Very Short Answer Questions (PYQ)

Q. 1. Write the expression, in a vector form, for the Lorentz magnetic force $\underset{F}{\rightarrow}$ due to a charge moving with velocity $\underset{V}{\rightarrow}$ in a magnetic field $\underset{B}{\rightarrow}$. What is the direction of the magnetic force? [CBSE Delhi 2014]

Ans. Force, $\vec{F} = q(\vec{v} \times \vec{B})$

Obviously, the force on charged particle is perpendicular to both velocity $\underset{V}{\rightarrow}$ and magnetic field $\underset{B}{\rightarrow}$.

Q. 2. When a charged particle moving with velocity $\underset{V}{\rightarrow}$ is subjected to magnetic field $\underset{B}{\rightarrow}$ the force acting on it is non-zero. Would the particle gain any energy? [CBSE (F) 2013]

Ans. No. (i) This is because the charge particle moves on a circular path.

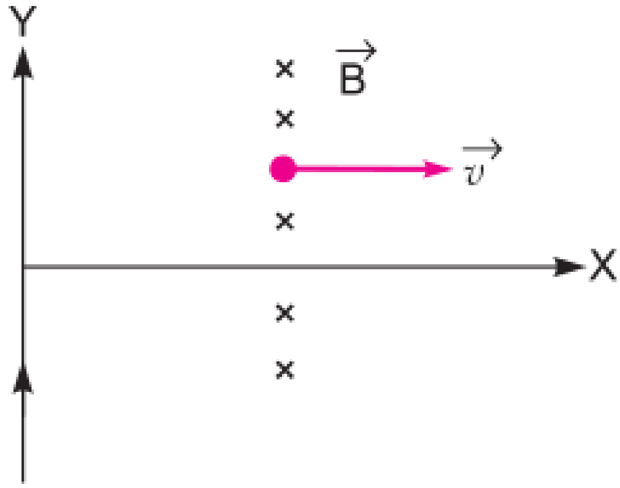
(ii) $\vec{F} = q(\vec{v} \times \vec{B})$

and power dissipated $P = \vec{F} \cdot \vec{v}$

$= q\left(\vec{v} \times \vec{B}\right) \cdot \vec{v}$

The particle does not gain any energy.

Q. 3. A long straight wire carries a steady current I along the positive Y-axis in a coordinate system. A particle of charge +Q is moving with a velocity $\underset{V}{\rightarrow}$ along the X-axis. In which direction will the particle experience a force? [CBSE (F) 2013]



Ans.

From relation $\vec{F} = \text{qvB}\,[\hat{i} \times (-\hat{k})] = +\text{qvB}\,(\hat{j})$

Magnetic force $\vec{F}$ along + Y axis.

Or

From Fleming's left hand rule, thumb points along+Y direction, so the direction of magnetic force will be along + Y axis (or in the direction of flow of current).

Q. 4. What can be the cause of helical motion of a charged particle?
[CBSE North 2016]

Ans. Charge particle moves inclined to the magnetic field. When there is an angle between velocity of charged particle and magnetic field, then the vertical component of velocity (v sin θ) will rotate the charge particle on circular path, but horizontal component (v cos θ) will move the charged particle in straight line. Hence path of the charge particle becomes helical.

Q. 5. In a certain region of space, electric field $\underset{E}{\rightarrow}$ and magnetic field $\underset{B}{\rightarrow}$ are perpendicular to each other. An electron enters in the region perpendicular to the directions of both $\underset{B}{\rightarrow}$ and $\underset{E}{\rightarrow}$ and moves undeflected. Find the velocity of the electron.
[Hots] [CBSE (F) 2013]

Ans.

Net force on electron moving in the combined electric field $\vec{E}$ and magnetic field $\vec{B}$ is

$$\vec{F} = -e[\vec{E} + \vec{v} \times \vec{B}]$$

Since electron moves undeflected then $\vec{F} = 0$.

$$\overrightarrow{E + \vec{v}} \times \vec{B} = 0$$

$$\Rightarrow \qquad |\vec{E}| = |\vec{v}| \times |\vec{B}| \qquad \Rightarrow \qquad |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}|}$$

Q. 6. Using the concept of force between two infinitely long parallel current carrying conductors, define one ampere of current. [CBSE (AI) 2014]

Ans. One ampere is the value of steady current which when maintained in each of the two very long, straight, parallel conductors of negligible cross-section and placed one metre apart in vacuum would exert a force of 2×10^{-7} N on 1 metre length of either wire.

Q. 7. Define one tesla using the expression for the magnetic force acting on a particle of charge 'q' moving with velocity $\vec{V}$ in a magnetic field $\vec{B}$. [CBSE (F) 2014]

Ans. One tesla is the magnetic field in which a charge of 1 C moving with a velocity of 1 ms^{-1}, normal to the magnetic field, experiences a force of 1 N.

$$B = \frac{F}{qv \sin\theta}$$

If $F = 1$ N, $q = 1$C, $v = 1$ ms^{-1}, $\theta = 90^\circ$

$$\text{then SI unit of } B = \frac{1N}{1C.1\ ms^{-1}\ .\ \sin 90^o}$$

$$= 1\ NA^{-1}\ m^{-1} = 1 \text{ tesla}$$

Q. 8. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the circular paths described by them?
[CBSE (F) 2011]
OR
A proton and a deuteron having equal momenta enter in a region of uniform magnetic field at right angle to the direction of the field. Find the ratio of the radii of curvature of the path of the particle. [CBSE Delhi 2013]

Ans. Charge on deutron (q_d) = charge on proton (q_p)

$q_d = q_p$

Radius of circular path $(r) = \frac{P}{Bq}$ $\left(\because qvB = \frac{mv^2}{r}\right)$

$r \propto \frac{1}{q}$ [for constant momentum (P)]

So, $\frac{r_p}{r_d} = \frac{q_d}{q_p} = \frac{q_p}{q_p} = 1$

Hence, $r_p : r_d = 1 : 1$

Q. 9. Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why? [CBSE Delhi 2009]

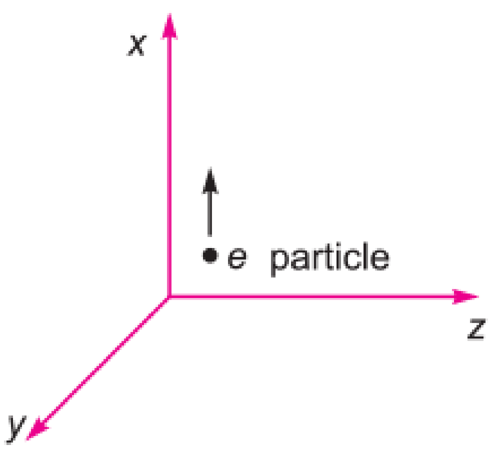
Ans. Magnetic field lines can be entirely confined within the core of a toroid because toroid has no ends. A solenoid is open ended and the field lines inside it which are parallel to the length of the solenoid, cannot form closed curves inside the solenoid.

Q. 10. An electron does not suffer any deflection while passing through a region of uniform magnetic field. What is the direction of the magnetic field? [CBSE (AI) 2009]

Ans. Magnetic field is parallel or antiparallel to velocity of electron i.e., angle between $\underset{V}{\rightarrow}$ and $\underset{B}{\rightarrow}$ is 0° or 180°.

Q. 11. A beam of particles projected along + x-axis, experiences a force due to a magnetic field along the + y-axis. What is the direction of the magnetic field?
[CBSE (AI) 2010]

Ans. By Fleming's left hand rule magnetic field must be along negative z-axis.



Very Short Answer Questions (OIQ)

Q. 1. Which physical quantity has the unit Wb/m^2 ? Is it a scalar or a vector quantity?

Ans. Magnetic field induction has the unit Wb/m^2. It is a vector quantity.

Q. 2. What will be the path of a charged particle moving along the direction of a uniform magnetic field?

Ans. The path of particle will remain unchanged (since magnetic force $F_m = qvB \sin \theta = 0$).

Q. 3. Under what condition an electron moving through a magnetic field experiences the maximum force?

Ans. $F_m = qvB \sin \theta$

Force is maximum when $\sin\theta = 1$ or $\theta = 90°$, that is, when electron is moving perpendicular to the direction of magnetic field.

Q. 4. Under what condition the force acting on a charge moving through a uniform magnetic field is minimum?

Ans. $F_m = qvB \sin \theta$; for minimum force $\sin \theta = 0$.

i.e., force is minimum when charged particle moves parallel or antiparallel to the field.

Q. 5. In what condition does a charged particle moving through a magnetic field follow a circular path?

Ans. The charged particle follows a circular path, when it moves perpendicular to the direction of a magnetic field.

Q. 6. A charged particle enters along the axis of a current carrying a long solenoid. How is its velocity affected? Will the particle be accelerated or decelerated?

Ans. The magnetic field due to a current in solenoid is along the axis, so when a charged particle enters along the axis ($\theta = 0$), the magnetic force on particle is $qvB \sin 0° = 0$; so the particle's velocity remains unchanged i.e., the particle remains unaccelerated.

Q. 7. Which types of fields are produced by a moving electron? If electron is at rest, then what type of field is produced?

Ans. A moving electron produces electric and magnetic fields both. A stationary electron produces electric field only.

Q. 8. A charged particle moving with a uniform velocity enters a magnetic field directed perpendicular to it, what will be the path of electron? How will its speed be affected?

Ans. In a perpendicular magnetic field a charged particle traverses a **circular path.** There will be no change in speed of particle.

Q. 9. When a charged particle moves in a magnetic field normally; what quantity changes — the particle's speed, particle's energy, path of motion of the particle?

Ans. Path of motion changes.

Q. 10. Give two factors by which the voltage sensitivity of a moving coil galvanometer can be increased.

Ans. The voltage sensitivity of a moving coil galvanometer is given by $S_V = \frac{\mathrm{NBA}}{\mathrm{GC}}$

Clearly, it can be increased by increasing the number of turns (N) and decreasing the torsional rigidity C of suspension wire.

Q. 11. What is the nature of magnetic field in a moving coil galvanometer?

Ans. The nature of magnetic field in a moving coil galvanometer is **radial.**

Q. 12. Why should an ammeter have a low resistance?

Ans. An ammeter is connected in series with the circuit to read the current. If it had large resistance, it will change the current in circuit which it has to measure correctly; hence ammeter reading will have significant error; so for correct reading an ammeter should have a very low resistance.

Q. 13. Why should a voltmeter have high resistance?

Ans. A voltmeter is connected in parallel. When connected in parallel, it should draw least current otherwise, the potential difference which it has to measure, will change.

Q. 14. What pre-information would you require to convert a galvanometer into ammeter or voltmeter?

Ans. Two information's are required (i) resistance of galvanometer and (ii) current in galvanometer for full scale deflection.

Q. 15. What is Bohr magneton?

Ans. Bohr magenton is a unit of atomic dipole moment. Its value is eh

$$\frac{\mathrm{eh}}{4\pi m} = 9.27 \times 10^{-24}\,\mathrm{Am}^2 .$$

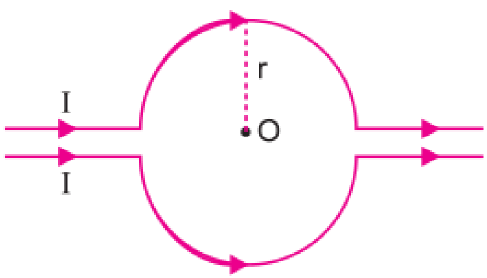
Q. 16. Equal currents are flowing through two infinitely long parallel wires. What will be the magnetic field at a point midway when the currents are flowing in the same direction? [HOTS]

Ans. Magnetic field at mid-point due to two wires will be equal and opposite, hence the net magnetic field at this point will be zero.

Q. 17. Two wires of equal length are bent in the form of two loops. One of the loops is square shaped whereas the other loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque? Give reasons. [HOTS]

Ans. Torque $\tau = IAB \sin \theta \propto A$. For given perimeter the area of circular loop is maximum, so a circular loop will experience greater torque.

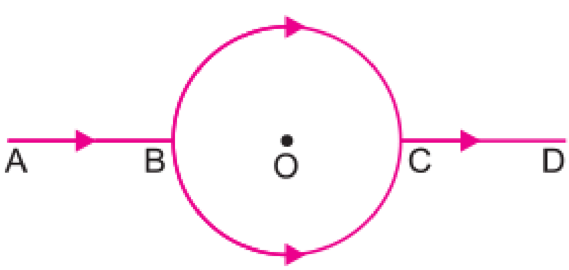
Q. 18. What is the value of magnetic field at point O due to current flowing in the wires? [HOTS]



Ans. Zero, because the upper and lower current carrying conductors are identical and so the magnetic fields caused by them at the centre O will be equal and opposite.

Q. 19. What is the magnetic field at point O due to current carrying wires shown in figure? [HOTS]

Ans.



Ans. The magnetic field due to straight wires AB and CD is zero since either $\theta = 0°$ or $180°$ and that due to a semi-circular arc are equal and opposite; hence net field at O is zero.

Q. 20. An electron, passing through a region is not deflected. Are you sure that there is no magnetic field in that region? [HOTS]

Ans. No, if an electron enters parallel to a magnetic field, no force acts and the electron remains undeflected.

Q. 21. Two identical charged particles moving with the same speed enter a region of uniform magnetic field. If one of these enters normal to the field direction and the other enters along a direction at 30° with the field, what would be the ratio of their angular frequencies. [HOTS]

Ans.

$\omega = \frac{qB}{m}$ independent of angle of entrance with the magnetic field.

Ratio $\omega_1 : \omega_2 = 1 : 1$

Short Answer Questions – I (PYQ)

Q. 1. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vector can be used to select charged particles of a particular speed. [CBSE (AI) 2017]

Ans. Consider a charge q moving with velocity v in the presence of electric and magnetic fields. The force on an electric charge q due to both of them is

$F= q\ [E\ (r)+v \times B(r)]$

$\Rightarrow F \equiv F_{electric} + F_{magnetic}$ ...(i)

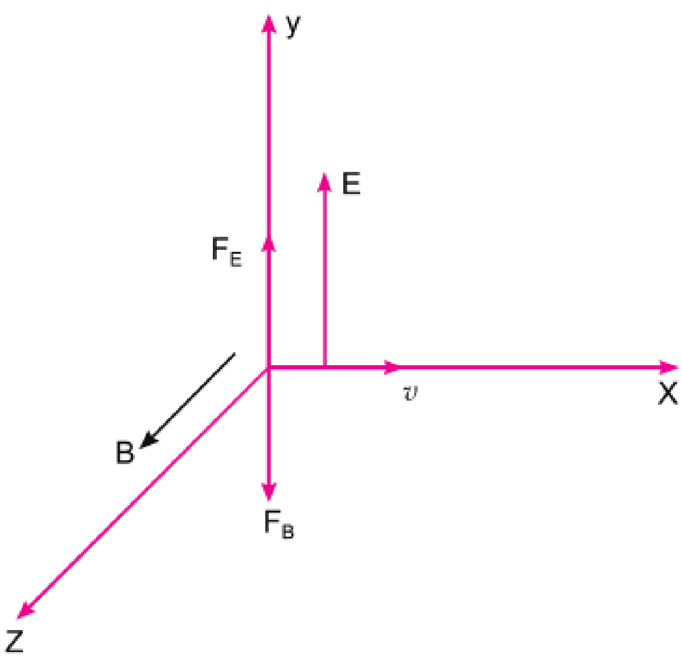
Where, v = velocity of the charge

r = location of the charge at a given time t

E(r) = Electric field

B(r) = Magnetic field

Let us consider a simple case in which electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle.



$$F_E = \mathrm{qE} = \mathrm{qE}\,\hat{j}$$

$$F_B = \mathrm{qv} \times B = q(v\hat{i} \times B\hat{k}) = -\ \mathrm{qvB}\,\hat{j}$$

$$\therefore \qquad F = q(E - vB)\ \hat{j}$$

Thus, electric and magnetic forces are in opposite directions.

Suppose we adjust the values of E and B such that magnitudes of the two forces are equal, then the total force on the charge is zero and the charge will move in the fields undeflected. This happens when

$$\mathrm{qE} = \mathrm{qvB} \qquad \text{or} \qquad v = \frac{E}{B}$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass). The crossed E and B fields therefore serve as a velocity selector.

Q. 2. Answer the following questions

(i) Write the expression for the magnetic force acting on a charged particle moving with velocity v in the presence of magnetic field B.

(ii) A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown. Trace their paths in the field and justify your answer. [CBSE Delhi 2016]

Ans. (i) $\vec{F} = q(\vec{v} \times \vec{B})$

(ii) Force on alpha particle and electron are opposite to each other, magnitude of charge of alpha particle is more than electron hence radius of alpha particle is more than radius of electron.

Q. 3. State the underlying principle of a cyclotron. Write briefly how this machine is used to accelerate charged particles to high energies. [CBSE Delhi 2014]

Ans. A cyclotron makes use of the principle that the energy of the charged particles can be increased to a high value by making it pass through an electric field repeatedly.

The magnetic field acts on the charged particle and makes them move in a circular path inside the dee. Every time the particle moves from one dee to another it is acted upon by the alternating electric field, and is accelerated by this field, which increases the energy of the particle.

Uses: (i) It is used to bombard nuclei with high energetic particles accelerated by cyclotron and study the resulting nuclear reaction.

(ii) It is used to implant ions into solids and modify their properties or even synthesize new materials.

Q. 4. Write the expression for Lorentz magnetic force on a particle of charge 'q' moving with velocity $\vec{V}$ in a magnetic field $\vec{B}$. Show that no work is done by this force on the charged particle. [CBSE (AI) 2011]

Ans.

Lorentz magnetic force, $\overrightarrow{F_m} = q\vec{v} \times \overrightarrow{B}$

Work done, $W = \overrightarrow{F_m}\vec{S} = \int \overrightarrow{F_m}\vec{v}\mathrm{dt} = \int q(\vec{v} \times \overrightarrow{B}).\vec{v}\ \mathrm{dt}$

As $(\vec{v} \times \overrightarrow{B}).\vec{v} = 0$

$\therefore$ Work, $W = 0$

Q. 5. A charged particle enters perpendicularly a region having either (i) magnetic field or (ii) an electric field. How can the trajectory followed by the charged

particle help us to know whether the region has an electric field or a magnetic field? Explain briefly.

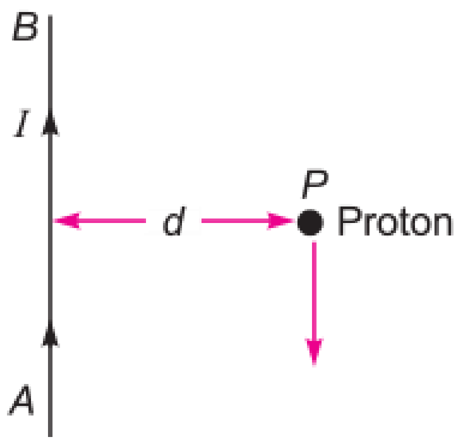
Ans. The path of the charged particle will be circular in a magnetic field. This is due to the reason that the force acting on the particle will be at right angles to the field as well as direction of motion, resulting in a circular trajectory.

In the case of electric field, the trajectory of the particle will be determined by the equation

$$s = \mathrm{ut} + \frac{1}{2}\left(\frac{\mathrm{qE}}{m}\right)t^2 \qquad \left(s = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2\right)$$

Where q and m are charge and mass of the particle, E is the electric field and s is the distance travelled by the particle in time t. Thus, the trajectory will be a parabolic path.

Q. 6. A long straight wire AB carries a current I. A proton P travels with a speed v, parallel to the wire, at a distance d from it in a direction opposite to the current as shown in the figure. What is the force experienced by the proton and what is its direction? [CBSE (AI) 2010]



Ans. Magnetic field due to current carrying wire is perpendicular to plane of paper – downward.

$$i.e., \ \vec{B} = -\frac{\mu_0 I}{2\pi d}\hat{k}$$

$$\text{Force } \vec{F} = q\vec{v} \times \vec{B} = e(-v\hat{j}) \times \left(-\frac{\mu_0 I}{2\pi d}\hat{k}\right) = \frac{\mu_0 \,\mathrm{evI}}{2\pi d}\hat{i}$$

That is the magnetic force has magnitude $\frac{\mu_0 \,\mathrm{evI}}{2\pi d}$ and is directed along positive X-axis *i.e.,* in the plane of paper perpendicular to direction of $\vec{v}$ and to the right.

Q. 7. Two long and parallel straight wires carrying currents of 2 A and 5 A in the opposite directions are separated by a distance of 1 cm. Find the nature and magnitude of the magnetic force between them. [CBSE (F) 2011]

Ans. I_1=2 A, I_2 =5A, a=1 cm = 1 × 10^{-2} m

Force between two parallel wires per unit length is given by

$$F = \frac{\mu_0}{2\pi} . \frac{I_1 I_2}{a} = 2 \times 10^{-7} \times \frac{2 \times 5}{1 \times 10^{-2}} = 20 \times 10^{-5} N(\text{Repulsive})$$

Q. 8. A short bar magnet of magnetic moment 0.9 J/T is placed with its axis at 30° to a uniform magnetic field. It experiences a torque of 0.063 J.

(i) Calculate the magnitude of the magnetic field.

(ii) In which orientation will the bar magnet be in stable equilibrium in the magnetic field? [CBSE (F) 2012]

Ans. (i)

We know $\vec{\tau} = \overrightarrow{M} \times \overrightarrow{B}$

or $\tau = M B \sin \theta$

$$0.063 = 0.9 \times B \times \sin 30^\circ$$

or $B = 0.14$ T

(ii) The position of minimum energy corresponds to position of stable equilibrium.

The energy (U) = –MB cos θ

When θ = 0° ⇒ U = – MB = Minimum energy

Hence, when the bar magnet is placed parallel to the magnetic field, it is the state of stable equilibrium.

Q. 9. A magnetised needle of magnetic moment 4.8 × 10–2 J T–1 is placed at 30° with the direction of uniform magnetic field of magnitude 3 × 10–2 T. Calculate the torque acting on the needle. [CBSE (F) 2012]

Ans.

We have, $\tau = M B \sin\theta$

where $\tau \rightarrow$ Torque acting on magnetic needle

$M \rightarrow$ Magnetic moment

$B \rightarrow$ Magnetic field strength

$$\text{Then}\ \tau = 4.8 \times 10^{-2} \times 3 \times 10^{-2} \sin 30^0 = 4.8 \times 10^{-2} \times 3 \times 10^{-2} \times \frac{1}{2}$$

$$\tau = 7.2 \times 10^{-4}\ \text{Nm}$$

Q. 10. State two reasons why a galvanometer cannot be used as such to measure current in a given circuit. [CBSE Delhi 2010]

OR

Can a galvanometer as such be used for measuring the current? Explain.

Ans. A galvanometer cannot be used as such to measure current due to following two reasons.

(i) A galvanometer has a finite large resistance and is connected in series in the circuit, so it will increase the resistance of circuit and hence change the value of current in the circuit.

(ii) A galvanometer is a very sensitive device, it gives a full scale deflection for the current of the order of microampere, hence if connected as such it will not measure current of the order of ampere.

Short Answer Questions – I (OIQ)

Q. 1. An electron beam passes through a region of crossed electric and magnetic fields of strength E and B respectively. For what value of electron-speed the beam will remain undeflected?

Ans. The electron beam will pass undeflected if electric force and magnetic force on electron is equal and opposite i.e., eE = evB

or $v = \frac{E}{B}$

Q. 2. An α–particle and a proton are moving in the plane of paper in a region where there is a uniform magnetic field directed normal to the plane of the paper. If the particles have equal linear momenta, what would be the ratio of the radii of their trajectories in the field?

Ans.

Radius of circular path of a charged particle, $r = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{P}{\mathrm{qB}}$.

For same linear momentum and magnetic field B,

$$r \propto \frac{1}{q}$$

$$\therefore \quad \frac{r_\alpha}{r_p} = \frac{q_p}{q_\alpha} = \frac{+e}{+2e} = \frac{1}{2}$$

Q. 3. A particle of mass 'm', with charge 'q' moving with a uniform speed 'v', normal to a uniform magnetic field 'B', describes a circular path of radius 'r'. Derive expressions for the (i) time period of revolution and (ii) kinetic energy of the particle.

Ans. (i) Motion of charged particle in perpendicular magnetic field: The magnetic force on charged particle qvB sin 90° provides the necessary centripetal force for a circular path, so

$$\mathrm{qvB} = \frac{\mathrm{mv}^2}{r} \Rightarrow v = \frac{\mathrm{qBr}}{m}$$

But $v = \frac{2\pi r}{T}$ where T is time period

$$\frac{2\pi r}{T} = \frac{\mathrm{qBr}}{m} \Rightarrow T = \frac{2\pi m}{\mathrm{qB}}$$

***(ii)* Kinetic energy of charged particle:**

$$\mathrm{KE} = \frac{1}{2}\mathrm{mv}^2 \qquad \Rightarrow \qquad \frac{1}{2}m\frac{q^2B^2r^2}{m^2} = \frac{q^2B^2r^2}{2m}$$

Q. 4. You are given a low resistance R_1 a high resistance R_2 and a moving coil galvanometer. Suggest how would you use these to have an instrument that will be able to measure

(i) Current
(ii) Potential difference.

Ans. (i) To measure current we shall connect low resistance R_1 in parallel with the coil of moving coil galvanometer. This arrangement is called ammeter.

(ii) To measure the potential difference we shall connect high resistance R_2 in series with the coil of galvanometer. This arrangement is called voltmeter.

Q. 5. If a particle of charge q is moving with velocity v along X-axis and the magnetic field B is acting along Y-axis, use the expression $\vec{F} = q(\vec{v} \times \vec{B})$ to find the direction of force $\vec{F}$ acting on it.

Ans.

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\text{Given } \vec{v} = v\, i,\ \vec{B} = B\,\hat{j}$$

$$\therefore \quad \vec{F} = q(v\hat{i} \times (B\hat{j}) = \text{qvB}\ \hat{k}$$

That is, force is acting along Z-axis.

Q. 6. The velocities of two α-particles A and B entering a uniform magnetic field are in the ratio 4 : 1. On entering the field they move in different circular paths. Give the ratio of the radii of curvature of the paths of the particles.

Ans.

$$\text{Radius of circular path } r = \frac{\text{mv}}{\text{qB}}\ \ i.e.\ \ r \propto v$$

$$\therefore \frac{r_A}{r_B} = \frac{v_A}{v_B} = \frac{4}{1}$$

Q. 7. An ammeter and a milliammeter are converted from the same galvanometer. Out of the two, which current measuring instrument has a higher resistance?

Ans.

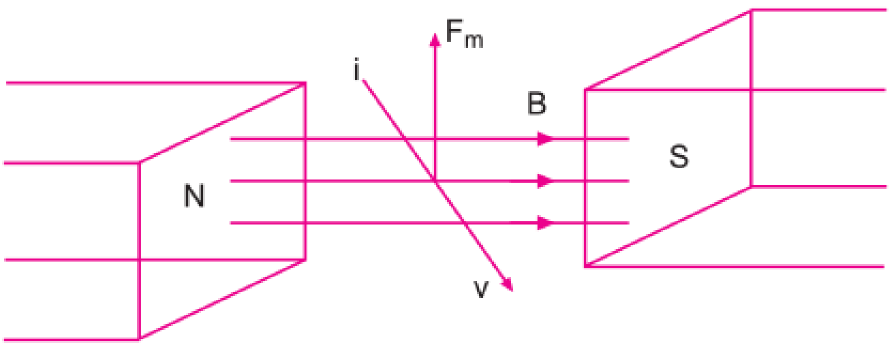
$$\text{Shunt resistance, } S = \frac{I_g}{I - I_g} G \approx \frac{I_g}{I} G$$

Clearly, smaller the value of range, larger is the shunt resistance. Obviously, milliammeter will have a larger shunt resistance and hence it will have a higher resistance.

$$\frac{1}{R_A} = \frac{1}{G} + \frac{1}{S}$$

Higher the S, higher the R_A for given G.

Q. 8. A charged particle enters into a uniform magnetic field and experiences an upward force as indicated in the figure. What is the charge sign on the particle? [HOTS]



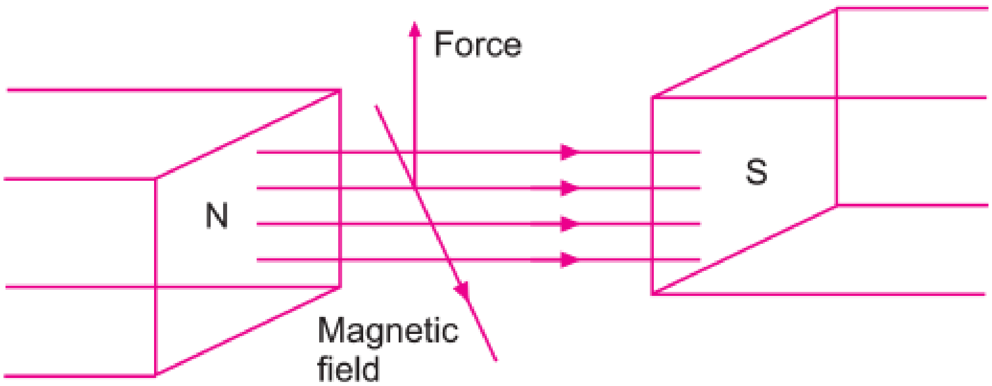
Ans. Positive Charge: By Fleming left hand rule the direction of current is along positive Z-axis. By vector method

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

$$F_m \; \hat{j} = q\, v\, \hat{k} \times B\, \hat{i}$$

$$F_m \; \hat{j} = \text{qvB} \; \hat{j}$$

This shows that q is positive.



Q. 9. A given galvanometer is to be converted into (i) an ammeter (ii) a milliammeter (iii) a voltmeter. In which case will the required resistance be (i) least (ii) highest and why? [HOTS]

Ans. The required resistance has least value in the case of an ammeter and maximum value in the case of a voltmeter.

This is due to the reason that the shunt resistance required to convert a galvanometer into ammeter or milliammeter has the value

$$S = \frac{I_g}{I - I_g} \times R_g$$

Thus, the shunt required in the case of milliammeter has higher value.

Similarly, since the voltmeter should have a high resistance, the value of required resistance should be highest in the case of a voltmeter. This is connected in series with the coil of the galvanometer.

Short Answer Questions – II (PYQ)

Q. 1. Write any two important points of similarities and differences each between Coulomb's law for the electrostatic field and Biot-Savart's law for the magnetic field.
[CBSE (F) 2015]

Ans. Similarities:

Both electrostatic field and magnetic field:

Follows the principle of superposition.

Depends inversely on the square of distance from source to the point of interest.

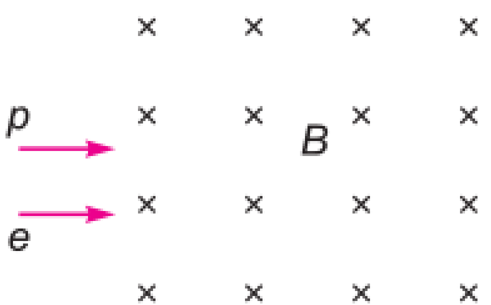
Differences:

(i) Electrostatic field is produced by a scalar source (q) and the magnetic field is produced by a vector (I $\overrightarrow{dI}$).

(ii) Electrostatic field is along the displacement vector between source and point of interest; while magnetic field is perpendicular to the plane, containing the displacement vector and vector source.

(iii) Electrostatic field is angle independent, while magnetic field is angle dependent between source vector and displacement vector.

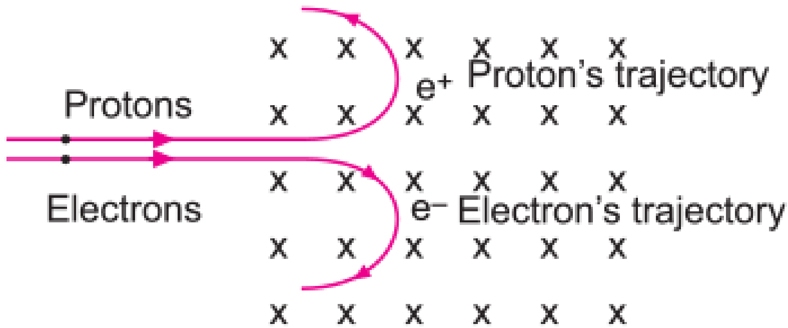
Q. 2. An electron and a proton enter a region of uniform magnetic field B with uniform speed v in a perpendicular direction (fig.).



(i) Show the trajectories followed by two particles.

(ii) What is the ratio of the radii of the circular paths of electron to proton?
[CBSE (F) 2010]

Ans. (i) Trajectories are shown in figure.



(ii)

As $r = \frac{\mathrm{mv}}{\mathrm{qB}} \rightarrow r \propto m$

Ratio of radii of electron path and proton path.

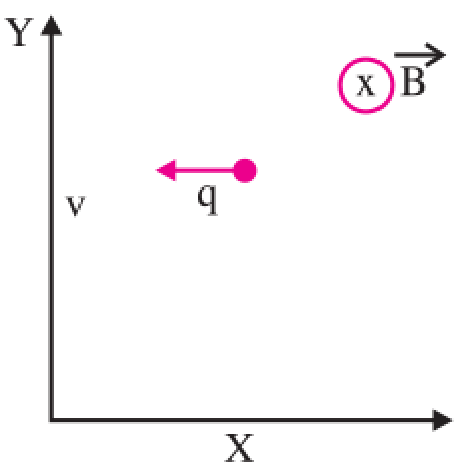
$$\frac{r_e}{r_p} = \frac{m_e}{m_p}$$

As mass of proton $m_p \approx 1840\times$ mass of electron (m_e)

$$\therefore \frac{m_e}{m_p} \approx \frac{1}{1840} \quad \text{and} \quad \frac{r_e}{r_p} = \frac{1}{1840}$$

Q. 3. Answer the following questions

(i) A point charge q moving with speed v enters a uniform magnetic field B that is acting into the plane of the paper as shown. What is the path followed by the charge q and in which plane does it move?



(ii) How does the path followed by the charge get affected if its velocity has a component parallel to $\vec{B}$?

(iii) If an electric fieldis also applied such that the particle continues moving along the original straight line path, what should be the magnitude and direction of the electric field [CBSE (F) 2016]

Ans. (i) The force experienced by the charge particle is given by $\vec{F}$ = q ($\vec{V} \times \vec{B}$)

When $\vec{V}$ is perpendicular to $\vec{B}$ the force on the charge particle acts as the centripetal force and makes it move along a circular path. Path followed by charge is anticlockwise in X-Y plane. The point charge moves in the plane perpendicular to both $\vec{V}$ and $\vec{B}$.

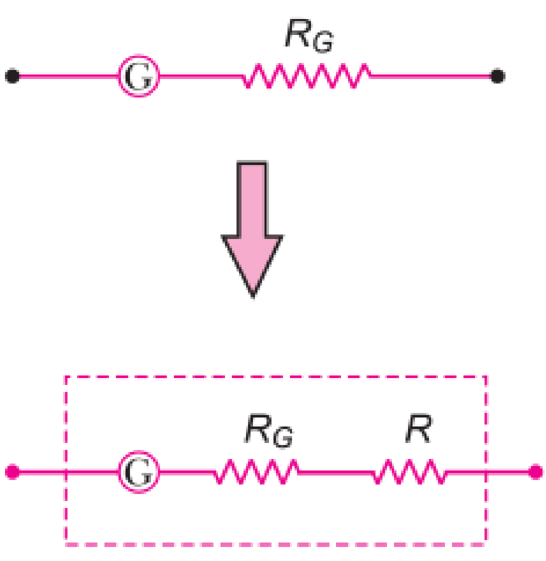
Q. 4. How is a galvanometer converted into a voltmeter and an ammeter? Draw the relevant diagrams and find the resistance of the arrangement in each case. Take resistance of galvanometer as G. [CBSE East 2016]

Ans. A galvanometer is converted into a voltmeter by connecting a high resistance 'R' in series with it.

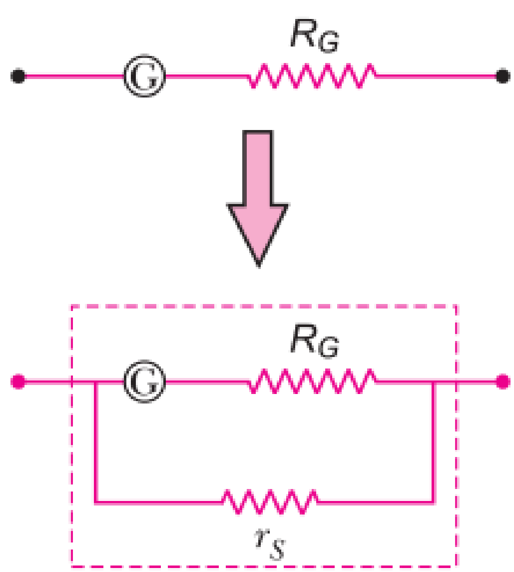
A galvanometer is converted into an ammeter by connecting a small resistance (called shunt) in parallel with it.

Resistance of voltmeter, R_V = G + R

Resistance of Ammeter, $R_A = \frac{Gr_s}{G+r_s}$



Voltmeter

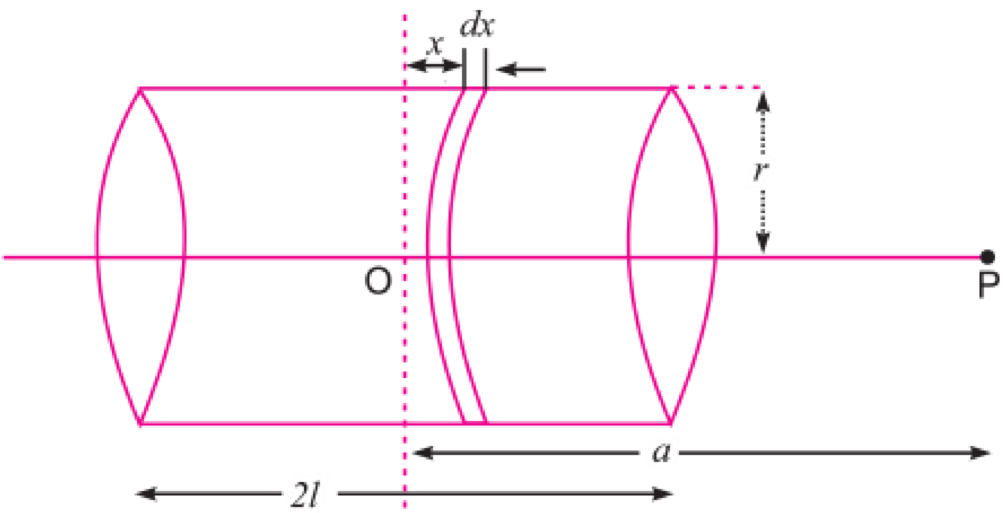


Ammeter

Q. 5. Derive an expression for the axial magnetic field of a finite solenoid of length 2l and radius r carrying current I. Under what condition does the field become equivalent to that produced by a bar magnet? [CBSE South 2016]

Ans. Consider a solenoid of length 2l, radius r and carrying a current I and having n turns per unit length.

Consider a point P at a distance a from the centre O of solenoid. Consider an element of solenoid of length dx at a distance x from its centre. This element is a circular current loop having (ndx) turns. The magnetic field at axial point P due to this current loop is



$$dB = \frac{\mu_0 (\text{ndx}) \text{Ir}^2}{2\{r^2+(a-x)^2\}^{3/2}}$$

The total magnetic field due to entire solenoid is

$$\therefore \quad B = \int_{-l}^{+l} \frac{\mu_0 (\text{ndx}) \text{Ir}^2}{2\{r^2+(a-x)^2\}^{3/2}}$$

For $a >> l$ and $a >> r$, we have $\{r^2+(a-x)^2\}^{3/2} = a^3$

$$\therefore \quad B = \frac{\mu_0 \text{nIr}^2}{2a^3} \int_{-l}^{+l} \text{dx} = \frac{\mu_0 \text{nIr}^2 (2l)}{2a^3}$$

The magnetic moment of solenoid $m \ (= NIA) = (n2l)I \,.\, \pi r^2$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{2m}{a^3}$$

This is also the far axial magnetic field of a bar magnet. Hence, the magnetic field, due to current carrying solenoid along its axial line is similar to that of a bar magnet for far off axial points.

Q. 6. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, calculate the kinetic energy (in MeV) of the proton beam produced by the accelerator.
[CBSE Ajmer 2015]

Ans. The oscillator frequency should be same as proton cyclotron frequency, then

Magnetic field,

$$B = \frac{2\pi\ \mathrm{mv}}{q}$$

$$= \frac{2\times3.14\times1.67\times10^{-27}\times10^{7}}{1.6\times10^{-19}} = 0.66\,T$$

$$v = r\omega = r \times 2\pi\nu$$

$$= 0.6 \times 2 \times 3.14 \times 10^{7} = 3.78 \times 10^{7}\,\mathrm{m/s}$$

$$\text{So,}\quad \text{Kinetic energy,}\ \ \mathrm{KE} = \frac{1}{2}\mathrm{mv}^2$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (3.78 \times 10^{7})^2 J$$

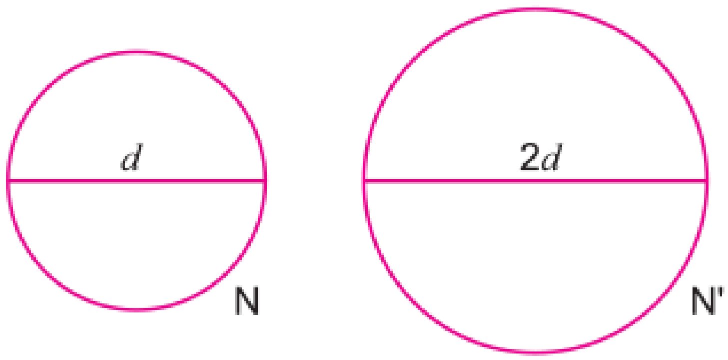
$$= \frac{1}{2} \times \frac{1.67\times10^{-27}\times14.3\times10^{14}}{1.6\times10^{-19}\times10^{6}}\,\mathrm{MeV} = 7.4\ \ \mathrm{MeV}$$

Q. 7. A circular coil of 'N' turns and diameter 'd' carries a current 'I'. It is unwound and rewound to make another coil of diameter '2d', current 'I' remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.
[CBSE (AI) 2012]

Ans. We know,

Magnetic moment (m) = NIA

Where N = Number of turns



Then, length of wire remains same

$$\text{Thus, } N \times \left[2\pi\left(\frac{d}{2}\right)\right] = N'\left[2\pi\left(\frac{2d}{2}\right)\right]$$

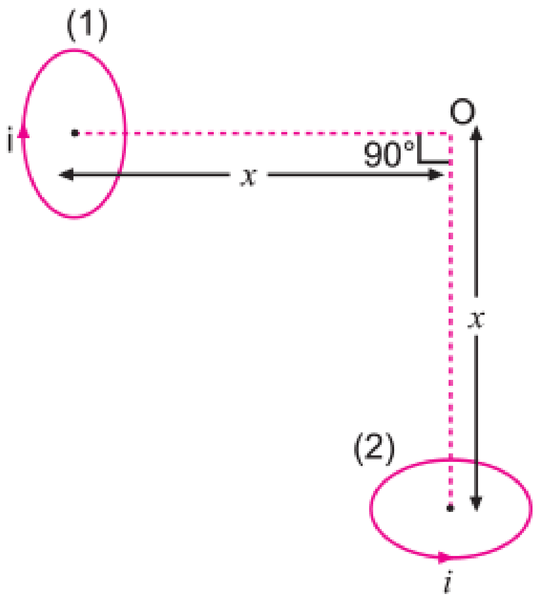
$$\Rightarrow \quad N' = \frac{N}{2}$$

$$\text{Now, } m_A = \text{NIA}_A = \text{NI}\left(\pi r_A^2\right) = \frac{1}{4}\text{NI}\,\pi d^2$$

$$\text{Similarly } m_B = N'I\,A_B = \frac{\text{NI}}{2}\left(\pi r_B^2\right) = \frac{1}{2}\left(\text{NI}\,\pi d^2\right)$$

$$\frac{m_B}{m_A} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{2}{1} \quad \Rightarrow \quad \frac{m_B}{m_A} = \frac{2}{1}$$

Q. 8. Two small identical circular loops, marked (1) and (2), carrying equal currents, are placed with the geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O. [CBSE (F) 2013, 2014]



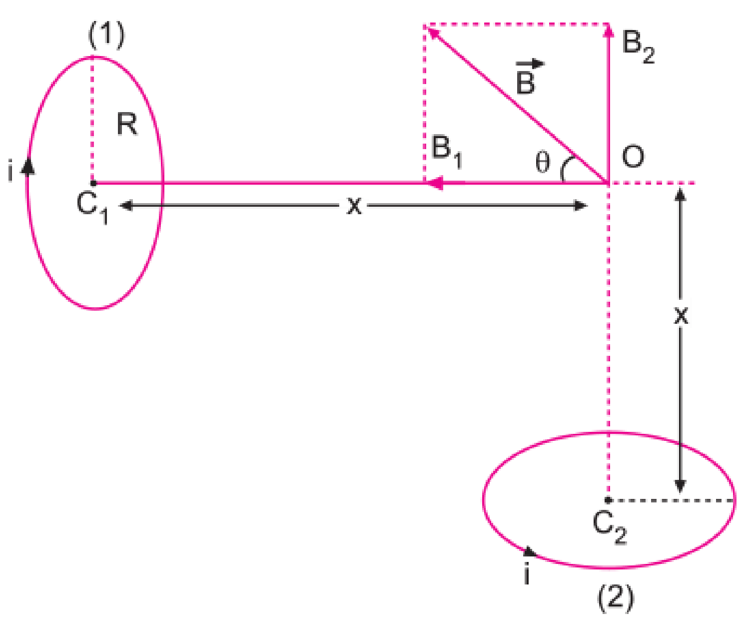
Ans. Magnetic field due to coil 1 at point O

$$\overrightarrow{B_1} = \frac{\mu_0 \text{ iR}^2}{2(R^2+x^2)^{3/2}} \text{ along } \overrightarrow{\text{OC}_1}$$

Magnetic field due to coil 2 at point O

$$\overrightarrow{B_2} = \frac{\mu_0 \text{ iR}^2}{2(R^2+x^2)^{3/2}} \text{along } \overrightarrow{C_2 O}$$

Both $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are mutually perpendicular, so the net magnetic field at O is



$$B=\sqrt{B_1^2+B_2^2}=\sqrt{2}B_1\,(\text{as } B_1=B_2)$$

$$=\sqrt{2}\frac{\mu_0\,\mathrm{i}R^2}{2(R^2+x^2)^{3/2}}$$

As $R << x$

$$B=\frac{\sqrt{2}\mu_0\,\mathrm{i}R^2}{2.x^3}=\frac{\mu_0}{4\pi}.\ \frac{2\sqrt{2}.\mu_0 i\,(\pi R^2)}{x^3}$$

$$=\frac{\mu_0}{4\pi}\ \frac{2\sqrt{2}.\mu_0 i\,A}{x^3}$$

where $A=\pi R^2$ is area of loop.

$$\tan\,\theta=\frac{B_2}{B_1}\quad\Rightarrow\quad\tan\,\theta=1\qquad(\because B_2=B_1)$$

$$\Rightarrow\qquad\theta=\frac{\pi}{4}$$

where $A=\pi R^2$ is area of loop.

$$\tan\,\theta=\frac{B_2}{B_1}\quad\Rightarrow\quad\tan\,\theta=1\qquad(\because B_2=B_1)$$

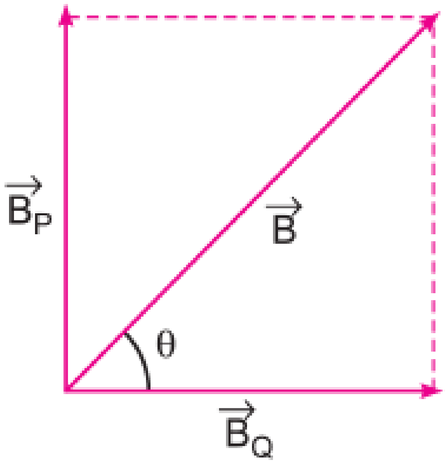
$$\Rightarrow\qquad\theta=\frac{\pi}{4}$$

$\therefore\ \overrightarrow{B}$ is directed at an angle $\frac{\pi}{4}$ with the direction of magnetic field $\overrightarrow{B_1}$.

Q. 9. Two identical coils P and Q each of radius R are lying in perpendicular planes such that they have a common centre. Find the magnitude and direction of magnetic field at the common centre of the two coils, if they carry currents equal to I and $-\sqrt{3}$ I respectively. [CBSE (F) 2016] [HOTS]

Ans. Given that two identical coils are lying in perpendicular planes and having common centre. P and Q carry current I and $\sqrt{3}$ I respectively.

Now, magnetic field at the centre of P due to its current, I



$$\overrightarrow{B_P} = \frac{\mu_0 I}{2R}$$

And, magnetic field at centre of Q, due to its current $\sqrt{3}\ I$

$$\overrightarrow{B_Q} = \frac{\mu_0 \sqrt{3} I}{2R}$$

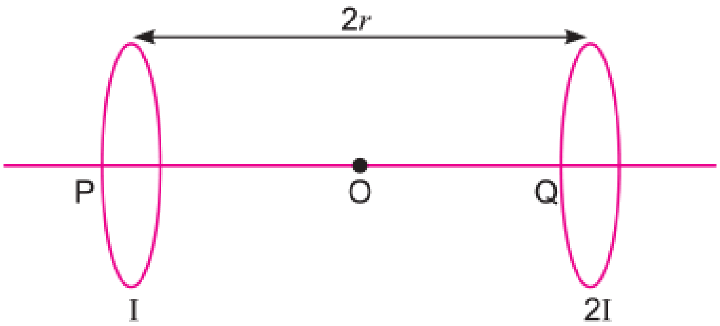
$$\therefore\ \overrightarrow{\overrightarrow{B_{\text{net}}}} = \overrightarrow{B_P} + \overrightarrow{B_Q} = \frac{\mu_0 I}{2R} + \frac{\mu_0 \sqrt{3} I}{2R}$$

$$\therefore\ |\overrightarrow{\overrightarrow{B_{\text{net}}}}| = \sqrt{\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0 \sqrt{3} I}{2R}\right)^2} = \frac{\mu_0 I}{2R} \times 2 = \frac{\mu_0 I}{R}$$

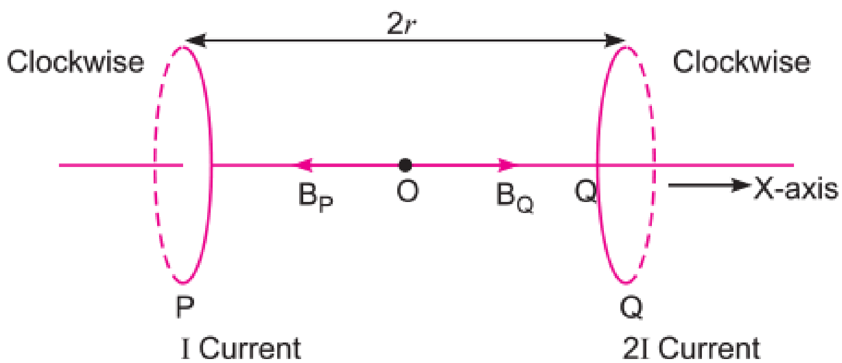
$$\therefore\quad \tan\theta = \frac{|\overrightarrow{B_P}|}{|\overrightarrow{B_Q}|} = \left(\frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 \sqrt{3} I}{2R}}\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow\qquad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Q. 10. Two identical circular loops, P and Q, each of radius r and carrying currents I and 2I respectively are lying in parallel planes such that they have a common axis. The direction of current in both the loops is clockwise as seen from O which is equidistant from the both loops. Find the magnitude of the net magnetic field at point O.
[CBSE (AI) 2012] [HOTS]



Ans.



$$|\overrightarrow{B_P}| = \frac{\mu_0 r^2 I}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}r} \text{ Pointing towards } P$$

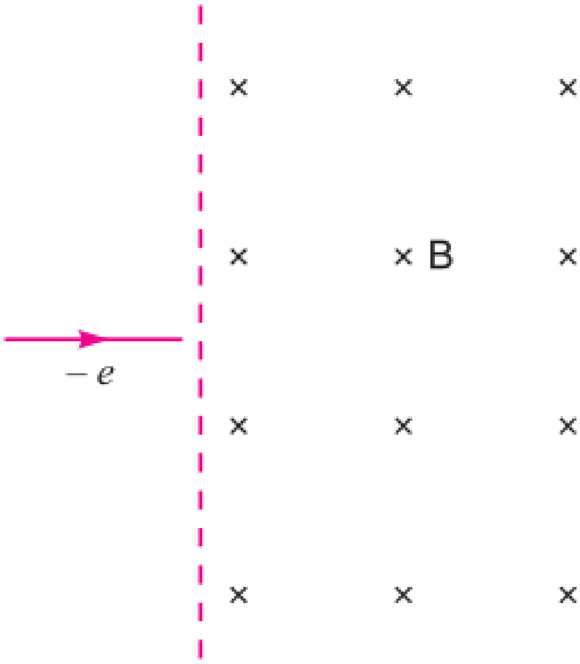
$$|\overrightarrow{B_Q}| = \frac{\mu_0 (2I) r^2}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 2I}{4\sqrt{2}r} \text{ Pointing towards } Q$$

$$|\vec{B}| = |\overrightarrow{B_Q}| - |\overrightarrow{B_P}| = \frac{\mu_0 I}{4\sqrt{2}r}$$

So, magnetic field at point O has a magnitude $\frac{\mu_0 I}{4\sqrt{2}r}$.

Q. 11. Answer the following questions

(i) An electron moving horizontally with a velocity of 4 × 10^4 m/s enters a region of uniform magnetic field of 10^{-5} T acting vertically upward as shown in the figure. Draw its trajectory and find out the time it takes to come out of the region of magnetic field.



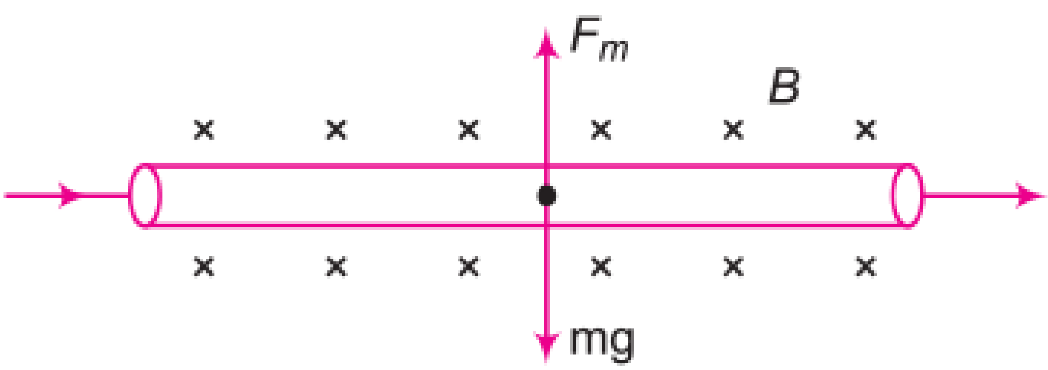
(ii) A straight wire of mass 200 g and length 1.5 m carries a current of 2A. It is suspended in midair by a uniform magnetic field B. What is the magnitude of the magnetic field? [CBSE (F) 2015] [HOTS]

Ans. If Ampere's force acts in upward direction and balances the weight, that is,

$F_m = mg$

$$B = \frac{mg}{Il} = \frac{0.2\times10}{2\times1.5} = \frac{2}{3} = 0.67$$

$$BIl = mg$$

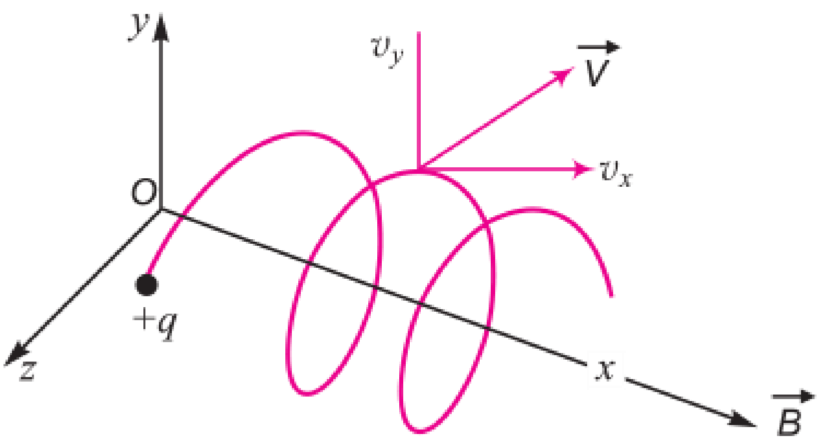


$$B = \frac{mg}{Il} = \frac{0.2\times10}{2\times1.5} = \frac{2}{3} = 0.67$$

Q. 12. A uniform magnetic field $\vec{B}$ is set up along the positive x-axis. A particle of charge 'q' and mass 'm' moving with a velocity $\vec{V}$ enters the field at the origin in X-Y plane such that it has velocity components both along and perpendicular to the magnetic field $\vec{B}$. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. [CBSE Allahabad 2015] [HOTS]

Ans. If component v_x of the velocity vector is along the magnetic field, and remain constant, the charge particle will follow a helical trajectory; as shown in fig.

If the velocity component v_y is perpendicular to the magnetic field B, the magnetic force acts like a centripetal force $qv_y B$.



So, $qv_yB = \frac{mv_y^2}{r} \quad \Rightarrow \quad v_y = \frac{qBr}{m}$

Since tangent velocity $v_y = r\omega$

$\Rightarrow \quad r\omega = \frac{qBr}{m} \quad \Rightarrow \quad \omega = \frac{qB}{m}$

Time taken for one revolution, $T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$

and the distance moved along the magnetic field in the helical path is

$x = v_x . T = v_x . \frac{2\pi m}{qB}$

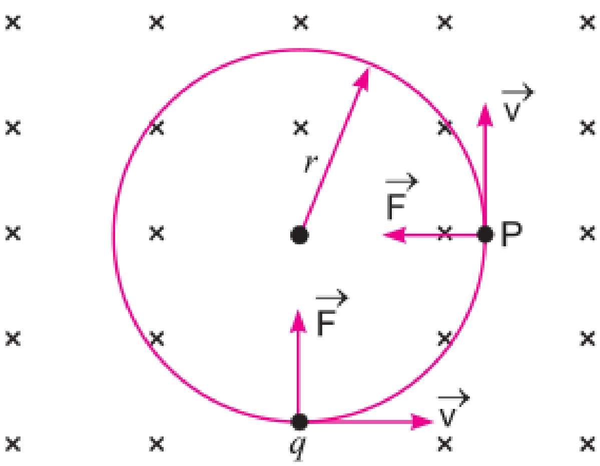
Q. 13. A particle of charge 'q' and mass 'm' is moving with velocity $\underset{V}{\rightarrow}$. It is subjected to a uniform magnetic field $\underset{B}{\rightarrow}$ directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius. [CBSE (F) 2012] [HOTS]

Ans. When a particle of charge 'q' of mass 'm' is directed to move perpendicular to the uniform magnetic field 'B' with velocity '$\underset{V}{\rightarrow}$'

The force on the charge

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This magnetic force acts always perpendicular to the velocity of charged particle. Hence magnitude of velocity remains constant but direction changes continuously. Consequently the path of the charged particle in a perpendicular magnetic field becomes circular. The magnetic force (qvB) provides the necessary centripetal force to move along a circular path. Then,



$$\mathrm{qvB} = \frac{\mathrm{mv}^2}{r} \quad \Rightarrow \quad r = \frac{\mathrm{mv}}{\mathrm{qB}}$$

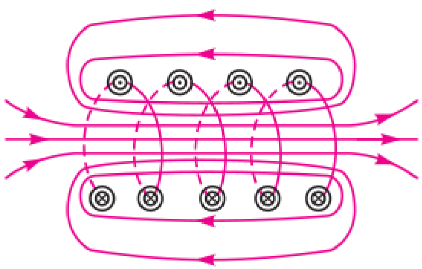
Here r = radius of the circular path followed by the charge.

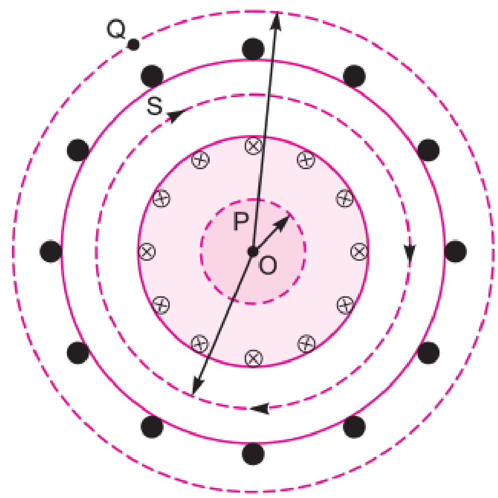
Q. 14. Answer the following questions

(i) In what respect is a toroid different from a solenoid? Draw and compare the pattern of the magnetic field lines in the two cases. [CBSE (AI) 2011]

(ii) How is the magnetic field inside a given solenoid made strong? [CBSE (AI) 2011]

Ans. (i) A toroid is a solenoid bent into the form of a closed ring. The magnetic field lines of solenoid are straight lines parallel to the axis inside the solenoid.





(ii) The magnetic field lines of toroid are circular having common centre.

Inside a given solenoid, the magnetic field may be made strong by (i) passing large current and (ii) using laminated coil of soft iron.

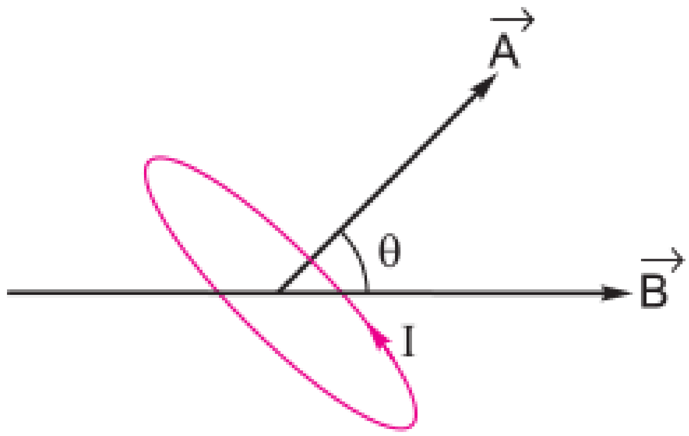
Q. 15. Answer the following questions.

(i) (a) A circular loop of area $\vec{B}$, carrying a current I is placed in a uniform magnetic field $\vec{B}$ Write the expression for the torque $\vec{t}$ acting on it in a vector form.

(b) If the loop is free to turn, what would be its orientation of stable equilibrium? Show that in this orientation, the flux of net field (external field + the field produced by the loop) is maximum.

(ii) Find out the expression for the magnetic field due to a long solenoid carrying a current I and having n number of turns per unit length.

Ans. (i)



i. Torque acting on the current loop $\vec{\tau} = \vec{m} \times \vec{B} = I(\vec{A} \times \vec{B})$

ii. If magnetic moment $\vec{m} = I\vec{A}$ is in the direction of external magnetic field *i.e.,* θ=0°.

Magnetic flux $\varphi_B = (\overrightarrow{B^{\text{ext}}} + B_C) \cdot \vec{A}$

$$\varphi_{\max} = \left[\left| \overrightarrow{B^{\text{ext}}} \right| + \frac{\mu_0 I}{2r} \right] |A| \cos 0^\circ$$

where r is radius of the loop.

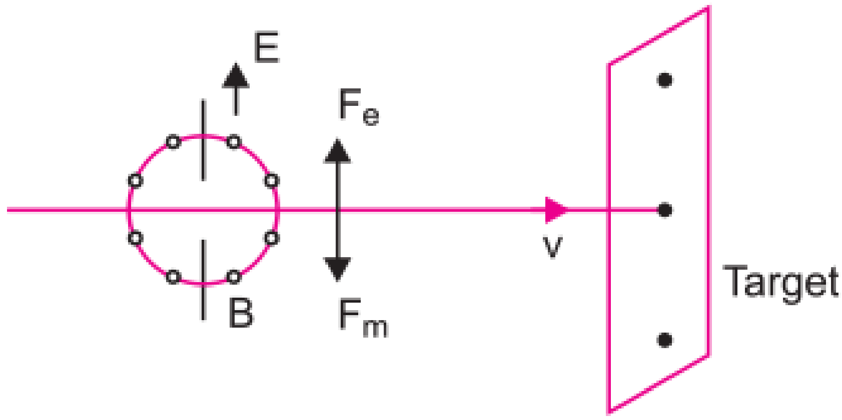
Q. 16. A beam of protons passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of the beam. If the magnitudes of the electric and magnetic fields are 50 kV/m and 100 mT respectively; calculate the [CBSE (AI) 2008] [HOTS]

(i) Velocity of the beam.

(ii) Force with which it strikes a target on the screen, if the proton beam current is equal to 0.80 mA.

Ans. (i) For a beam of charged particles to pass undeflected crossed electric and magnetic fields, the condition is that electric and magnetic forces on the beam must be equal and opposite i.e.,

$$\text{eE} = \text{evB} \quad \Rightarrow \quad v = \frac{E}{B}$$



Given, $E = 50\text{kV/m} = 50\times10^{3}\ \text{V/m}$

$B = 100\ \text{mT} = 100 \times 10^{-3}\ \text{T}$

$$\therefore \quad v = \frac{50\times10^{3}}{100\times10^{-3}} = 5\times10^{5}\ \text{ms}^{-1}$$

Q. 17. Answer the following questions.

(i) Obtain the expression for the cyclotron frequency.

(ii) A deuteron and a proton are accelerated by the cyclotron. Can both be accelerated with the same oscillator frequency? Give reason to justify your answer.
[CBSE Delhi 2017]

Ans. (i) Suppose the positive charge ion with charge q moves in a dee with a velocity v, then

$$\frac{\text{mv}^2}{r} = \text{qvB} \qquad \Rightarrow r = \frac{\text{mv}}{\text{qB}}$$

$$\text{Frequency of revolution } \nu = \frac{1}{\text{Time Period } (T)} = \frac{v}{2\pi r} \qquad \left(\because \ T = \frac{2\pi r}{v}\right)$$

$$\nu = \frac{\text{qB}}{2\pi m}$$

(ii) No.

The mass of the two particles, i.e., deuteron and proton, is different. Since cyclotron frequency depends inversely on the mass, they cannot be accelerated by the same oscillator frequency.

Q. 18. Answer the following questions

(i) Write the expression for the force $\underset{F}{\rightarrow}$ acting on a particle of mass m and charge q moving with velocity $\underset{V}{\rightarrow}$ in a magnetic field $\underset{B}{\rightarrow}$ Under what conditions will it move in (a) a circular path and (b) a helical path? [CBSE Delhi 2017]

(ii) Show that the kinetic energy of the particle moving in magnetic field remains constant. [CBSE Delhi 2017]

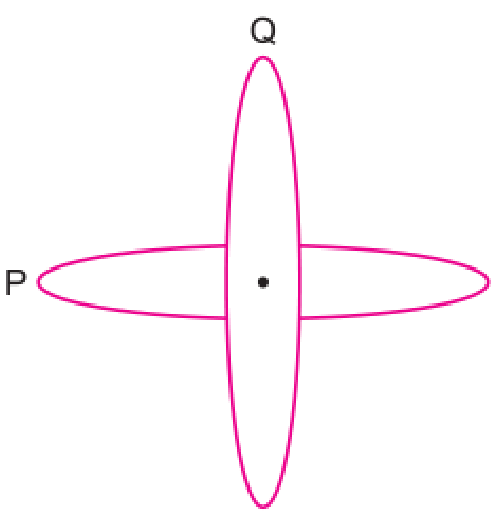
Ans. (i) i. $\overrightarrow{F} = q(\overrightarrow{v} \times \overrightarrow{B})$

(a) When velocity of charged particle and magnetic field are perpendicular to each other charged particle will move in circular path.

(b) When velocity is neither parallel nor perpendicular to the magnetic field charged particle will move in a helical path.

(ii) The force, experienced by the charged particle is perpendicular to the instantaneous velocity, $\underset{V}{\rightarrow}$ at all instants. Hence the magnetic force cannot bring any change in the speed of the charged particle. Since speed remain constant, the kinetic energy also remains constant.

Q. 19. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3A and 4A respectively. [CBSE (AI) 2017]



Ans.

Magnetic field at the centre of a circular coil = $\frac{\mu_0 I}{2R}$

Magnetic field due to coil P,

$$B_P = \frac{\mu_0 \times 3}{2 \times 5 \times 10^{-2}} T = 12\pi \times 10^{-6} T$$

Magnetic field due to coil Q,

$$B_Q = \frac{\mu_0 \times 4}{2 \times 5 \times 10^{-2}} T = 16\pi \times 10^{-6} T$$

Net magnetic field, $B = \sqrt{B_P^2 + B_Q^2}$

$$= \sqrt{(12\pi \times 10^{-6})^2 + (16\pi \times 10^{-6})^2}$$

$$= \pi\sqrt{144 + 256} \times 10^{-6} T = 20\pi \times 10^{-6} T$$

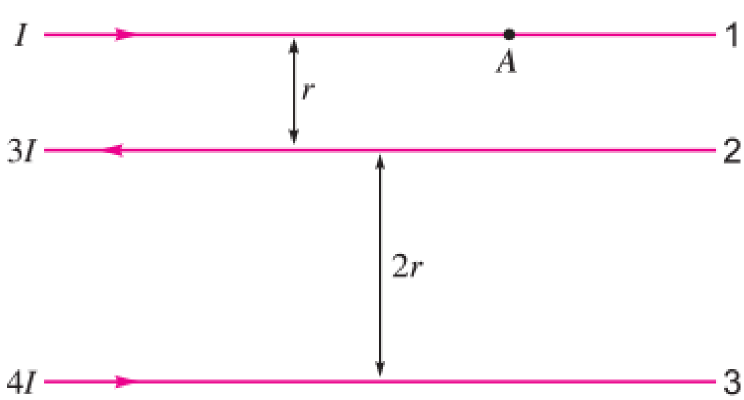
Let the field make an angle θ with the magnetic field due to Q.

$$\tan\theta = \frac{12\pi \times 10^{-6}}{16\pi \times 10^{-6}} = \frac{3}{4} \qquad \Rightarrow \theta = \tan^{-1} = \frac{3}{4}$$

Q. 20. The figure shows three infinitely long straight parallel current carrying conductors. Find the

(i) Magnitude and direction of the net magnetic field at point A lying on conductor 1,

(ii) Magnetic force on conductor 2.



Ans. (i)

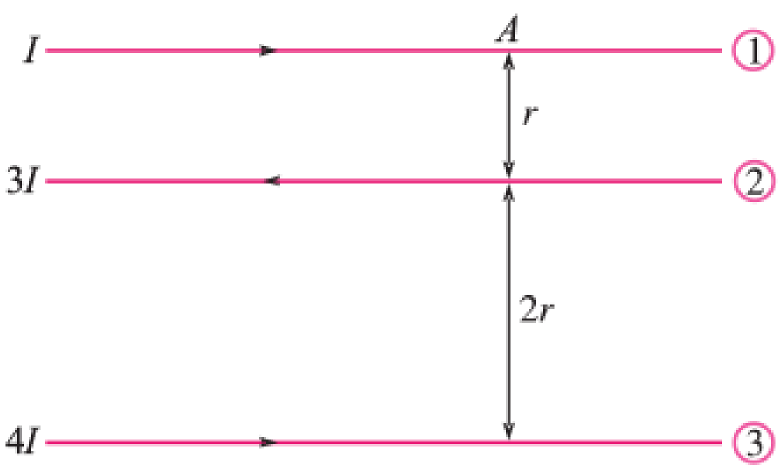
$$B_2 = \frac{\mu_0}{4\pi}\frac{2(3I)}{r} = \frac{\mu_0(6I)}{4\pi r} \text{ into the plane of the } paper.$$

$$B_3 = \frac{\mu_0}{4\pi}\frac{2(4I)}{3r} = \frac{\mu_0}{4\pi}\left(\frac{8I}{3r}\right) \text{ out of the plane of the paper.}$$

$B_A = B_2 - B_3$ into the paper.

$$= \frac{\mu_0}{4\pi}\frac{10I}{3r} \text{ into the } paper.$$

(ii)



Magnetic force per unit length on wire (2)

$$F = \frac{\mu_0}{2\pi r}.3I^2 - \frac{\mu_0 12I^2}{2\pi(2r)}$$

$$= \frac{3}{2}\frac{\mu_0 I^2}{\pi r} - 3\frac{\mu_0 I^2}{\pi r} = -\frac{3}{2}\frac{\mu_0 I^2}{\pi r}$$

Hence, $F = \frac{3}{2}\frac{\mu_0 I^2}{\pi r}$ in the direction of wire 1.

Q. 21. Answer the following questions.

(i) State the condition under which a charged particle moving with velocity v goes undeflected in a magnetic field B.

(ii) An electron, after being accelerated through a potential difference of 10^4 V, enter a uniform magnetic field of 0.04T, perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory. [CBSE (AI) 2017]

Ans. (i)

Force in magnetic field on a charged particle

$$\vec{F} = q(\vec{v} \times \vec{B}) \qquad \Rightarrow F \; \mathrm{qvB} \; \sin \theta$$

If $F = 0$,

$$\Rightarrow \qquad 0 = qvB \sin \theta$$

$$\Rightarrow \sin \theta = 0 \qquad \theta = \pm n\pi$$

So, magnetic field will be parallel or antiparallel to the velocity of charged particle.

(ii)

For a charged particle moving in a constant magnetic field and $\vec{v} \perp \vec{B}$

$$\frac{\mathrm{mv}^2}{r} = \mathrm{qvB} \qquad \Rightarrow \qquad r = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{P}{\mathrm{qB}} \qquad \ldots(i)$$

If e is accelerated through a potential difference of 10^4 V, then

K. E of electron $= eV$

$$\Rightarrow \qquad \frac{P^2}{2m} = \mathrm{eV} \qquad \Rightarrow P = \sqrt{2\,\mathrm{meV}} \quad \ldots(ii)$$

From (i) & (ii)

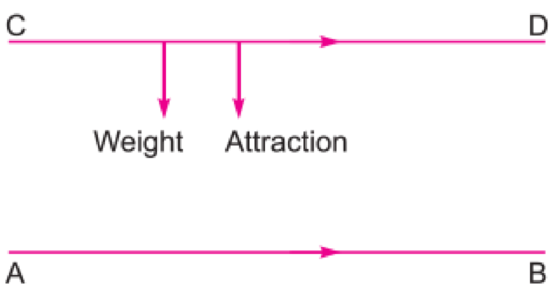
$$\Rightarrow \qquad r = \frac{\sqrt{2\,\mathrm{meV}}}{\mathrm{qB}}$$

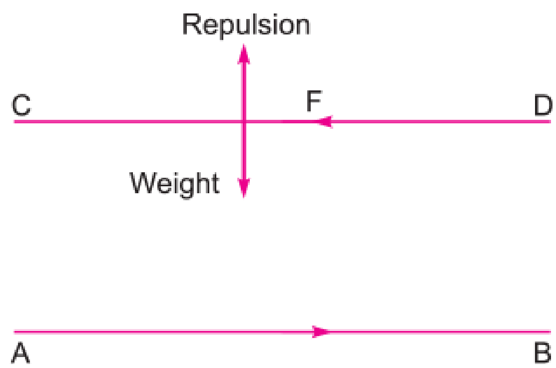
$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^4}}{1.6 \times 10^{-19} \times 0.04}$$

$$= \frac{5.39 \times 10^{-23}}{6.4 \times 10^{-21}} m = 8.4 \times 10^{-3} m$$

Q. 22. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB. [Take the value of g = 10 ms^{-2}] [CBSE (AI) 2013]

Ans. (i) Current carrying conductors repel each other, if current flows in the opposite direction.





(ii) Current carrying conductors attract each other if current flows in the same direction.

If wire CD remain suspended above AB then

$$F_{repulsion} = \text{Weight}$$

$$\frac{\mu_0 I_1 I_2 l}{2\pi r} = mg$$

where r = Separation between the wires

$$\frac{m}{l} = \frac{\mu_0 I_1 I_2}{2\pi\, rg}$$

$$= \frac{2 \times 10^{-7} \times 12 \times 5}{1 \times 10^{-3} \times 10}$$

1.2×10^{-3} kg / m

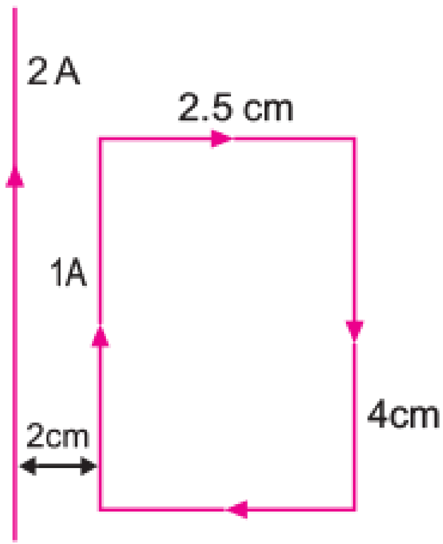
Current in CD should be in opposite direction to that in AB.

Q. 23. A rectangular loop of wire of size 2.5 cm × 4 cm carries a steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of the force on the loop due to the current carrying wire. [CBSE Delhi 2012]

Ans.

(*i*) Torque on the loop 'τ' = $MB \sin\theta = \overrightarrow{M} \times \overrightarrow{B}$

$\tau = 0$ $\qquad\qquad$ $[\therefore \overrightarrow{M}$ and $\overrightarrow{B}$ are parallel$]$



(ii) Magnitude of force

$$\left|\overrightarrow{F}\right| = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

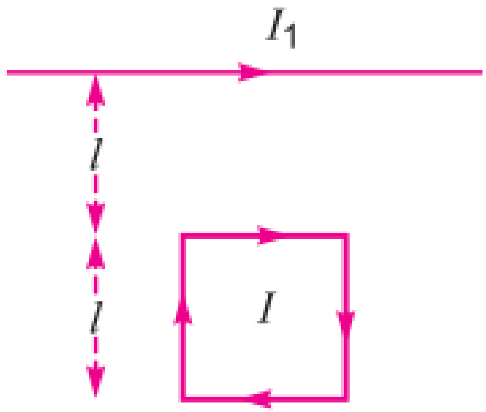
$$= 2 \; ? \; 10^{-7} ?2?1?4?10^{-2}\left[\frac{1}{2?10^{-2}} - \frac{1}{4.5 \; ? \; 10^{-2}}\right]$$

$$= 16 \times 10^{-7} \times \left[\frac{4.5 - 2}{2 \times 4.5}\right] = \frac{8 \times 5 \times 10^{-7}}{9} = 4.44 \times 10^{-7} N$$

Direction of force is towards conductor (attractive).

Q. 24. Write the expression for the magnetic moment ($\vec{m}$) due to a planar square loop of side 'l' carrying a steady current I in a vector form.

In the given figure this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop. [HOTS][CBSE Delhi 2010]

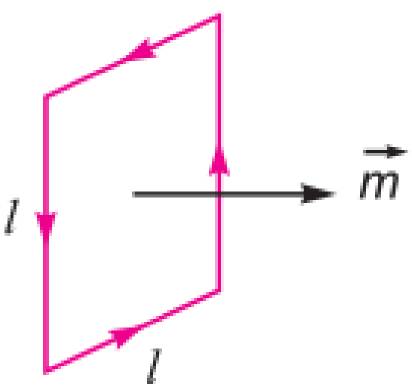


Ans. Magnetic moment due to a planar square loop of side l carrying current I is

$$\vec{m} = I\,\vec{A}$$

For square loop $A = l^2$

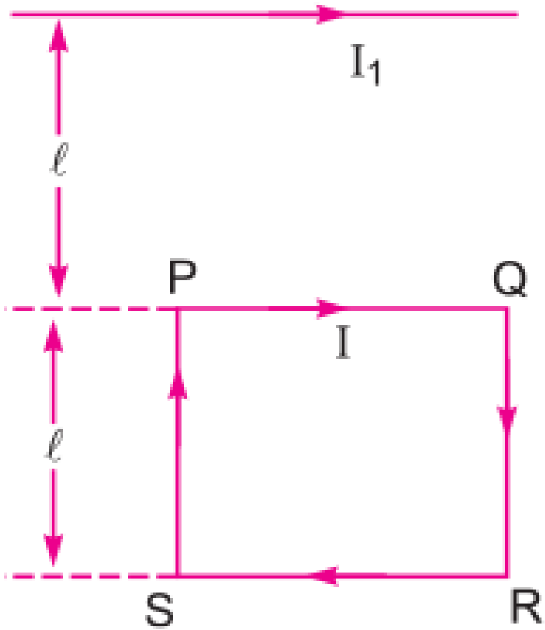
$$\therefore \qquad \vec{m} = I\,l^2\,\hat{n}$$



Where $\hat{n}$ is unit vector normal to loop.

Magnetic field due to current carrying wire at the location of loop is directed downward perpendicular to plane of loop. Since the magnetic moment is parallel to area vector hence torque is zero.

Force on QR and SP are equal and opposite, so net force on these sides is zero.



Force on side PQ, $\overrightarrow{F}_{PQ} = I\ \overrightarrow{l} \times \overrightarrow{B}_1\ l\hat{i}$

$$= \text{Il}\ \hat{i} \times \frac{u_0 I_1}{2\pi l}(-\hat{k})$$

$$= \frac{\mu_0 \text{II}_1}{2\pi}\ \hat{j}\ ;$$

Force on side *RS*, $\overrightarrow{F_{RS}} = I\ l\ (-\hat{i}) \times \frac{\mu_0 I_1}{2\pi(2l)}\ (-\hat{k})$

Net force $\overrightarrow{F} = \overrightarrow{F_{PQ}} - \overrightarrow{F_{RS}} = \frac{\mu_0 \text{II}_1}{4\pi}\ \hat{j}\ ;$

That is loop experiences a repulsive force but no torque.

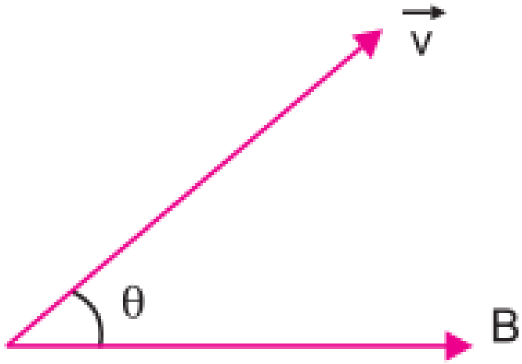
Short Answer Questions –II (OIQ)

Q. 1. Write the expression for the force on a charge moving in a magnetic field. Use this expression to define the SI unit of magnetic field.

Ans. Force on a charge (q) moving in a magnetic field B with velocity $\underset{v}{\rightarrow}$ making an angle θ (with the direction of magnetic field ($\underset{B}{\rightarrow}$) is given by

$F_m = qvB \sin\theta$

When $\theta=90° \Rightarrow \sin\theta=1$, so



$F_m = qvB$

or $B = \frac{F_m}{qv}$

If v=1 m/s, $B = \frac{F_m}{q}$ newton/coulomb.

SI unit of magnetic field is tesla.

Thus, **1 tesla is the magnetic field in which a charged particle moving with velocity 1 m/s perpendicular to velocity experiences a force of 1 newton/coulomb.**

Q. 2. Define the term magnetic moment of a current loop. Write the expression for the magnetic moment when an electron revolves at a speed v around an orbit of radius ‘r’ in hydrogen atom.

Ans. Magnetic moment of a current loop: The torque on current loop is

$\tau = MB \sin\theta$, where θ is angle between magnetic moment and magnetic field.

$$\Rightarrow \quad M = \frac{\tau}{B \sin\theta}$$

If B or 1 T, $\sin\theta = 1$ or $\theta = 90°$ then $M = \tau$.

That is ***the magnetic moment of a current loop is defined as the torque acting on the loop when placed in a magnetic field of 1 T such that the loop is oriented with its area vector normal to the magnetic field.***

Also, $M = IA$

i.e., magnetic moment of a current loop is the product of current flowing in the loop and area of loop. Its direction is perpendicular to the plane of the loop and determined by using right hand thumb rule.

Magnetic moment of revolving electron,

$$M = \frac{\mathrm{evr}}{2}$$

Q. 3. Which of the following will describe the smallest circle when projected with the same velocity perpendicular to the magnetic field B (i) α-particle and (ii) β-particle?

Ans. Radius of circular path in transverse magnetic field

$r = \frac{\mathrm{mv}}{\mathrm{qB}} \propto \frac{m}{q}$ for same v and B

For α-particle $\left(\frac{m}{q}\right)_{\alpha} = \frac{4m_p}{2e} = \frac{2m_p}{e}$ where m_p is mass of proton.

For β-particle $\left(\frac{m}{q}\right)_{\beta} = \frac{\frac{1}{1840}m_p}{e} = \frac{1}{1840}\left(\frac{m_p}{e}\right)$

Clearly β-particle has smallest value of $\frac{m}{q}$; so β-particle will describe the smallest circle.

Q. 4. A solenoid of length 1.0 m, radius 1 cm and total turns 1000 wound on it, carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 104 ms^{-1} along the axis of this current carrying solenoid, what would be the force experienced by this electron?

Ans. Magnetic field inside a solenoid,

$B = \mu_0 nI$

$n = \frac{N}{l} = \frac{1000 \text{ turns}}{1.0 \ m} = 1000 \text{ turns/m}$

$I = 5\text{A}$

$\therefore \quad B = (4\pi \times 10^{-7}) \times 1000 \times 5$

$= 20 \times 3.14 \times 10^{-4}$ J= $\mathbf{6.28 \times 10^{-3}}$ **T**, along the axis

Force experienced by electron

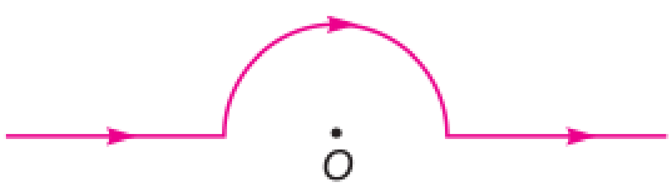
$F_m = qvB \sin\theta$

Here $q = -e, v = 10^4$ m/s,

θ = angle between $\vec{v}$ and $\vec{B} = 0$

$\therefore \quad F_m = -evB \sin 0° = 0$ (zero)

Q. 5. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown. What is the magnetic field at O due to (i) straight segments (ii) the semi-circular arc?



Ans. Magnetic field due to a current carrying element.

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi} \frac{I\overrightarrow{\delta l}\times\vec{r}}{r^3}$$

i. For straight segments $\theta = 0$ or $\pi \Rightarrow \quad \overrightarrow{\delta l}\times\vec{r} = \delta l\ r\ \sin\ 0\ \widehat{n} = 0 \quad \therefore\ B_1 = 0$

ii. For semicircular arc $\Sigma\ \mathrm{dl} = \pi r, \theta = \frac{\pi}{2}$

$$\therefore \overrightarrow{B_2} = \frac{\mu_0}{4\pi} \frac{\Sigma I\ \overrightarrow{\delta l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I\ \Sigma\ \delta l \sin\frac{\pi}{2}}{r^2}\widehat{n}$$

$$= \frac{\mu_0}{4\pi} \frac{I\pi r}{r^2}\widehat{n} = \frac{\mu_0 I}{4r},$$

directed perpendicular to plane of paper downward.

Q. 6. A semi-circular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of magnetic field at the centre of the arc.

Ans. The magnetic field due to a semi-circular arc of radius 'r' carrying current (I) at centre is given by

$$\Delta B = \frac{\mu_0}{4\pi} \frac{I\Delta l \sin 90^o}{r^2} = \frac{\mu_0}{4\pi} \frac{I\Delta}{r^2}$$

The net magnetic field due to whole length of arc l will be

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \sum\ \Delta l$$

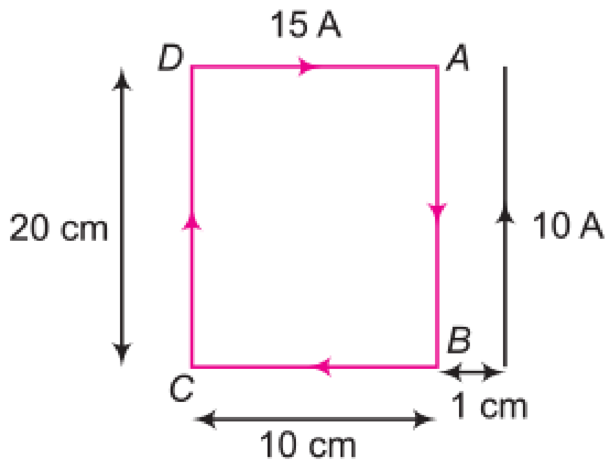
For semi-circular arc $\sum \Delta l = \pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{r^2}(\pi r) = \frac{\mu_0 I}{4r}$$

Given $I = 10$ A, $r = 20$m $= 0.20$ m

$$\therefore \quad B = \frac{4\pi\times10^{-7}\times10}{4\times0.20} = \frac{4\times3.14\times10^{-7}\times10}{4\times0.20} = 1.57\times10^{-5}\ T$$

Q. 7. A rectangular current carrying loop is placed 1 cm away from a long straight current-carrying conductor as shown.



(i) Will the net force acting on the loop due to straight conductor be attractive or repulsive in nature? Justify your answer. Calculate the magnitude of this force.

Ans. The net force acting on the loop will be repulsive in nature.

Justification: Part AB of the loop will experience a force of repulsion whereas part CD will experience attraction. Parts BC and AD will not experience any force. Thus, the overall force will be a force of repulsion because AB is closer to the straight conductor than CD and the force between two current carrying conductors is inversely proportional to the distance between them.

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r} l$$

Net Force

Force F_1 on AB due to current in the straight conductor

$$F_1 = \frac{\mu_0}{2\pi} \times \frac{2\times10\times15}{1\times10^{-2}} \times 20 \times 10^{-2}$$

$$= 3 \times 10^{-4}\ N \text{ towards left} \quad \left(\frac{\mu_0}{4\pi} = 10^{-7}\right)$$

Force F_2 on CD due to current in the straight conductor

$$F_2 = \frac{\mu_0}{2\pi} \times \frac{2\times10\times15}{11\times10^{-2}} \times 20 \times 10^{-2}$$

$$= 0.2725 \times 10^{-4} \text{ N towards right}$$

Hence, net force on the loop = 2.72 N towards left.

Q. 8. The magnitude F of the force between two straight parallel current carrying conductors kept at a distance d apart in air is given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Where I_1 and I_2 are the currents flowing through the two wires.

Use this expression, and the sign convention that the:

"Force of attraction is assigned a negative sign and Force of repulsion is assigned a positive sign".

Draw graphs showing dependence of F on

(i) I_1 I_2 when d is kept constant

(ii) d when the product I_1 I_2 is maintained at a constant positive value.

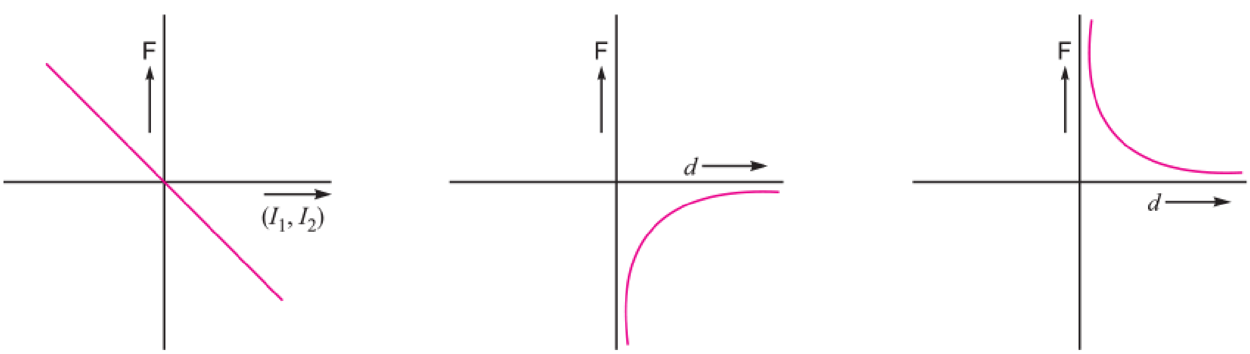
(iii) d when the product I_1 I_2 is maintained at a constant negative value.
[CBSE Sample Paper] [HOTS]

Ans. We know that F is an **attractive (–ve)** force when the currents I_1 and I_2 are 'like' currents i.e. when the product I_1 I_2 is positive.

Similarly F is a **repulsive (+ve)** force when the currents I_1 and I_2 are 'unlike' currents, i.e. when the product I_1 I_2 is negative.

Now $F \propto (I_1 I_2)$, when d is kept constant and $\mathbf{F \propto 1/d}$ when $I_1 I_2$ is kept constant.

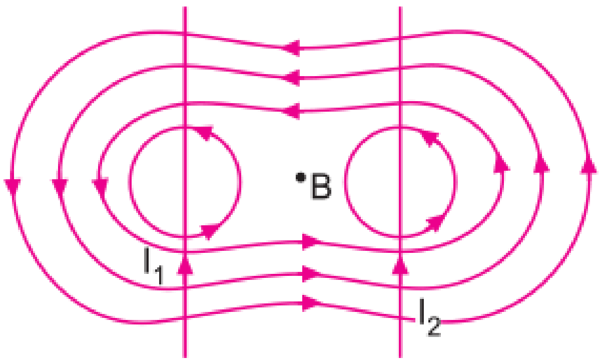
The required graphs, therefore, have the forms shown below:



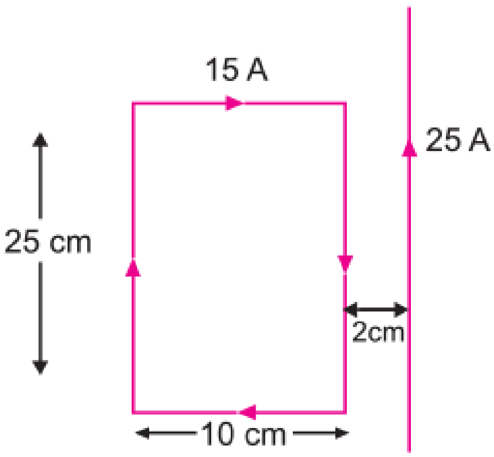
(i) (ii) (iii)

Q. 9. Answer the following questions.

(i) Draw the magnetic field lines due to two straight, long, parallel conductors carrying currents I_1 and I_2 in the same direction. Write an expression for the force acting per unit length on one conductor due to other. Is this force attractive or repulsive?



(ii) Figure shows a rectangular current-carrying loop placed 2 cm away from a long, straight, current-carrying conductor. What is the direction and magnitude of the net force acting on the loop? [HOTS]



Ans. (i) The magnetic field lies due to two current carrying parallel wires are shown in figure. The force between parallel wires $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} N/m$

(ii) We know that parallel currents attract and opposite currents repel and F∝1/r. As wire of loop carrying opposite current is nearer, so the net force acting on the loop is **repulsive.**

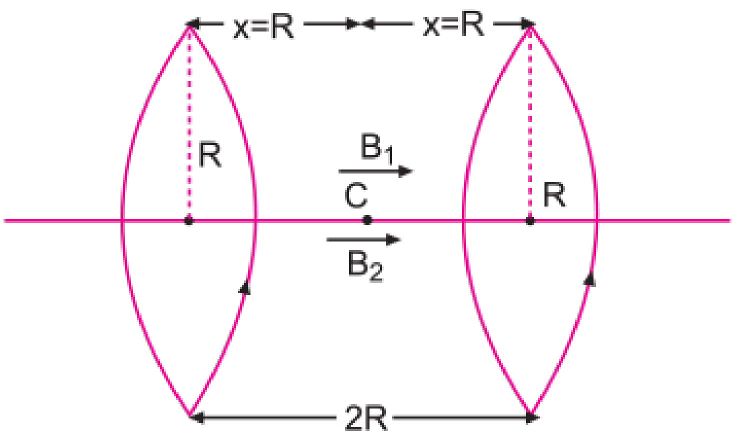
Q. 10. Two parallel coaxial circular coils of equal radius 'R' and equal number of turns 'N', carry equal currents 'I' in the same direction and are separated by a distance '2R'. Find the magnitude and direction of the net magnetic field produced at the mid-point of the line joining their centres. [HOTS]

Ans. Magnetic field due to a circular coil of radius 'R' at a distance x from centre is

$$B = \frac{\mu_0 \, NIR^2}{2(R^2+x^2)^{3/2}}$$

Here $x = R$

$$\therefore \qquad B = \frac{\mu_0 \, NIR^2}{2(R^2+R^2)^{3/2}} = \frac{\mu_0 \, NIR^2}{2.2\sqrt{2}R^3} = \frac{\mu_0 \, NI}{4\sqrt{2}R}$$



Total magnetic field at centre C due to both coils

$$B = B_1 + B_2 = 2 \times \frac{\mu_0 \, NI}{4\sqrt{2}R} = \frac{\mu_0 \, NI}{2\sqrt{2}R}$$

Q. 11. A rectangular loop of sides 25 cm and 10 cm carrying a current of 15 A is placed with its longer side parallel to a long straight conductor 2.0 cm apart carrying a current of 25 A (fig). What is the net force on the loop? [HOTS]

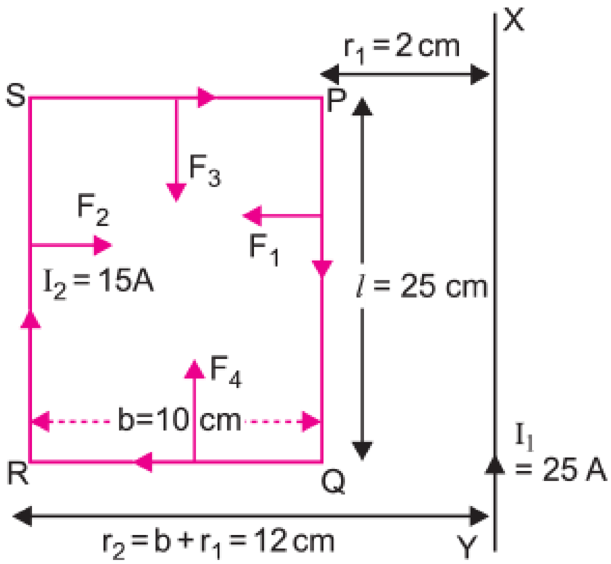
Ans. Rectangular loop PQRS is placed near a long straight wire as shown.

l = PQ = RS = 25 cm = 0.25 m

b = QR = PS =10 cm = 0.10 m

r_1 = 2 cm = 0.02 m,

r_2 = 12 cm = 0.12 m



The currents in PQ and XY are antiparallel, so PQ is repelled away from wire XY. This repulsive force is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} l$$

$$= \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0.25}{2\pi \times 0.02} = 9.375 \times 10^{-4}\ N$$

The currents in XY and RS are in the same direction, so wire RS is attracted towards wire XY. This attractive force is

$$F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0.25}{2\pi \times 0.12} = 1.563 \times 10^{-4}\ N$$

The currents in PS and QR are equal and opposite. By symmetry they exert equal and opposite forces (F_3 and F_4) and hence net force on these sides is zero.

∴ Net force on rectangular loop

$F = F_1 - F_2$ (repulsive)

$= 9.375 \times 10^{-4} - 1.563 \times 10_{-4}$

$= 7.812 \times 10^{-4}$ N (repulsive)

The net force is directed away from long wire XY.

Q. 12. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change? [HOTS]

Ans.

Current sensitivity, $S_C = \frac{\theta}{I} = \frac{NAB}{C}$

Voltage sensitivity, $S_V = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{S_C}{R}$

When current sensitivity is increased by 50%, the resistance is made twice.

$\therefore$ New current sensitivity $S_C' = S_C + \frac{50}{100} S_C = 1.5 S_C$

New resistance $R' = 2R$

$\therefore$ New Voltage sensitivity, $S_V' = \frac{S_C'}{R'} = \frac{1.5\, S_C}{2R} = 0.75\, S_V$

Clearly, $S_V' < S_V$, i.e, voltage sensitivity decreases

% decrease in voltage sensitivity $= \frac{S_V - S_V'}{S_V} \times 100\%$

$= \frac{S_V - 0.75 S_V}{S_V} \times 100\% = 25\%$

Q. 13. Write the expression for the force, $\vec{F}$ acting on a charged particle of charge 'q', moving with a velocity $\vec{V}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields.

Ans.

Electric force on particle, $\overrightarrow{F_e} = q\overrightarrow{E}$

Magnetic force on particle, = $\overrightarrow{F_m} = q(\vec{v} + \overrightarrow{B})$

Total force, $\overrightarrow{F} = \overrightarrow{F_e} + \overrightarrow{F_m}$

$\overrightarrow{F} = q(\overrightarrow{E} + \vec{v} \times \overrightarrow{B})$

If a charge particle enter's perpendicular to both the electric and magnetic fields then it may happen that the electric and magnetic forces cancel each other and so particle will pass undeflected.

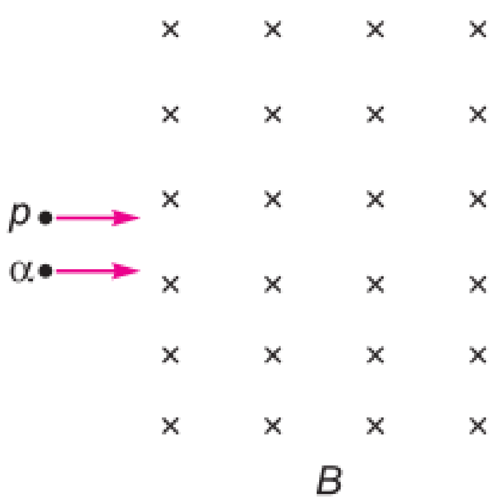
In such a case, $\overrightarrow{F} = 0$

$$\Rightarrow \quad q(\overrightarrow{E} + \vec{v} \times \overrightarrow{B}) = 0 \Rightarrow \quad \overrightarrow{E} = -(\vec{v} \times \overrightarrow{B})$$

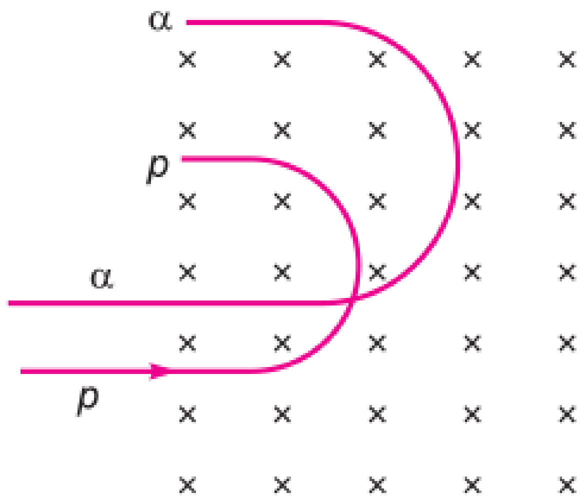
$$\Rightarrow \quad \overrightarrow{E} = \overrightarrow{B} \times \vec{v} \quad \Rightarrow \quad \overrightarrow{E} = \mathrm{Bv}\sin\theta = \mathrm{Bv} \quad (\text{when } \theta = 90^\circ)$$

$$\Rightarrow \quad v = \frac{E}{B} \text{ (when } v, E \text{ and } B \text{ are mutually perpendicular)}$$

Q. 14. An α-particle and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectories followed by the two particles in the region of the magnetic field. Find the ratio of the radii of the circular paths which the two particles may describe.



Ans. Radius of charged particle in magnetic field



$$r = \frac{\mathrm{mv}}{\mathrm{qB}}$$

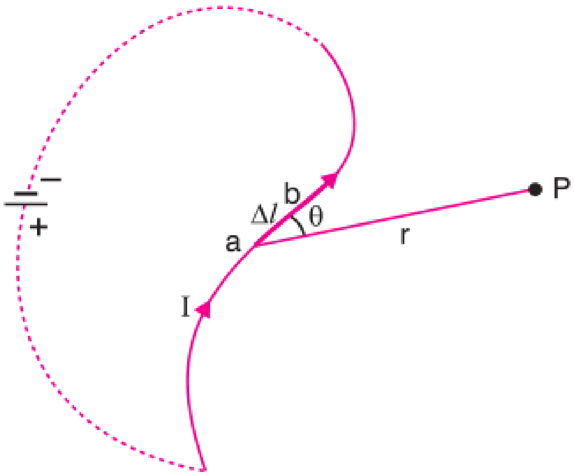
$$r \propto \frac{m}{q} \text{ for same } v \text{ and } B.$$

$$\frac{r_p}{r_\alpha} = \frac{(m/q)_p}{(m/q)_\alpha}$$

$$= \frac{(m_p/e)}{((4m_p)/2e)} = \frac{1}{2}$$

Long Answer Questions

Q. 1. State and explain Biot-Savart law. Use it to derive an expression for the magnetic field produced at a point near a long current carrying wire.



Ans. Biot-Savart law: Suppose the current I is flowing in a conductor and there is a small current element 'ab' of length Δl. According to Biot-Savart the magnetic field (ΔB) produced due to this current element at a point P distant r from the element is given by

$$\Delta B \propto \frac{I\Delta l \sin\theta}{r^2} \text{ or } \Delta B = \frac{\mu}{4\pi}\frac{I\ \Delta l\ \sin\theta}{r^2} \qquad ...(i)$$

Where $\frac{\mu}{4\pi}$ is a constant of proportionality. It depends on the medium between the current element and point of observation (P).μ is called the permeability of medium. Equation (i) is called Biot-Savart law. The product of current (I) and length element (Δl) (i.e., I Δl) is called the current element. Current element is a vector quantity, its direction is along the direction of current. If the conductor be placed in vacuum (or air), then μ is replaced by μ0; where μ0 is called the permeability of free space (or air). In S.I. system μ0= 4π ×10^{-7} weber/ampere-metre (or newton/ampere^2).

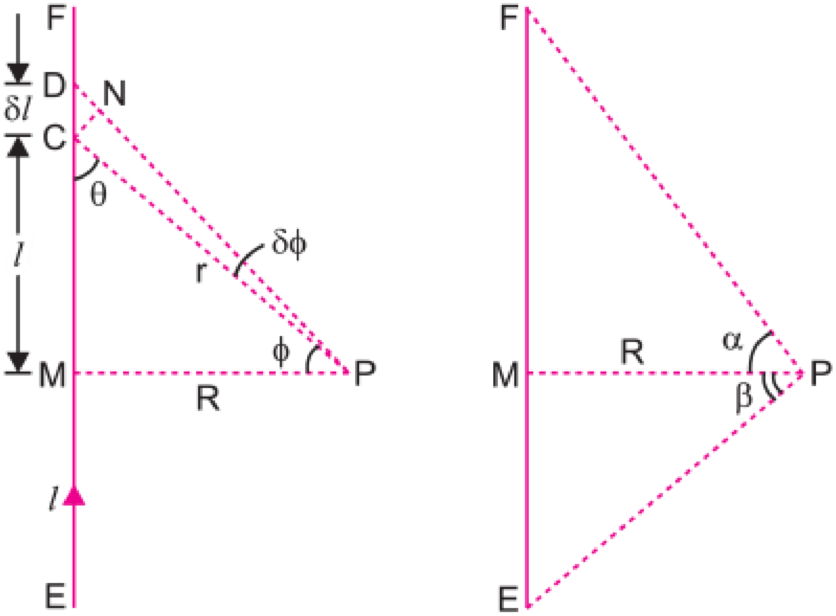
$$\text{Thus } \frac{\mu_0}{4\pi} = 10^{-7} \text{ weber/ampere} \times \text{metre}$$

As in most cases the medium surrounding the conductor is air, therefore, in general, Biot-Savart law is written as

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\Delta l\ \sin\theta}{r^2}$$

The direction of magnetic field is perpendicular to the plane containing current element and the line joining point of observation to current element. So in vector form the expression for magnetic field takes the form

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\ \frac{I\Delta\vec{l} \times \vec{r}}{r^3}$$



(a) (b)

Derivation of formula for magnetic field due to a current carrying wire using Biot-Savart law: Consider a wire EF carrying current I in upward direction. The point of observation is P at a finite distance R from the wire. If PM is perpendicular dropped from P on wire; then PM = R. The wire may be supposed to be formed of a large number of small current elements. Consider a small element CD of length δl at a distance l from M.

Let $\angle CPM = \varphi$

And $\angle CPD = \delta\varphi$, $\angle PDM = \theta$

The length δl is very small, so that ∠PCM may also be taken equal to θ.

The perpendicular dropped from C on PD is CN. The angle formed between elements

$I\ \overrightarrow{\delta l}$ and $\vec{r}\ (=\overrightarrow{\mathrm{CP}})$ is $(\pi - \theta)$. Therefore according to Biot-Savart law, the magnetic field due to current element $I\,\overrightarrow{\delta l}$ at P is

$$\delta B = \frac{\mu_0}{4\pi}\,\frac{I\ \delta l\ \sin\ (\pi-\theta)}{r^2} = \frac{\mu_0}{4\pi}\,\frac{I\ \delta l\ \sin\ \theta}{r^2} \qquad \ldots(i)$$

But in $\Delta\ CND$, $\sin\theta = \sin\,(\angle \mathrm{CDN}) = \frac{\mathrm{CN}}{\mathrm{CD}} = \frac{r\ \delta\varphi}{\delta l}$

or $\quad \delta l \sin\theta = r\,\delta\varphi$

$\therefore \quad$ From equation (i)

$$\delta B = \frac{\mu_0}{4\pi}\,\frac{I\ r\ \delta\varphi}{r^2} = \frac{\mu_0}{4\pi}\,\frac{I\ \delta\ \varphi}{r} \qquad \ldots(ii)$$

Again from fig.

$$\cos\ \varphi = \frac{R}{r} \Rightarrow r = \frac{R}{\cos\ \varphi}$$

From equation (ii)

$$\delta\ B = \frac{\mu_0}{4\pi}\,\frac{I\ \cos\ \varphi\ \delta\varphi}{R} \qquad \ldots(iii)$$

If the wire is of finite length and its ends make angles α and β with line MP, then net magnetic field (B) at P is obtained by summing over magnetic fields due to all current elements, i.e.,

$$B = \int_{-\beta}^{\alpha} \frac{\mu_0}{4\pi}\,\frac{I\ \cos\ \varphi\ d\varphi}{R} = \frac{\mu_0 I}{4\mu R}\int_{-\beta}^{\alpha} \cos\ \varphi\ d\varphi$$

$$\frac{\mu_0 I}{4\pi R}\,[\sin\ \varphi]_{-\beta}^{\alpha} = \frac{\mu_0 I}{4\pi R}\,[\sin\ \alpha - \sin\,(-\beta)]$$

$$i.e.,\ \ B = \frac{\mu_0 I}{4\pi R}\,(\sin\ \alpha + \sin\ \beta)$$

This is expression for magnetic field due to current carrying wire of finite length.

If the wir e is of infinite length (or very long), then α = β ⇒ π/2

$$\therefore \quad B = \frac{\mu_0 I}{4\pi R}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = \frac{\mu_0 I}{4\pi R}[1+1] \quad \text{or} \quad B = \frac{\mu_0 I}{2\pi R}$$

Q. 2. Answer the following questions.

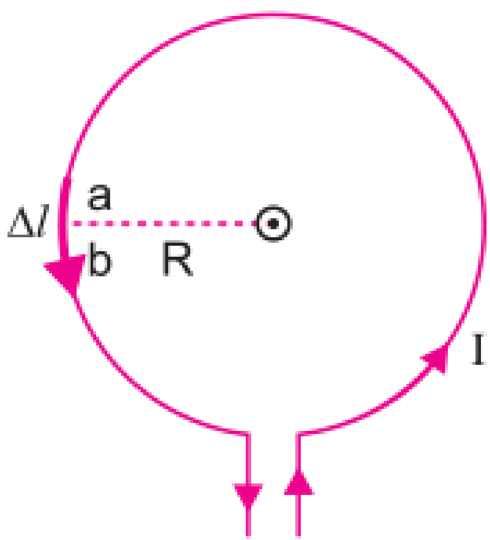
(i) State Biot-Savart Law. Using this law, find an expression for the magnetic field at the centre of a circular coil of N-turns, radius r, carrying current I.

(ii) Sketch the magnetic field for a circular current loop, clearly indicating the direction of the field. [CBSE (F) 2010, Central 2016]

Ans. (i) Biot-Savart Law: Refer to above question

Magnetic field at the centre of circular loop: Consider a circular coil of radius R carrying current I in anticlockwise direction. Say, O is the centre of coil, at which magnetic field is to be computed. The coil may be supposed to be formed of a large number of current elements. Consider a small current element 'ab' of length Δl. According to Biot Savart law the magnetic field due to current element 'ab' at centre O is

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\ \Delta l\ \sin\theta}{R^2}$$



Where θ is angle between current element ab and the line joining the element to the centre O. Here θ =90° because current element at each point of circular path is perpendicular to the radius. Therefore magnetic field produced at O, due to current element ab is

$$\Delta B = \frac{\mu_0}{4\pi}\frac{I\ \Delta l}{R^2}$$

According to Maxwell's right hand rule, the direction of magnetic field at O is upward, perpendicular to the plane of coil. The direction of magnetic field due to all current elements is the same. Therefore the resultant magnetic field at the centre will be the sum of magnetic fields due to all current elements. Thus

$$B = \sum \Delta B = \sum \frac{\mu_0}{4\pi}\frac{I\ \Delta l}{R^2} = \frac{\mu_0}{4\pi}\frac{I}{R^2}\sum\ \Delta l$$

But $\sum \Delta l$ = total length of circular coil $= 2\pi R$ (for one-turn)

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{I}{R^2}.2\pi R \quad \text{or} \quad B = \frac{\mu_0 I}{2R}$$

If the coil contains N–turns, then $\sum \Delta l = N.\ 2\pi\ R$

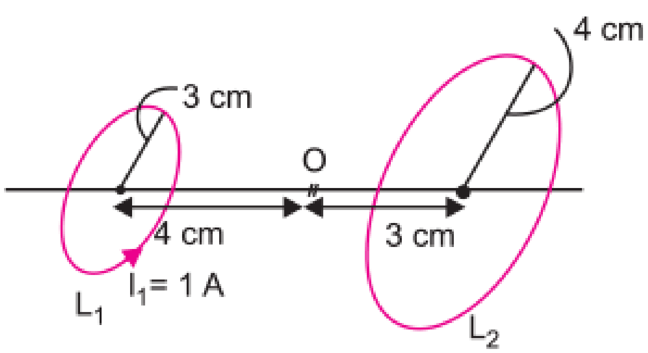
$$B = \frac{\mu_0 I}{4\pi R^2}.N.2\pi R \quad \text{or} \quad B = \frac{\mu_0\ NI}{2R}$$

Here current in the coil is anticlockwise and the direction of magnetic field is perpendicular to the plane of coil upward; but if the current in the coil is clockwise, then the direction of magnetic field will be perpendicular to the plane of coil downward.

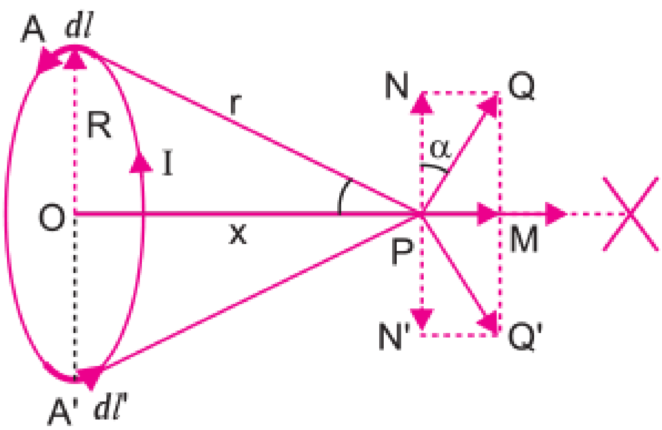
Q. 3. Answer the following questions.

(i) Derive an expression for the magnetic field at a point on the axis of a current carrying circular loop. [CBSE (AI) 2013; (F) 2010]

(ii) Two co-axial circular loops L_1 and L_2 of radii 3 cm and 4 cm are placed as shown. What should be the magnitude and direction of the current in the loop L_2 so that the net magnetic field at the point O be zero?



Ans. (i) Magnetic field at the axis of a circular loop: Consider a circular loop of radius R carrying current I, with its plane perpendicular to the plane of paper. Let P be a point of observation on the axis of this circular loop at a distance x from its centre O. Consider a small element of length dl of the coil at point A. The magnitude of the magnetic induction $\overrightarrow{dB}$ at point P due to this element is given by



$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I\delta l \sin\alpha}{r^2} \qquad \ldots(i)$$

The direction of $\overrightarrow{dB}$ is perpendicular to the plane containing $\overrightarrow{dI}$ and $\overrightarrow{r}$ and is given by right hand screw rule. As the angle between I $\overrightarrow{dI}$ and $\overrightarrow{r}$ and is 90°, the magnitude of the magnetic induction $\overrightarrow{dB}$ is given by,

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{dl}\ \sin\ 90^o}{r^2} = \frac{\mu_0 I\ \mathrm{dl}}{4\pi r^2}. \qquad \ldots(ii)$$

If we consider the magnetic induction produced by the whole of the circular coil, then by symmetry the components of magnetic induction perpendicular to the axis will be cancelled out, while those parallel to the axis will be added up. Thus the resultant magnetic induction $\overrightarrow{B}$ at axial point P is along the axis and may be evaluated as follows:

The component of $\overrightarrow{dB}$ along the axis,

$$\overrightarrow{dB_x} = \frac{\mu_0 I \, dl}{4\pi \, r^2} \sin \ \alpha \qquad ...(iii)$$

But $\sin \ \alpha = \frac{R}{r}$ and $r = (R^2 + x^2)^{1/2}$

$$\therefore \quad \overrightarrow{dB_x} = \frac{\mu_0 I \, dl}{4\pi \, r^2} \cdot \frac{R}{r} = \frac{\mu_0 \, IR}{4\pi r^3} dl = \frac{\mu_0 \, IR}{4\pi \, (R^2+x^2)^{3/2}} dl \qquad ..(iv)$$

Therefore the magnitude of resultant magnetic induction at axial point P due to the whole circular coil is given by

$$\overrightarrow{B} = \oint \frac{\mu_0 \, IR}{4\pi (R^2+x^2)^{3/2}} dl = \frac{\mu_0 \, IR}{4\pi (R^2+x^2)^{3/2}} \oint dl$$

But $\oint dl$ = length of the loop = $2\pi R$ $\qquad ...(v)$

$$\text{Therefore,} \quad B = \frac{\mu_0 \, IR}{4\pi (R^2+x^2)^{3/2}} (2\pi R)$$

$$\overrightarrow{B} = B_x \ \hat{i} = \frac{\mu_0 \, IR^2}{2(R^2+x^2)^{3/2}} \ \hat{i}. \ \left[\text{At centre}, x = 0, \overrightarrow{B} = \frac{\mu_0 I}{2R}\right]$$

If the coil contains N turns, then

$$B = \frac{\mu_0 \, NIR^2}{2(R^2+x^2)^{3/2}} \quad \text{tesla} \qquad ....(vi)$$

Q. 4. Answer the following questions.

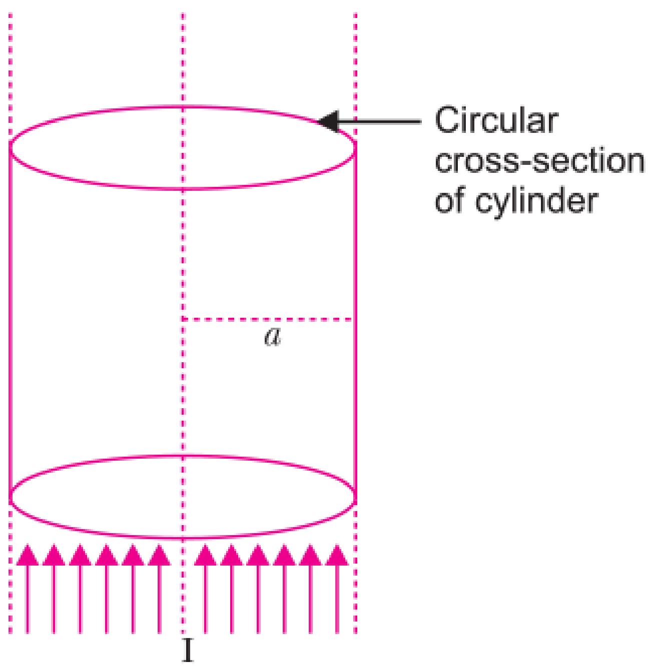
(i) A straight thick long wire of uniform circular cross-section of radius ‘a’ is carrying a steady current I. The current is uniformly distributed across the cross-section. Use Ampere’s circuital law to obtain a relation showing the variation of the magnetic field (Br) inside and outside the wire with distance r, (r ≤ a) and (r > a) of the field point from the centre of its cross-section. What is the magnetic field at the surface of this wire? Plot a graph showing the nature of this variation.

(ii) Calculate the ratio of magnetic field at a point above the surface of the wire to that at a point below its surface. What is the maximum value of the field of this

wire?
[CBSE Delhi 2010; Chennai 2015]

Ans. (i) Magnetic field due to a straight thick wire of uniform cross-section: Consider an infinitely long cylindrical wire of radius a, carrying current I. Suppose that the current is uniformly distributed over whole cross-section of the wire. The cross-section of wire is circular. Current per unit cross-sectional area.

$$i = \frac{I}{\pi a^2} \qquad \ldots(\mathrm{i})$$



Magnetic field at external points (r > a): We consider a circular path of radius r (> a) passing through external point P concentric with circular cross-section of wire. By symmetry the strength of magnetic field at every point of circular path is same and the direction of magnetic field is tangential to path at every point. So line integral of magnetic field $\underset{B}{\rightarrow}$ around the circular path

$$\oint \overrightarrow{B}\,.\,\overrightarrow{\mathrm{dl}} = \oint B\ \mathrm{dl}\ \cos\ 0^o = B\ 2\pi r$$

Current enclosed by path = Total current on circular cross-section of cylinder = I

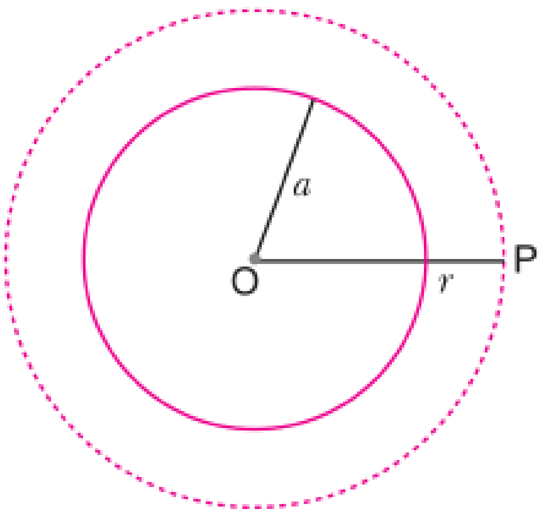
By Ampere's circuital law

$$\oint \overrightarrow{B} \, . \, \overrightarrow{\mathrm{dl}} = \mu \ \times \text{current enclosed by path}$$

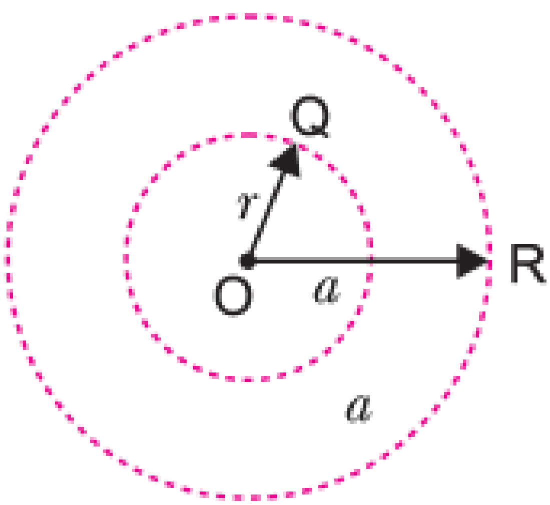
$$\Rightarrow B\ 2\pi r = \mu_0 \times I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

This expression is same as the magnetic field due to a long current carrying straight wire.

This shows that **for external points the current flowing in wire may be supposed to be concerned at the axis of cylinder.**



Magnetic Field at Internal Points (r < a) : Consider a circular path of radius r (<a), passing through internal point Q concentric with circular cross-section of the wire. In this case the assumed circular path encloses only a path of current carrying circular cross-section of the wire.



∴ Current enclosed by path = current per unit cross-section × cross section of assumed circular path

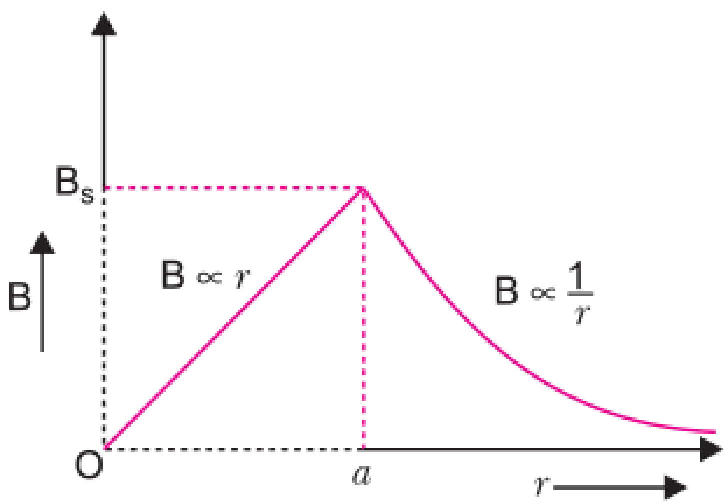
$$= i \times \pi r^2 = \left(\frac{I}{\pi a^2}\right) \times \pi r^2 = \frac{\mathrm{I}r^2}{a^2}$$

∴ By Ampere's circuital law

$$\oint \vec{B} \cdot \vec{\mathrm{dl}} = \mu_0 \times \text{ current closed by path}$$

$$\Rightarrow \quad B \cdot 2\pi r = \mu_0 \times \frac{\mathrm{I}r^2}{a^2} \quad \Rightarrow \quad B = \frac{\mu_0 \, \mathrm{I}r}{2\pi a^2}$$

Clearly, magnetic field strength inside the current carrying wire is directly proportional to distance of the point from the axis of wire.



At surface of cylinder r = a, so magnetic field at surface of wire

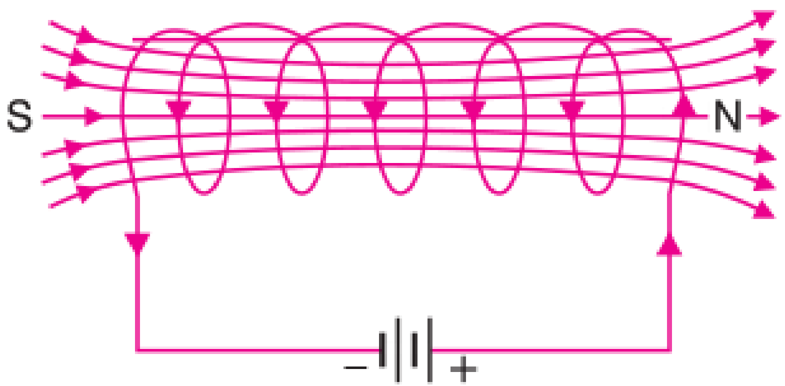
$$B_S = \frac{\mu_0 I}{2\pi a} \text{ (maximum value)}$$

The variation of magnetic field strength (B) with distance (r) from the axis of wire for internal and external points is shown in figure.

Q. 5. Using Ampere's circuital law find an expression for the magnetic field at a point on the axis of a long solenoid with closely wound turns. [CBSE (F) 2010]

Ans. Magnetic field due to a current carrying long solenoid:

A solenoid is a long wire wound in the form of a close-packed helix, carrying current. To construct a solenoid a large number of closely packed turns of insulated copper wire are wound on a cylindrical tube of card-board or china clay. When an electric current is passed through the solenoid, a magnetic field is produced within the solenoid. If the solenoid is long and the successive insulated copper turns have no gaps, then the magnetic field within the solenoid is uniform; with practically no magnetic field outside it. The reason is that the solenoid may be supposed to be formed of a large number of circular current elements. The magnetic field due to a circular loop is along its axis and the current in upper and lower straight parts of solenoid is equal and opposite. Due to this the magnetic field in a direction perpendicular to the axis of solenoid is zero and so the resultant magnetic field is along the axis of the solenoid.



If there are 'n' number of turns per metre length of solenoid and I amperes is the current flowing, then magnetic field at axis of long solenoid

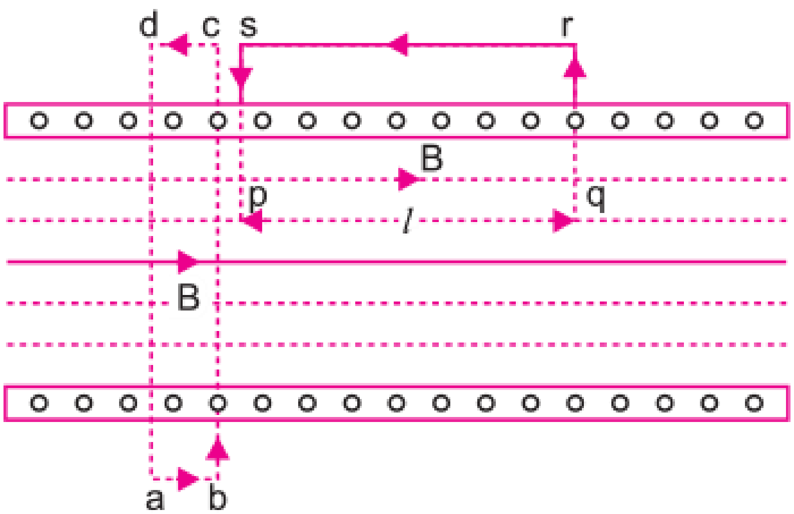
$B = \mu_0 nI$

If there are N turns in length l of wire, then

$$n = \frac{N}{l} \quad \text{or} \quad B = \frac{\mu_0 \, NI}{l}$$

Derivation: Consider a symmetrical long solenoid having number of turns per unit length equal to n.

Let I be the current flowing in the solenoid, then by right hand rule, the magnetic field is parallel to the axis of the solenoid.



Field outside the solenoid: Consider a closed path abcd Applying Ampere's law to this path

$$\oint \vec{B} . \vec{dl} = \mu \times 0 \text{ (since net current enclosed by path is zero)}$$

As $dl \neq 0 \therefore B = 0$

This means that the magnetic field outside the solenoid is zero.

Field inside the solenoid: Consider a closed path pqrs The line integral of magnetic field along path pqrs is

$$\oint_{pqrs} \vec{B}.\vec{dl} = \int_{pq} \vec{B}.\vec{dl} + \int_{qr} \vec{B}.\vec{dl} + \int_{rs} \vec{B}.\vec{dl} + \int_{sp} \vec{B}.\vec{dl} \quad ...(i)$$

For path *pq*, $\vec{B}$ and $\vec{dl}$ are along the same direction,

$$\therefore \quad \int_{pq} \vec{B}.\vec{dl} = \int B \ dl = Bl \qquad (pq = l \text{ say})$$

For paths *qr* and *sp*, $\vec{B}$ and $d\vec{l}$ are mutually perpendicular.

$$\therefore \quad \int_{qr} \vec{B}.\vec{dl} = \int_{sp} \vec{B}.d\vec{l} = \int B \ dl \ \cos 90^o = 0$$

For path *rs*, $B = 0$ (since field is zero outside a solenoid)

$$\therefore \quad \int_{rs} \vec{B}.\vec{dl} = 0$$

In view of these, equation (i) gives

In view of these, equation (i) gives

$$\therefore \quad \oint_{pqrs} \vec{B}.\vec{dl} = \int_{pq} \vec{B}.\vec{dl} = Bl \qquad ...(ii)$$

By Ampere's law $\oint \vec{B}.\vec{dl} = \mu_0 \times$ net current enclosed by path

$\therefore Bl = \mu_0 \ (nl \ I) \quad \therefore B = \mu_0 \ nI$

This is the well known result.

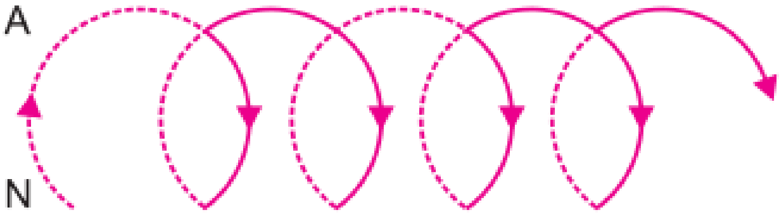
Q. 6. Using Ampere's circuital law, derive an expression for the magnetic field along the axis of a toroidal solenoid. [CBSE (AI) 2013]

OR

(a) State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius 'r', having 'n' turns per unit length and carrying a steady current I.

(b) An observer to the left of a solenoid of N turns each of cross section area 'A' observes that a steady current I in it flows in the clockwise direction. Depict the

magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment m = NIA. [CBSE Delhi 2015]



Ans. Magnetic field due to a toroidal solenoid: A long solenoid shaped in the form of closed ring is called a toroidal solenoid (or endless solenoid).

Let n be the number of turns per unit length of toroid and I the current flowing through it. The current causes the magnetic field inside the turns of the solenoid. The magnetic lines of force inside the toroid are in the form of concentric circles. By symmetry the magnetic field has the same magnitude at each point of circle and is along the tangent at every point on the circle.

(i) For points inside the core of toroid

Consider a circle of radius r in the region enclosed by turns of toroid. Now we apply Ampere's circuital law to this circular path, i.e.,

$$\oint \vec{B} . d\vec{l} = \mu_0 I \qquad \text{...(i)}$$

$$\oint \vec{B} . d\vec{l} = \oint \text{Bdl} \ \cos 0 = B.2\pi r$$

Length of toroid $= 2\pi r$

N = Number of turns in toroid = $n(2\pi r)$ and current in one-turn $= I$

$\therefore$ Current enclosed by circular path $= (n\ 2\pi r).\ I$

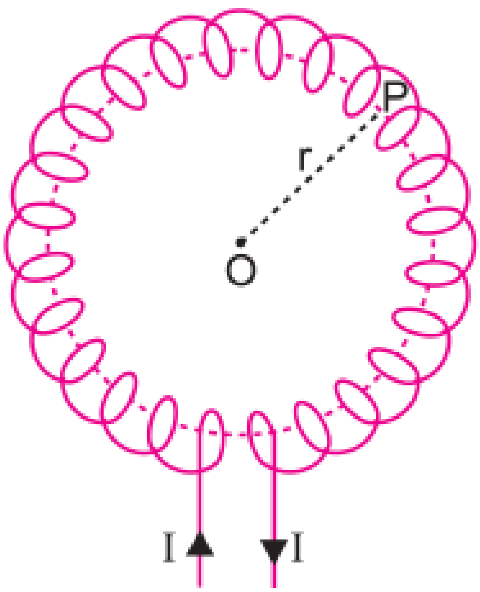
$\therefore$ Equation (i) gives

$$B\ 2\pi r = \mu_0\ (n\ 2\pi r I) \quad \Rightarrow \quad B = \mu_0\ nI$$

(ii) For points in the open space **inside the toroid:** No current flows through the Amperian loop, so I = 0

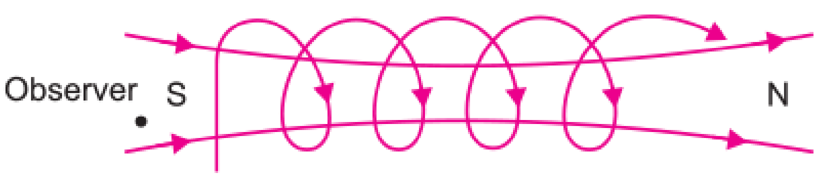
$$\oint \vec{B} . \vec{\text{dl}} = \mu_0 I = 0 \quad \Rightarrow \quad |B|_{\text{inside}} = 0$$

(iii) For points in the open space **exterior to the toroid :** The net current entering the plane of the toroid is exactly cancelled by the net current leaving the plane of the toroid.

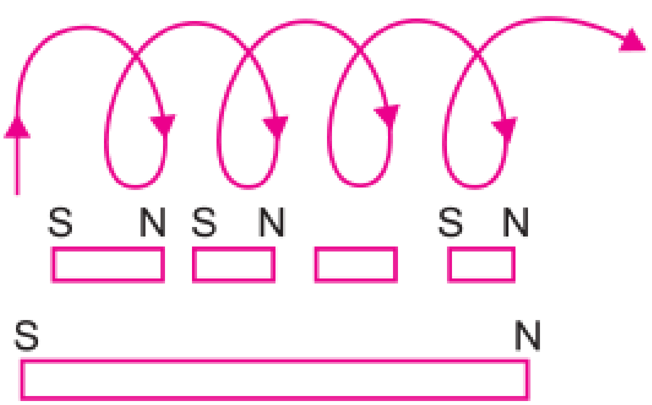


$$\oint \vec{B} . \vec{dl} = 0 \quad \Rightarrow \quad |B|_{\text{exterior}} = 0$$

For observer, current is flowing in clockwise direction hence we will see magnetic field lines going towards South Pole.



The solenoid can be regarded as a combination of circular loops placed side by side, each behaving like a magnet of magnetic moment IA, where I is the current and A area of the loop.



These magnets neutralise each other except at the ends where south and north poles appear.

Magnetic moment of bar magnet = NIA

Q. 7. (i) Draw a neat labeled diagram of a cyclotron.

(ii) Show that time period of ions in cyclotron is independent of both the speed of ion and radius of circular path. What is the significance of this property?

(ii) An electron after being accelerated through a potential difference of 100 V enters a uniform magnetic field of 0.004 T perpendicular to its direction of motion. Calculate the radius of the path described by the electron. [CBSE East 2016]

OR

(a) Explain with the help of a labelled diagram construction, principle and working of a cyclotron stating clearly the functions of electric and magnetic fields on a charged particle. Derive an expression for time period of revolution and cyclotron frequency. Show that it is independent of the speed of the charged particles and radius of the circular path. [CBSE (AI) 2009]

(b) What is resonance condition? How is it used to accelerated charged particles? [CBSE (AI) 2009]

(c) Also find the total KE attained by the charged particle.

(d) Is there an upper limit on the energy acquires by the article? Give reason.

OR

With the help of a labelled diagram, state the underlying principle of a cyclotron. Explain clearly how it works to accelerate the charged particles.

Show that cyclotron frequency is independent of energy of the particle. Is there an upper limit on the energy acquired by the particle? Give reason.
[CBSE Delhi 2011, 2014]

OR

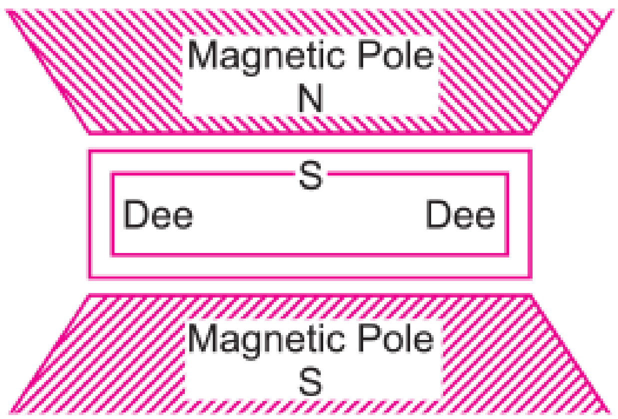
(a) Draw a schematic sketch of a cyclotron. Explain clearly the role of crossed electric and magnetic field in accelerating the charge. Hence derive the expression for the kinetic energy acquired by the particles.

(b) An α–particle and a proton are released from the centre of the cyclotron and made to accelerate.

(i) Can both be accelerated at the same cyclotron frequency? Give reason to justify your answer.

(ii) When they are accelerated in turn, which of the two will have higher velocity at the exit slit of the dees? [CBSE (AI) 2013]

Ans. Cyclotron: The cyclotron, devised by Lawrence and Livingston, is a device for accelerating charged particles to high speed by the repeated application of accelerating potentials.



Construction: The cyclotron consists of two flat semi - circular metal boxes called 'dees' and are arranged with a small gap between them. A source of ions is located near the mid-point of the gap between the dees (fig.). The dees are connected to the terminals of a radio frequency oscillator, so that a high frequency alternating potential of several million cycles per second exists between the dees. Thus dees act as electrodes. The dees are enclosed in an insulated metal box containing gas at low pressure. The whole apparatus is placed between the poles of a strong electromagnet which provides a magnetic field perpendicular to the plane of the dees.

Working: The principle of action of the apparatus is shown in figure. The positive ions produced from a source S at the centre are accelerated by a dee which is at negative potential at that moment. Due to the presence of perpendicular magnetic field the ion will move in a circular path inside the dees. The magnetic field and the frequency of the applied voltages are so chosen that as the ion comes out of a dee, the dees change their polarity (positive becoming negative and vice-versa) and the ion is further accelerated and moves with higher velocity along a circular path of greater radius. The phenomenon is continued till the ion reaches at the periphery of the dees where an auxiliary negative electrode (deflecting plate) deflects the accelerated ion on the target to be bombarded

Role of electric field.

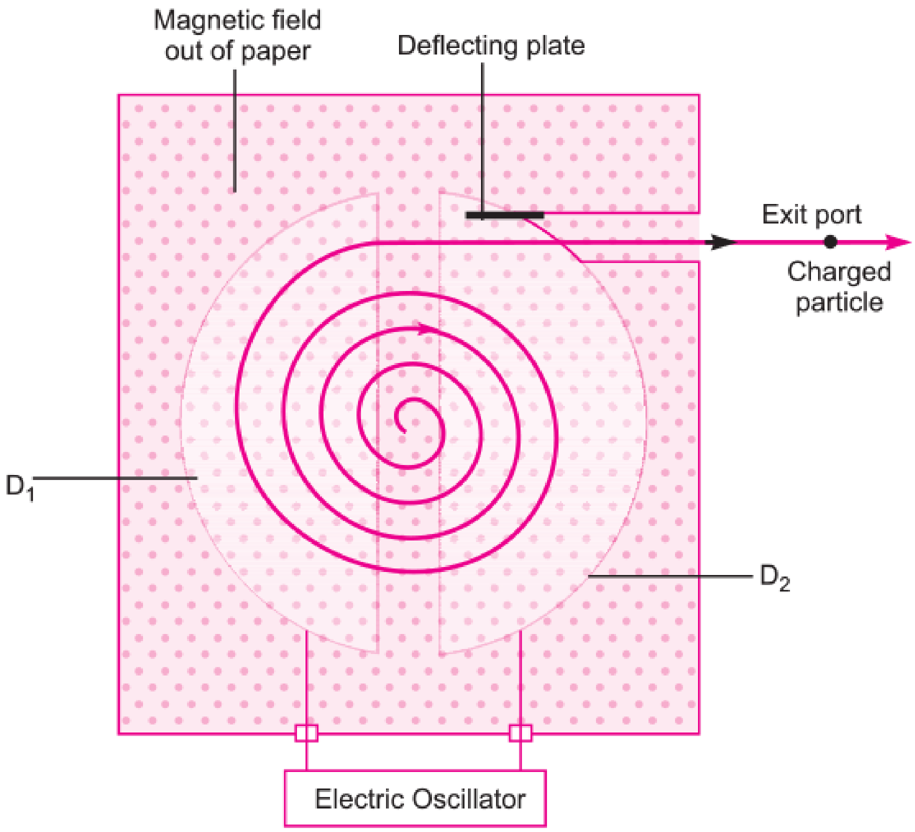
Electric field accelerates the charge particle passing through the gap.

Role of magnetic field

As the accelerated charge particle enters normally to the uniform magnetic field, it exerts a magnetic force in the form of centripetal force and charge particle moves on a semicircular path of increasing radii in each dee (D_1 or D_2) alternatively.

Expression for period of revolution and frequency:

Suppose the positive ion with charge q moves in a dee with a velocity v then,



$$qvB = \frac{mv^2}{r}$$

$$\text{or} \quad r = \frac{mv}{qB} \qquad \ldots(i)$$

Where m is the mass and r the radius of the path of ion in the dee and B is the strength of the magnetic field.

The angular velocity ω of the ion is given by,

$$\omega = \frac{v}{r} = \frac{qB}{m} \qquad (\text{from } eq.\ i) \qquad \ldots(ii)$$

The time taken by the ion in describing a semi-circle, i.e., in turning through an angle π is,

$$t = \frac{\pi}{\omega} = \frac{\pi m}{\mathrm{Bq}} \quad ...(iii)$$

Thus the time is independent of the speed of the ion i.e., although the speed of the ion goes on increasing with increase in the radius (from eq. i) when it moves from one dee to the other, yet it takes the same time in each dee.

From eq. (iii) it is clear that for a particular ion, m/q being known, B can be calculated for producing resonance with the high frequency alternating potential.

Significance: The applied voltage is adjusted so that the polarity of dees is reversed in the same time that it takes the ion to complete one half of the revolution.

Resonance condition: The condition of working of cyclotron is that the frequency of radio frequency alternating potential must be equal to the frequency of revolution of charged particles within the dees. This is called resonance condition.

Now for the cyclotron to work, the applied alternating potential should also have the same semi-periodic time (T/2) as that taken by the ion to cross either dee, i.e.,

$$\frac{T}{2} = t = \frac{\pi m}{\mathrm{qB}} \quad ...(iv)$$

$$\text{or} \quad T = \frac{2\pi m}{\mathrm{qB}} \qquad ...(v)$$

This is the expression for period of revolution.

Obviously, period of revolution is independent of speed of charged particle and radius of circular path.

∴ Frequency of revolution of particles

$$f = \frac{1}{T} = \frac{\mathrm{qB}}{2\pi m}$$

This frequency is called the **cyclotron frequency.** Clearly the cyclotron frequency is independent of speed of particle.

Expression for KE attained

If R be the radius of the path and v_{max} the velocity of the ion when it leaves the periphery, then in accordance with eq. (ii)

$$v_{\max} = \frac{\mathrm{qBR}}{m} \quad \text{...}(vi)$$

The kinetic energy of the ion when it leaves the apparatus is,

$$\mathrm{KE} = \frac{1}{2}\mathrm{mv}_{\max} = \frac{q^2B^2R^2}{2m} \quad \text{...}(vii)$$

When charged particle crosses the gap between dees it gains KE = qV

In one revolution, it crosses the gap twice, therefore if it completes n-revolutions before emerging the dees,

The kinetic energy gained = 2nqV ...(viii)

$$\text{Thus } \mathrm{KE} = \frac{q^2B^2R^2}{2m} = 2\,\mathrm{nqV}$$

No, from equation (i) $v = \frac{\mathrm{qBr}}{m}$

$$\Rightarrow \quad v = r\omega = \frac{\mathrm{qBr}}{m} \quad \Rightarrow \quad 2\pi v = \frac{\mathrm{qB}}{m} \quad \Rightarrow \quad v = \frac{\mathrm{qB}}{2\pi m}$$

Cyclotron frequency depends on $\left(\frac{q}{m}\right)$ ratio, since

$$\left(\frac{q}{m}\right)_\alpha < \left(\frac{q}{m}\right)_p$$

or $$f_\alpha < f_p$$

Kinetic energy, $KE = eV = \frac{mv^2}{2}$ for one revolution

$$v = \sqrt{\frac{2\ eV}{m}}$$

$v \propto \frac{e}{m}$ for proton $= \frac{e}{m_p}$ and for $\alpha -$particle, $\frac{2e}{4m} = \frac{e}{2m}$

Hence, proton acquires higher velocity as compared to a-particle.

$r = \frac{mv}{qB}$, here $v \propto r$ for fixed q, B, m.

Hence, the upper limit of energy depends upon the maximum radius of dees of cyclotron.

So, proton acquires higher velocity at the exit slit for fixed radius r ≤ R, where R is the radius of the dee.

$$r = \frac{mv}{qB} = \frac{\sqrt{2\,mqV}}{qB}$$

$$r = \frac{\sqrt{2\times 9.1\times 10^{-31}\times 1.6\times 10^{-19}\times 100}}{1.6\times 10^{-19}\times 0.004} m$$

$$r = \frac{5.4\times 10^{-24}}{6.4\times 10^{-22}} m = 8.4\times 10^{-3} m$$

Q. 8. Answer the following questions.

(i) Consider a beam of charged particles moving with varying speeds. Show how crossed electric and magnetic fields can be used to select charged particles of a particular velocity?

(ii) Name another device/machine which uses crossed electric and magnetic fields. What does this machine do and what are the functions of magnetic and electric fields in this machine? Where do these field exist in this machine? Write about their natures.
[CBSE South 2016]

Ans. (i) If we adjust the value of $\underset{E}{\rightarrow}$ and $\underset{B}{\rightarrow}$ such that magnitude of the two forces are equal, then total force on the charge is zero and the charge will move in the fields undeflected. This happen when

$$qE = Bqv \qquad \text{or} \qquad v = \frac{E}{B}$$

(ii) Name of the device: Cyclotron

It accelerates charged particles or ions.

Electric field accelerates the charged particles.

Magnetic field makes particles to move in circle.

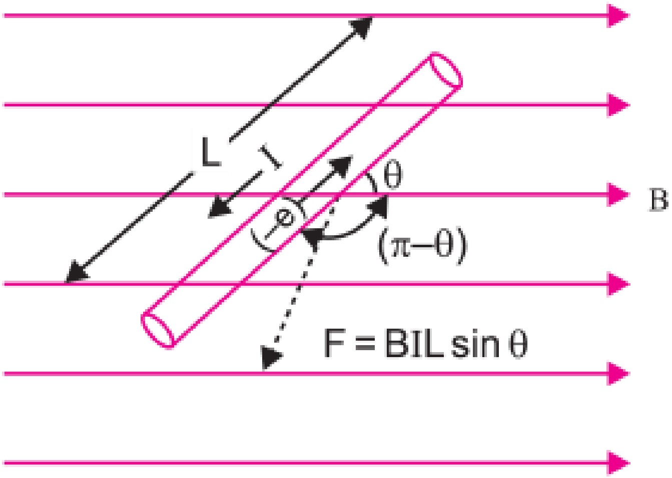
Electric field exists between the Dees.

Magnetic field exists both inside and outside the dees.

Magnetic field is uniform.

Electric field is alternating in nature.

Q. 9. Derive an expression for the force experienced by a current carrying straight conductor placed in a magnetic field. Under what condition is this force maximum?



Ans. Force on a current carrying conductor on the basis of force on a moving charge: Consider a metallic conductor of length L, cross-sectional area A placed in a uniform magnetic field B and its length makes an angle θ with the direction of magnetic field B. The current in the conductor is I.

According to free electron model of metals, the current in a metal is due to the motion of free electrons. When a conductor is placed in a magnetic field, the magnetic field exerts a force on every free-electron. The sum of forces acting on all electrons is the net force acting on the conductor. If vd is the drift velocity of free electrons, then

Current I = $neAv_d$ ...(i)

Where n is number of free electrons per unit volume.

Magnetic force on each electron $= ev_d B \sin\theta$ ...(ii)

Its direction is perpendicular to both $\underset{Vd}{\rightarrow}$ and $\underset{B}{\rightarrow}$

Volume of conductor V = AL

Therefore, the total number of free electrons in the conductor = nAL

Net magnetic force on each conductor

F = (force on one electron) × (number of electrons)

$= (ev_d B \sin\theta) . (nAL) = (neAv_d). BL \sin\theta$

Using equation (i) $F = IBL \sin\theta$ ...(iii)

$\therefore$ $F = ILB \sin\theta$

This is the general formula for the force acting on a current carrying conductor.

In vector form $\vec{F} = I\vec{L} \times \vec{B}$ ...(iv)

Force will be maximum when $\sin\theta = 1$ or $\theta = 90°$. That is when length of conductor is perpendicular to magnetic field.

Q. 10. Two long straight parallel conductors carry steady current I_1 and I_2 separated by a distance d. If the currents are flowing in the same direction, show how the magnetic field set up in one produces an attractive force on the other. Obtain the expression for this force. Hence define one ampere. [CBSE Delhi 2016]

OR

Derive an expression for the force per unit length between two long straight parallel current carrying conductors. Hence define SI unit of current (ampere). [CBSE (AI) 2009, 2010, 2012, Patna 2015]

Ans. Suppose two long thin straight conductors (or wires) PQ and RS are placed parallel to each other in vacuum (or air) carrying currents I_1 and I_2 respectively. It has been observed experimentally that when the currents in the wire are in the same direction, they experience an attractive force (fig. a) and when they carry currents in opposite directions, they experience a repulsive force (fig. b).

Let the conductors PQ and RS carry currents I_1 and I_2 in same direction and placed at separation r.

Consider a current–element ‘ab’ of length ΔL of wire RS. The magnetic field produced by current-carrying conductor PQ at the location of other wire RS

$$B_1 = \frac{\mu_0 I_1}{2\pi\, r} \qquad \ldots(i)$$

According to Maxwell’s right hand rule or right hand palm rule number 1, the direction of B_1 will be perpendicular to the plane of paper and directed downward. Due to this magnetic field, each element of other wire experiences a force. The direction of current element is perpendicular to the magnetic field; therefore the magnetic force on element ab of length ΔL

$$\Delta F = B_1 I_2\ \Delta L\ \sin\ 90^o = \frac{\mu_0 I_1}{2\pi\, r} I_2\ \Delta L$$

∴ The total force on conductor of length L will be

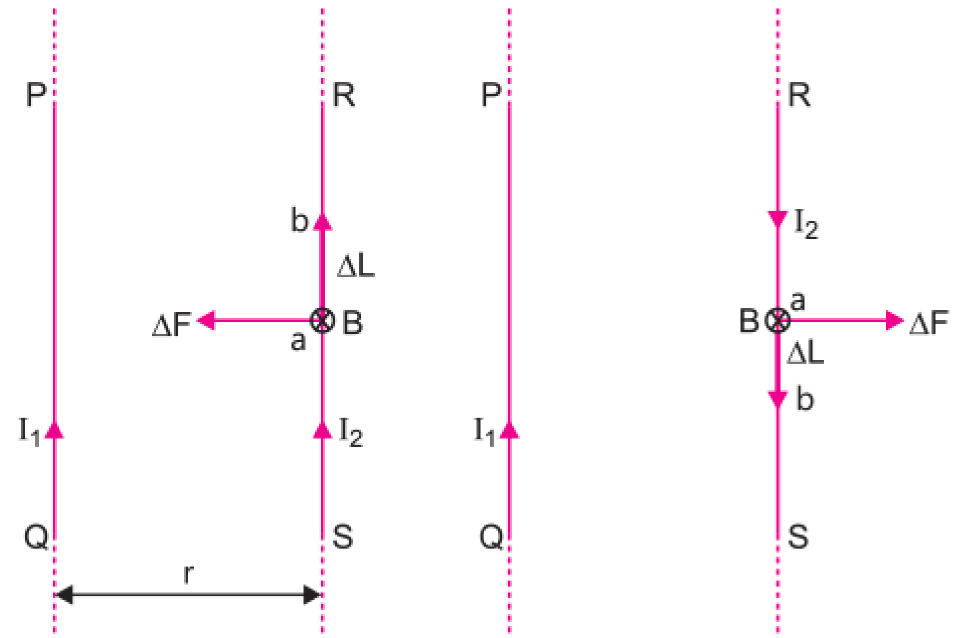
$$F = \frac{\mu_0 I_1 I_2}{2\pi\, r} \sum\ \Delta L = \frac{\mu_0 I_1\ I_2}{2\pi\, r} L$$

∴ Force acting per unit length of conductor

$$f = \frac{F}{L} = \frac{\mu_0 I_1\, I_2}{2\pi\, r} N\,/\,m \qquad \ldots(ii)$$

According to Fleming’s left hand rule, the direction of magnetic force will be towards PQ i.e. the force will be attractive.

On the other hand if the currents I_1 and I_2 in wires are in opposite directions, the force will be repulsive. The magnitude of force in each case remains the same.



Definition of SI unit of Current (ampere): In SI system of fundamental unit of current 'ampere' is defined assuming the force between the two current carrying wires as standard.

The force between two parallel current carrying conductors of separation r is

$$f = \frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} N/m$$

If $I_1 = I_2 = 1\text{A}$, $r = 1$ m, then

$$f = \frac{\mu_0}{2\pi} = 2 \times 10^{-2} N/m$$

Thus 1 ampere is the current which when flowing in each of parallel conductors placed at separation 1 m in vacuum exert a force of 2×10^{-7} on 1 m length of either wire.

Q. 11. Derive an expression for torque acting on a rectangular current carrying loop kept in a uniform magnetic field B. Indicate the direction of torque acting on the loop.
[CBSE Delhi 2013; (F) 2009]

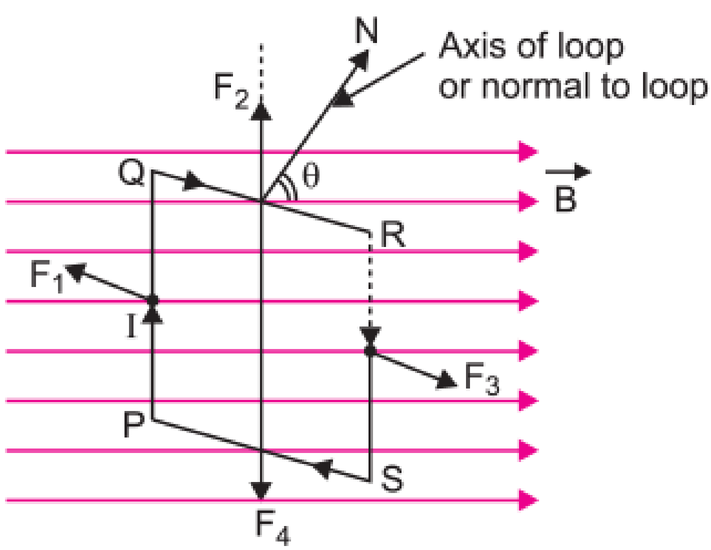
OR

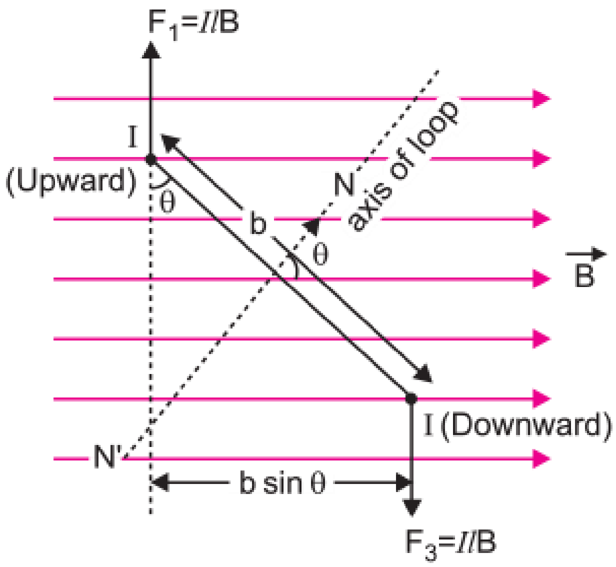
Deduce the expression for the torque $\vec{t}$ acting on a planar loop of area $\vec{A}$ and carrying current I placed in a uniform magnetic field $\vec{B}$

If the loop is free to rotate, what would be its orientation in stable equilibrium? [CBSE Ajmer 2015]

Ans. Torque on a current carrying loop: Consider a rectangular loop PQRS of length l, breadth b suspended in a uniform magnetic field $\vec{B}$ The length of loop = PQ = RS= l and breadth QR = SP = b. Let at any instant the normal to the plane of loop make an angle θ with the direction of magnetic field $\vec{B}$ and I be the current in the loop. We know that a force acts on a current carrying wire placed in a magnetic field. Therefore, each side of the loop will experience a force. The net force and torque acting on the loop will be determined by the forces acting on all sides of the loop. Suppose that the forces on sides PQ, QR, RS and SP are $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$ and $\vec{F}_4$ respectively. The sides QR and SP make angle (90°– θ) with the direction of magnetic field. Therefore each of the forces $\vec{F}_2$ and $\vec{F}_4$ acting on these sides has same magnitude F′ = Blb sin (90°– θ) = Blb cos θ. According to Fleming's left hand rule the forces are equal and opposite but their line of action is same. Therefore these forces cancel each other i.e. the resultant of $\vec{F}_2$ and $\vec{F}_4$ is zero.

The sides PQ and RS of current loop are perpendicular to the magnetic field, therefore the magnitude of each of forces $\vec{F}_1$ and $\vec{F}_3$ acting on sides PQ and RS are equal and opposite, but their lines of action are different; therefore the resultant force of $\vec{F}_1$ and $\vec{F}_3$ is zero, but they form a couple called the **deflecting couple.** When the normal to plane of loop makes an angle with the direction of magnetic field the perpendicular distance between F_1 and F_3 is b sin θ.





∴ Moment of couple or Torque,

τ = (Magnitude of one force F) × perpendicular distance = (BIl). (b sin θ)=I (lb) B sin θ

But lb = area of loop =A (say)

∴. Torque, τ = IAB sin θ

If the loop contains N-turns, then τ = NI AB sin θ

In vector form $\vec{\tau} = \mathrm{NI}\,\vec{A} \times \vec{B}$

The magnetic dipole moment of rectangular current loop = M = NIA

$$\therefore \vec{\tau} = \vec{M} \times \vec{B}$$

Direction of torque is perpendicular to direction of area of loop as well as the direction of magnetic field i.e., along $I\vec{A} \times \vec{B}.$

The current loop would be in stable equilibrium, if magnetic dipole moment is in the direction of the magnetic field ($\vec{B}$).

Q. 12. Answer the following questions.

(i) What is the relationship between the current and the magnetic moment of a current carrying circular loop?

(ii) Deduce an expression for magnetic dipole moment of an electron revolving around a nucleus in a circular orbit. Indicate the direction of magnetic dipole moment? Use the expression to derive the relation between the magnetic moment of an electron moving in a circle and its related angular momentum? [CBSE (AI) 2010; (F) 2009]

(iii) A muon is a particle that has the same charge as an electron but is 200 times heavier than it. If we had an atom in which the muon revolves around a proton instead of an electron, what would be the magnetic moment of the muon in the ground state of such an atom?

Ans. (i) Relation between current and magnetic moment:

Magnetic moment, for a current carrying coil is M = IA

For circular coil of radius r, A = πr^2

$$M = I.\ \pi r^2$$

(ii) Magnetic moment of an electron moving in a circle:

Consider an electron revolving around a nucleus (N) in circular path of radius r with speed v. The revolving electron is equivalent to electric current

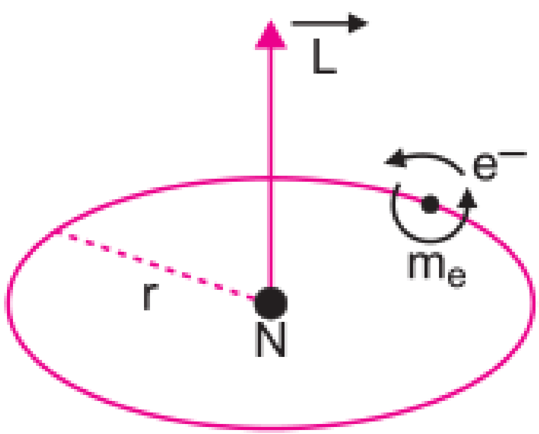
$$I = \frac{e}{T}$$

where T is period of revolution $= \frac{2\pi r}{v}$

$$I = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r} \qquad ...(i)$$

Area of current loop (electron orbit), $A = \pi r^2$

Magnetic moment due to orbital motion,



$$M_l = IA = \frac{ev}{2\pi r} = (2\pi r^2) = \frac{evr}{2} ...(ii)$$

This equation gives the magnetic dipole moment of a revolving electron. The direction of magnetic moment is along the axis.

Relation between magnetic moment and angular momentum

Orbital angular momentum of electron

$L = m_e vr$ ...(iii)

Where m_e is mass of electron,

Dividing (ii) by (iii), we get

$$\frac{M_l}{L} = \frac{\text{ev } r/2}{m_e v r} = \frac{e}{2m_e}$$

$$\text{Magnetic moment } \overrightarrow{M_l} = -\frac{e}{2m_e}\vec{L} \qquad \ldots(iv)$$

This is expression of magnetic moment of revolving electron in terms of angular momentum of electron.

$$\text{In vector form } \overrightarrow{M_l} = -\frac{e}{2m_e}\vec{L} \qquad \ldots(v)$$

Q. 13. Draw the labelled diagram of a moving coil galvanometer. Prove that in a radial magnetic field, the deflection of the coil is directly proportional to the current flowing in the coil. [CBSE (F) 2012]

(a) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(b) Answer the following:

(i) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

(ii) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain, giving reason. [CBSE (AI) 2014]

OR

Explain, using a labelled diagram, the principle and working of a moving coil galvanometer. What is the function of (i) uniform radial magnetic field, (ii) soft iron core?

Define the terms (i) current sensitivity and (ii) voltage sensitivity of a galvanometer. Why does increasing the current sensitivity not necessarily increase voltage sensitivity? [CBSE Allahabad 2015]

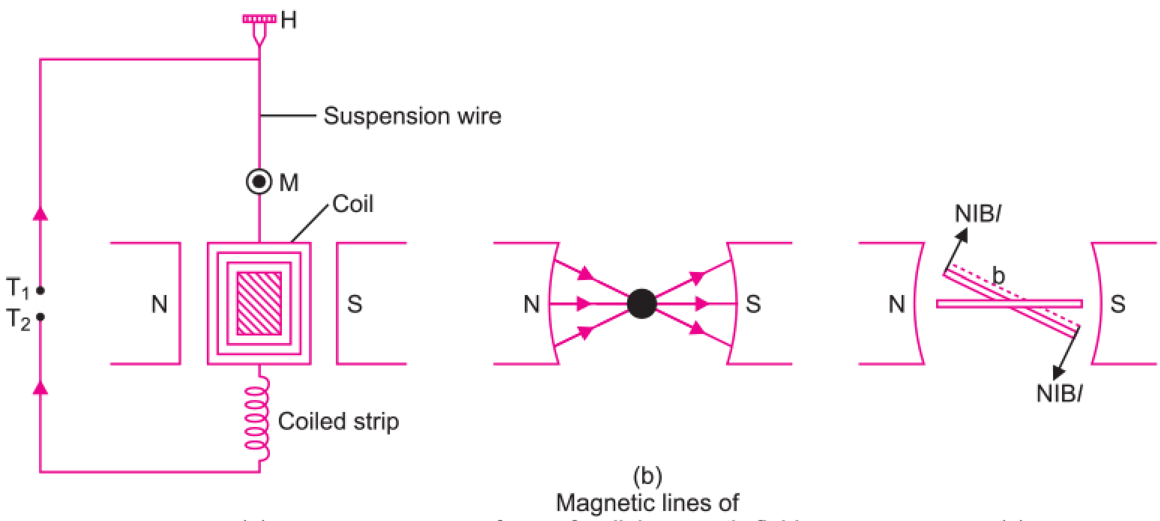
Ans. Moving coil galvanometer: A galvanometer is used to detect current in a circuit.

Construction: It consists of a rectangular coil wound on a non-conducting metallic frame and is suspended by phosphor bronze strip between the pole-pieces (N and S) of a strong permanent magnet.

A soft iron core in cylindrical form is placed between the coil.

One end of coil is attached to suspension wire which also serves as one terminal (T1) of galvanometer. The other end of coil is connected to a loosely coiled strip, which serves as the other terminal (T2). The other end of the suspension is attached to a torsion head which can be rotated to set the coil in zero position. A mirror (M) is fixed on the phosphor bronze strip by means of which the deflection of the coil is measured by the lamp and scale arrangement. The levelling screws are also provided at the base of the instrument.

The pole pieces of the permanent magnet are cylindrical so that the magnetic field is radial at any position of the coil.



(a) (b) Magnetic lines of force of radial magnetic field (c)

Principle and working: When current (I) is passed in the coil, torque τ acts on the coil, given by

$$\tau = NIAB \sin\theta$$

Where θ is the angle between the normal to plane of coil and the magnetic field of strength B, N is the number of turns in a coil.

A current carrying coil, in the presence of a magnetic field, experiences a torque, which produces proportionate deflection.

i.e., Deflection, $\theta \propto \tau$ (Torque)

When the magnetic field is radial, as in the case of cylindrical pole pieces and soft iron core, then in every position of coil the plane of the coil, is parallel to the magnetic field lines, so that θ =90-° and sin 90°=1. The coil experiences a uniform coupler.

Deflecting torque, τ = NIAB

If C is the torsional rigidity of the wire and is the twist of suspension strip, then restoring torque =C θ

For equilibrium, deflecting torque = restoring torque

i.e. $$NIAB = C\theta$$

∴ $$\theta = \frac{\mathrm{NAB}}{C} I \qquad ...(i)$$

i.e. $$\theta \propto I$$

Deflection of coil is directly proportional to current flowing in the coil and hence we can construct a linear scale.

Importance (or function) of uniform radial magnetic field: Torque for current carrying coil in a magnetic field is τ = NIAB sin θ

In radial magnetic field sinθ = 1, so torque is τ = NIAB

This makes the deflection (θ) proportional to current. In other words, the radial magnetic field makes the **scale linear.**

The cylindrical, soft iron core makes the field radial and increases the strength of the magnetic field, i.e., the magnitude of the torque.

Sensitivity of galvanometer:

Current sensitivity: It is defined as the deflection of coil per unit current flowing in it.

$$\text{Sensitivity, } S_I = \left(\frac{\theta}{I}\right) = \frac{\mathrm{NAB}}{C} \quad ...(i)$$

Voltage sensitivity: It is defined as the deflection of coil per unit potential difference across its ends

i.e., $$S_V = \frac{\theta}{V} = \frac{\mathrm{NAB}}{R_g . C}, \qquad ...(ii)$$

Where R_g is resistance of galvanometer.

Clearly for greater sensitivity number of turns N, area A and magnetic field strength B should be large and torsional rigidity C of suspension should be small.

Dividing (ii) by (i)

$$\frac{S_V}{S_I} = \frac{1}{G} \quad \Rightarrow \quad S_V = \frac{1}{G} S_I$$

Clearly the voltage sensitivity depends on current sensitivity and the resistance of galvanometer. If we increase current sensitivity then it is not certain that voltage sensitivity will be increased. Thus, the increase of current sensitivity does not imply the increase of voltage sensitivity.

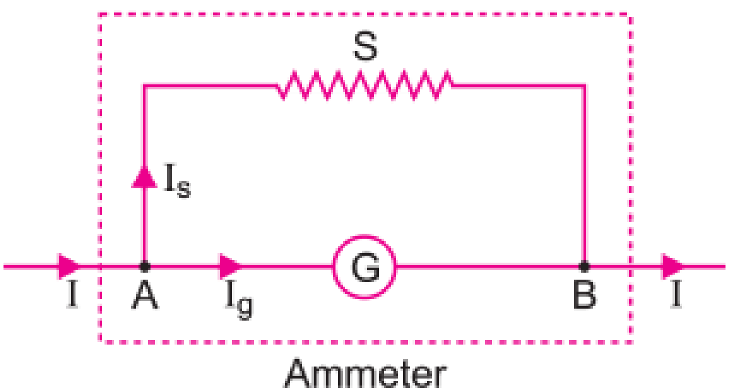
Q. 14. With the help of a circuit, show how a moving coil galvanometer can be converted into an ammeter of a given range. Write the necessary mathematical formula.

Ans. Conversion of galvanometer into ammeter

An ammeter is a low resistance galvanometer and is connected in series in a circuit to read current directly.

The resistance of an ammeter is to be made as low as possible so that it may read current without any appreciable error. Therefore to convert a galvanometer into ammeter a **shunt resistance.** (I.e. small resistance in parallel) is connected across the coil of galvanometer.

Let G be the resistance of galvanometer and I_g the current required for full scale deflection. Suppose this galvanometer is to converted into ammeter of range I ampere and the value of shunt required is S. If It I_s current in shunt, then from fig.



$I = I_g + I_S \Rightarrow I_S = (I - I_g)$ ...(i)

Also potential difference across A and B

$$(V_{AB}) = I_S . S = I_g . G$$

Substituting value of I_S from (i), we get

Or $(I - I_g) S = I_g G$

Or $I_S - I_g S = I_g G$ or $I_S = I_g (S+G)$

or $I_g = \frac{S}{S+G} I$ ...(ii)

i.e. required shunt, $S = \frac{GI_g}{I - I_g}$ ...(iii)

This is the working equation of conversion of galvanometer into ammeter.

The resistance (R_A) of ammeter so formed is given by

$$\frac{1}{R_A} = \frac{1}{S} + \frac{1}{G} \quad \text{or} \quad \frac{1}{R_A} = \frac{S+G}{SG} \quad \Rightarrow \quad R_A = \frac{SG}{S+G}$$

If k is figure of merit of the galvanometer and n is the number of scale divisions, then I_g = nk. Out of the total main current I amperes, only a small permissible value Ig flows through the galvanometer and the rest $I_S = (I - I_g)$ passes through the shunt.

Remark: An ideal ammeter has zero resistance.

Q. 15. A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto V/2 volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2.
[CBSE Delhi 2015]

Ans. Let I_g be the current through galvanometer at full deflection

To measure V volts, $V = I_g (G + R1)$ ...(i)

$\frac{V}{2}$ volts, $\quad \frac{V}{2} = I_g(G + R_2)$ ...(*ii*)

2 V volts, $\quad 2V = I_g(G + R_3)$ ...(*iii*)

To measure for conversion of range dividing (*i*) by (*ii*),

$$2 = \frac{G+R_1}{G+R_2} \quad \Rightarrow \quad G = R_1 - 2R_2$$

Putting the value of G in (*i*), we have

$$I_g = \frac{V}{R_1 - 2R_2 + R_1} \quad \Rightarrow \quad I_g = \frac{V}{2R_1 - 2R_2}$$

Substituting the value of G and I_g in equation (*iii*), we have

$$2V = \frac{V}{2R_1 - 2R_2}(R_1 - 2R_2 + R_3)$$

$$4R_1 - 4R_2 = R_1 - 2R_2 + R_3$$

$$R_3 = 3R_1 - 2R_2$$