 32cm

$\therefore$ Volume of hollow cylinder $= \pi h[R^2 - r^2]$

$= \pi \times 32 \times [(5)^2 - x^2]$

$= 32\pi\ [25 - x^2]$

Solid sphere is melted and casted into hollow cylinder

$\therefore$ Volume of sphere = Volume of hollow cylinder

$\Rightarrow 4 \times \pi \times 2 \times 36 = 32\pi\ [25 - x^2]$

$\Rightarrow \frac{8 \times 36}{32} = 25 - x^2$

$\Rightarrow 9 - 25 = -x^2$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

$\Rightarrow x = 4cm$

Hence, the inner radius of the cylinder is 4cm

So, the thickness of the cylinder = external radius – inner radius

= 5 – 4

= 1cm

10. Question

The diameter of a copper sphere is 6cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 36cm, find its radius [Take π=2.14]

Answer

Diameter of a copper sphere = 6 cm

So, Radius of copper sphere = 3cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times (3)^3$

$= 36\pi\ cm^3$

Length of wire = 36cm

Let the radius of wire be R cm

Volume of wire = $\pi R^2 h$

$= \pi R^2 \times 36$

But the volume of wire = volume of sphere

$\Rightarrow 36\pi R^2 = 36\pi$

$\Rightarrow R^2 = 1$

$\Rightarrow R = 1cm$ [taking positive root, because radius can't be negative]

Hence, the radius of wire is 1cm

11. Question

A cylindrical container is filled with ice-cream. Its diameter is 12cm and height is 15cm. The whole ice-cream is distributed among 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

Answer

Let the radius of the base of conical ice cream = x cm

Then, height of the conical ice cream = 2 × diameter

= 2 × (2x)

= 4x cm

Volume of ice- cream cone

= Volume of conical portion + Volume of hemispherical portion

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi x^2(4x) + \frac{2}{3}\pi x^3$$

$$= \frac{4\pi x^3 + 2\pi x^3}{3}$$

$= 2\pi x^3\ cm^3$

Now,

Diameter of cylindrical container = 12cm

So, radius = 6cm

and height = 15cm

$\therefore$ Volume of cylindrical container = $\pi r^2 h$

$= \pi \times (6)^2 \times (15)$

$= 540\pi\ cm^3$

$$\text{Number of children} = \frac{\text{Volume of cylindrical container}}{\text{Volume of one ice} - \text{cream cone}}$$

$$\Rightarrow \; 10 = \frac{540 \times \pi}{2 \times \pi \times x^3}$$

$$\Rightarrow \; x^3 = \frac{540}{2 \times 10}$$

$\Rightarrow x^3 = 27$

$\Rightarrow x = 3$

So, the radius of the base of conical ice cream is 3cm

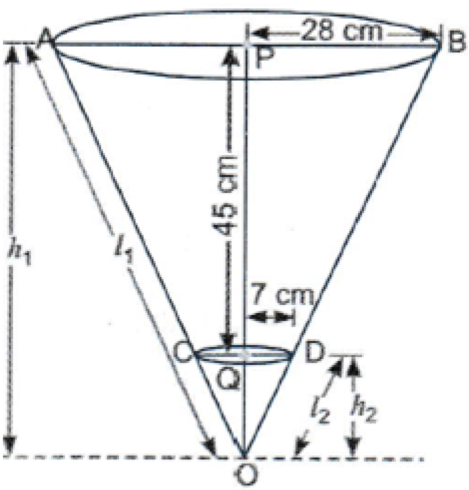
Hence, the diameter of the base of conical ice-cream = 2×3 = 6cm

Exercise 14.4

1. Question

The radii of the ends of a frustum of a cone 45 cm high are 28cm and 7cm. Find its volume, the curved surface area. [Take π=22/7]

Answer



Frustum = difference of two right circular cones OAB and OCD

Let the height of the cone OAB be h_1 and its slant height l_1

i.e. OA = OB = l_1 and OP = h_1.

Let h_2 be the height of cone OCD and l_2 its slant height

i.e. OC = OD = l_2 and OQ = h_2

We have , r_1 = 28cm and r_2 = 7cm

and height of frustum (h) = 45cm

Also,

$h_1 = 45 + h_2$ ...(i)

Now, we first need to determine the h_1 and h_2

∵ ΔOPB and OQD are similar, we have

$$\frac{h_1}{h_2} = \frac{28}{7} = \frac{4}{1}$$

$\Rightarrow h_1 = 4h_2$ ...(ii)

From (i) and (ii), we get

$4h_2 = 45 + h_2$

$\Rightarrow 3h_2 = 45$

$\Rightarrow h_2 = 15$cm

Putting the value of h_2 in eq. (ii), we get

$h_1 = 4 \times 15 = 60$cm

So, $h_1 = 60$cm and $h_2 = 15$cm

Now, the volume of frustum = Vol. of cone OAB – Vol. of cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3} \times \frac{22}{7}[28^2 \times 60 - 7^2 \times 15]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 15[28^2 \times 4 - 7^2]$$

$$= 5 \times \frac{22}{7}[3136 - 49]$$

$$= 5 \times \frac{22}{7} \times 3087$$

$= 5 \times 22 \times 441$

$= 48510$ cm^3

Now, we first have to find the slant height l_1 and l_2

$l_1 = \sqrt{\{(28)^2+(60)^2\}}$

$\Rightarrow l_1 = \sqrt{\{(4\times7)^2 + (4\times15)^2}$

$\Rightarrow l_1 = 4\sqrt{\{(7)^2 + (15)^2\}}$

$\Rightarrow l_1 = 4\sqrt{49 + 225}$

$\Rightarrow l_1 = 4\sqrt{(274)}$

$\Rightarrow l_1 = 4 \times 16.55$

$\Rightarrow l_1 = 66.20\text{cm}$

and $l_2 = \sqrt{\{(7)^2 + (15)^2\}}$

$\Rightarrow l_2 = \sqrt{49 + 225}$

$\Rightarrow l_2 = \sqrt{(274)}$

$\Rightarrow l_2 = 16.55$ cm

So, CSA of frustum = CSA of cone OAB – CSA of cone OCD

$= \pi r_1 l_1 - \pi r_2 l_2$

$$= \frac{22}{7}[28 \times 66.20 - 7 \times 16.55]$$

$= 22\ [4\times66.20 - 16.55]$

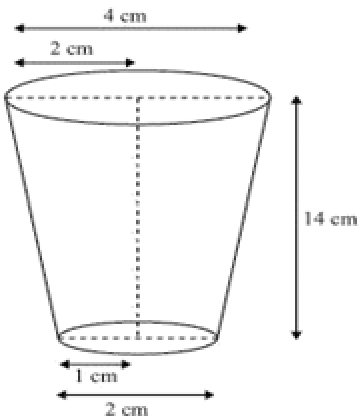
$= 22 \times 248.25$

$= 5461.5\text{cm}^2$

2. Question

A drinking glass is in the shape of a frustum of a cone of height 14cm. The diameters of its two circular ends are 4cm and 2cm. Find the capacity of the glass.

Answer



Given: Height of frustum = 14cm

Diameter of 1st circular end = 4cm

So, radius (R) = 2cm

Diameter of 2nd circular end = 2cm

So, radius (r) = 1cm

Capacity of the glass = Volume of frustum

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14[(2)^2 + (1)^2 + 2 \times 1]$$

$$= \frac{44}{3}[4 + 1 + 2]$$

$$= \frac{44 \times 7}{3}$$

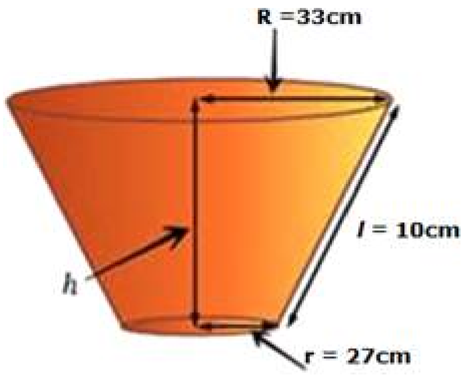
= 102.66 cm^3

Hence, the volume of glass = 102.66cm^3

3. Question

The radii of the circular ends of a solid frustum of a cone are 33 cm and 27cm, and its slant height is 10cm. Find its capacity and total surface area. [Take π=22/7]

Answer



Greater radius = R = 33 cm

Smaller radius = r = 27 cm

Slant height = l = 10 cm

Total Surface area of the frustum = $\pi R^2 + \pi r^2 + \pi l(R+r)$

$= \pi[(33)^2 + (27)^2 + 10\times(33+27)]$

$= \pi[1089 + 729 + 600]$

$$= \frac{22}{7} \times 2418$$

$= 7599.428\ cm^2$

$= 7599.43\ cm^2$

We first need to find the height, h

$l^2 = (R - r)^2 + h^2$

$\Rightarrow (10)^2 = (33 - 27)^2 + h^2$

$\Rightarrow 100 = (6)^2 + h^2$

$\Rightarrow 100 = 36 + h^2$

$\Rightarrow h^2 = 100 - 36$

$\Rightarrow h^2 = 64$

$\Rightarrow h = \pm 8$

$\Rightarrow h = 8cm$

Now, Capacity of a solid frustum of a cone = Volume of frustum

$$= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 8[(33)^2 + (27)^2 + 33 \times 27]$$

$$= \frac{22 \times 8}{21}[1089 + 729 + 891]$$

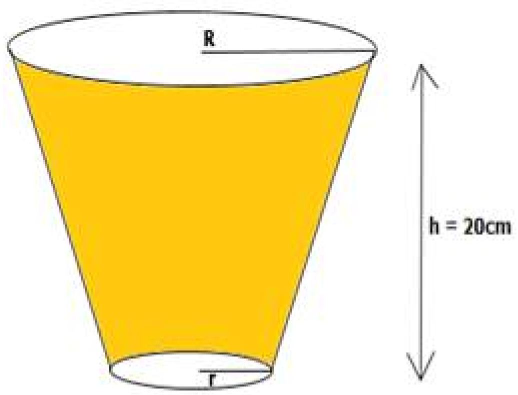
$$= \frac{22 \times 8 \times 2709}{21}$$

= 22704 cm^3

4. Question

The perimeters of the ends of the frustum of a cone are 96 cm and 68 cm. If the height of the frustum be 20 cm, find its radii, slant height, volume and total surface. [Take π=22/7]

Answer



Let radii of the circular ends of the frustum are R and r respectively.

Given that Perimeter of one end = 96cm

⇒ 2πR = 96

$$\Rightarrow \ R \ = \frac{96}{2\pi}$$

$$\Rightarrow \ R = \frac{96 \times 7}{2 \times 22}$$

⇒ R = 15.27 cm

Perimeter of other end = 68cm

⇒ 2πr = 68

$$\Rightarrow \ r \ = \frac{68}{2\pi}$$

$\Rightarrow\ r = \dfrac{68 \times 7}{2 \times 22}$

$\Rightarrow$ r = 10.82 cm

It is given that height of the frustum = 20cm

So, Slant height, l = $\sqrt{\{h^2 + (R - r)^2\}}$

= $\sqrt{\{(20)^2 + (15.27 - 10.82)^2}$

= $\sqrt{400 + (4.45)^2}$

= $\sqrt{400 + 19.80}$

= $\sqrt{419.80}$

= 20.49 cm

Now,

Volume of the frustum $= \dfrac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 20[(15.27)^2 + (10.82)^2 + 15.27 \times 10.82]$

$= \dfrac{22 \times 20}{21}[233.1729 + 117.0724 + 165.2214]$

$= \dfrac{22 \times 20 \times 515.4667}{21}$

= 10800.25 cm^3

Total Surface Area of the frustum = $\pi R^2 + \pi r^2 + \pi l(R+r)$

= $\pi[(15.27)^2 + (10.82)^2 + 20.49 \times (15.27+10.82)]$

= $\pi[233.1729 + 117.0724 + 534.5841]$

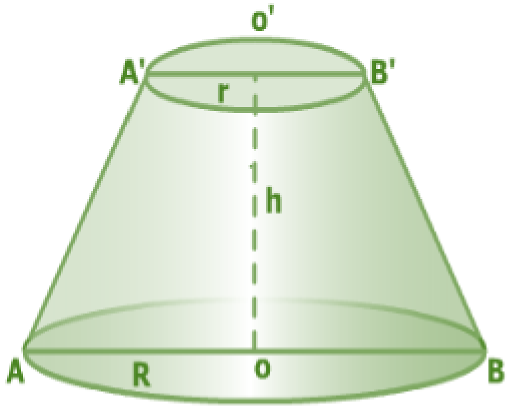
$= \dfrac{22}{7} \times 884.83$

= 2780.89 cm^2

5. Question

A friction clutch in the form of the frustum of a cone, the diameters of the ends being 8cm, and 10 cm and length 8 cm. Find its bearing surface and its volume. [Take π=3.14]

Answer



We have,

Diameter of one end = 10cm

So, radius (R) = 5cm

Diameter of other end = 8cm

So, radius (r) = 4cm

and slant height, l = 8cm

So, height, h = $\sqrt{\{l^2 - (R - r)^2\}}$

= $\sqrt{\{(8)^2 - (5 - 4)^2}$

= $\sqrt{64} - (1)^2$

= $\sqrt{64} - 1$

= $\sqrt{63}$

= 7.937 cm

Bearing surface of the clutch = CSA of the frustum

= $\pi l\ (R + r)$

= 3.14 × 8 (5 + 4)

= 25.12 (9)

= 226.08 cm^2

Volume of the frustum $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times 3.14 \times 7.93[(5)^2 + (4)^2 + 5 \times 4]$

$= \frac{3.14 \times 7.93}{3}[25 + 16 + 20]$

$= \frac{3.14 \times 7.937 \times 61}{3}$

= 506.75 cm^3

Hence, Volume of frustum = 506.75 cm^3

6. Question

A bucket is in the form of a frustum of a cone. Its depth is 15cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how many litres of water can the bucket hold. [Take π=22/7]

Answer

Greater diameter of the frustum = 56 cm

So, radius of the frustum = R = 28 cm

Smaller diameter of the frustum = 42 cm

So, Radius of the smaller end of the frustum = r = 21 cm

and Height of the frustum = h = 15 cm

Capacity of the frustum

$= \text{Volume of the frustum } = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 15 \times [28^2 + 21^2 + 28 \times 21]$

$= \frac{22 \times 15}{21}[784 + 441 + 588]$

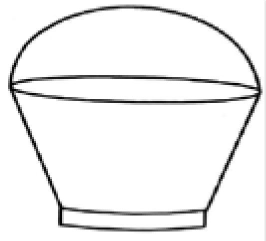
$= 22 \times \frac{5}{7} \times 1813$

= 28490 cm^3

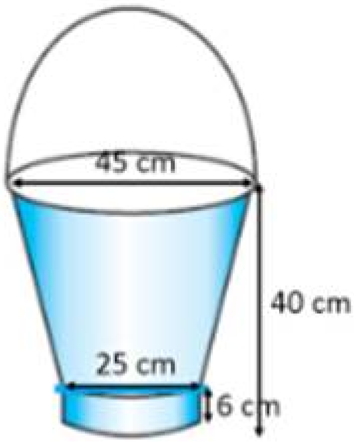
= 28.49 litres

7. Question

An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet (see Fig.) The diameters of the two circular ends of the bucket are 45 cm and 25cm, the total vertical height of the bucket is 40cm and that of the cylindrical base is 6cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold. [Take π=22/7]



Answer



Diameter of bigger circular end = 45cm

So, Radius (R) $= \frac{45}{2}$ cm

Diameter of smaller circular end = 25cm

So, Radius (r) $= \frac{25}{2}$ cm

Now,

Height of frustum = Total height of bucket – Height of cylinder

= 40 – 6

= 34cm

Also,

Slant height of the frustum, $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{34^2 + \left(\frac{45}{2} - \frac{25}{2}\right)^2}$

$= \sqrt{1156 + \left(\frac{20}{2}\right)^2}$

= √1156 + 100

= √ 1256

= 35.44cm

Curved Surface Area of frustum = πl(R + r)

$= \frac{22}{7} \times 35.44 \times \left(\frac{45}{2} + \frac{25}{2}\right)$

$= \frac{22}{7} \times 35.44 \times \frac{70}{2}$

= 22 × 35.44 × 5

= 3898.4 cm^2

Curved Surface area of cylinder = 2πrh

$= 2 \times \frac{22}{7} \times \frac{25}{2} \times 6$

= 471.428 cm^2

Area of circular base = πr^2

$= \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

= 491.07 cm^2

Now, Area of metallic sheet used

= CSA of frustum + Area of circular base + CSA of cylinder

= 3898.4 + 471.428 + 491.07

= 4860.89

= 4860.9 cm^2

Volume of water that bucket can hold $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times \frac{22}{7} \times 34[(22.5)^2 + (12.5)^2 + 22.5 \times 12.5]$

$= \frac{22 \times 34}{21}[506.25 + 156.25 + 281.25]$

$= \frac{22 \times 34 \times 943.75}{21}$

= 33615.47 cm^3

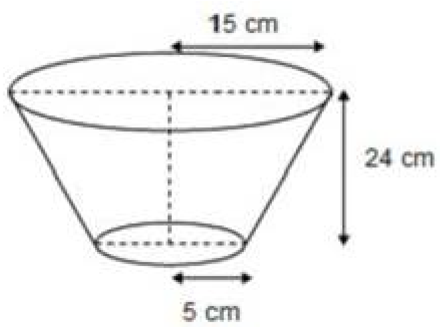
We know that 1 cm^3 = 0.001 litre

∴ Volume of water that bucket can hold = 33.62 litres (approx.)

8. Question

A bucket made up of a metal sheet is in the form of frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of Rs. 20 per litre and the cost of metal sheet used if it costs Rs. 10 per 100cm^2. [Use π=3.14]

Answer



Greater diameter of the bucket = 30 cm

Radius of the bigger end of the bucket = R = 15 cm

Diameter of the smaller end of the bucket = 10 cm

Radius of the smaller end of the bucket = r = 5 cm

Height of the bucket = 24 cm

Slant height, l = $\sqrt{\{h^2 + (R - r)^2\}}$

= $\sqrt{\{(24)^2 + (15 - 5)^2}$

$= \sqrt{576 + (10)^2}$

$= \sqrt{576 + 100}$

$= \sqrt{676}$

= 26 cm

Now,

Volume of the frustum $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$= \frac{1}{3} \times 3.14 \times 24[(15)^2 + (5)^2 + 15 \times 5]$

= 3.14 × 8 [225 + 25 + 75]

= 3.14 × 8 ×325

= 8164 cm^3

= 8.164 Litres

A litre of milk cost Rs 20

So, total cost of filling the bucket with milk = Rs 20 × 8.164

= Rs 163.28

Surface Area of the bucket

= CSA of the frustum + Area of circular base

$= \pi l(R+r) + \pi r^2$

$= \pi [\{26 \times (15+5)\} + (5)^2]$

$= \pi[520 + 25]$

= 3.14 × 545

= 1711.3 cm^2

Cost of 100cm^2 of metal sheet = Rs 10

Cost of 1 cm^2 of metal sheet $= \frac{10}{100}$

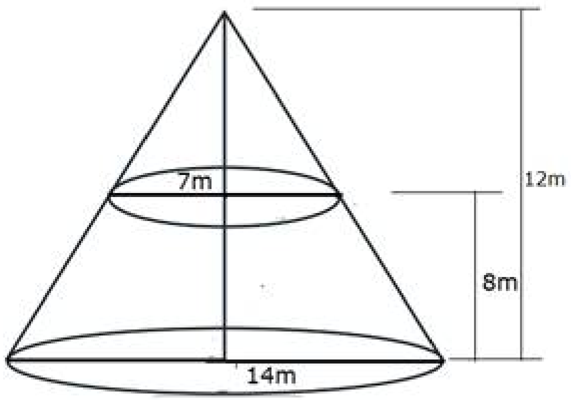
Cost of 1711.3 cm^2 of metal sheet $= \frac{1711.3 \times 10}{100}$

= Rs 171.13

9. Question

A tent is made in the form of a conic frustum, surmounted by a cone. The diameters of the base and top of the frustum are 14m and 7 m and its height is 8 m. The height of the tent is 12m. Find the quantity of canvas required. [Take π=22/7]

Answer



For the lower portion of the tent:

Diameter of the base = 14 m

Radius, R, of the base = 7 m

Diameter of the top end of the frustum = 7 m

Radius of the top end of the frustum = $r = \frac{7}{2}\ m = 3.5m$

Height of the frustum = h = 8 m

Slant height = l = $\sqrt{\{h^2 + (R - r)^2\}}$

= $\sqrt{\{(8)^2 + (7 - 3.5)^2}$

= $\sqrt{64} + (3.5)^2$

= $\sqrt{64} + 12.25$

= $\sqrt{76.25}$

= 8.73 m

For the conical part

Radius of the cone base = r = 3.5 m

Height of the cone = Height of the tent – height of frustum

= 12 – 8

= 4 m

Slant height of cone = L = $\sqrt{(4)^2+(3.5)^2}$

= $\sqrt{16 + 12.25}$

=$\sqrt{28.25}$

= 5.3 m

Total quantity of canvas = CSA of frustum + CSA of conical top

= πl(R + r) + πrL

= π [8.73(7 + 3.5) + 3.5 × 5.3]

$= \frac{22}{7}[91.66 + 18.6]$

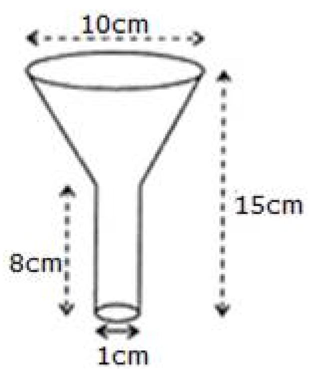
$= \frac{22}{7}[110.26]$

= 346.5 m^2

10. Question

An oil funnel of tin sheet consists of a cylindrical portion 8 cm along attached to a frustum of a cone. If the total height be 15cm, the diameter of the cylindrical portion 1 cm and diameter of the top of the funnel 10cm, find the area of the tin required. [Take π=22/7]

Answer



Diameter of top of funnel = 10cm

So, Radius (R) = 5cm

Diameter of cylindrical portion = 1cm

So, radius $(r) = \frac{1}{2} = 0.5\text{cm}$

Height of frustum = Total height – Height of cylindrical part

= 15 – 8

= 7cm

and Slant Height, $l = \sqrt{\{h^2 + (R - r)^2\}}$

$= \sqrt{\{(7)^2 + (5 - 0.5)^2}$

$= \sqrt{49} + (4.5)^2$

$= \sqrt{49} + 20.25$

$= \sqrt{69.25}$

= 8.32 cm

Now, CSA of frustum = $\pi l\ (R + r)$

$$= \frac{22}{7} \times 8.32 \times (5 + 0.5)$$

$$= \frac{22}{7} \times 8.32 \times (5.5)$$

= 143.81 cm^2

Height of cylinder, H = 8cm

Now, CSA of cylinder = $2\pi rH$

$$= 2 \times \frac{22}{7} \times 0.5 \times 8$$

= 25.14 cm^2

Area of tin required = CSA of frustum + CSA of cylinder

= 143.81 + 25.14

= 168.95 cm^2