Very Short Answer Questions (PYQ)

[1 Mark]

Q.1. If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

Ans.

Given $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$

$$\Rightarrow \quad A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$$

Q.2. If matrix *A* = [1, 2, 3], then write *AA'*, where *A'* is the transpose of matrix *A*.

Ans.

Given $A = (1 \ 2 \ 3)$

$$A' = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$AA' = (1 \times 1 + 2 \times 2 + 3 \times 3) = (14)$$

Q.3. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, then find the value of *x* and *y*.

Ans.

$$\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$$

Equating the corresponding elements, we get

$$x + 3y = 4, \ y = -1, \ 7 - x = 0$$

$$\Rightarrow \quad x = 7, \ y = -1$$

Q.4. Find the value of *x*, if $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$

Ans.

Given $\begin{bmatrix} 3x+y & -y \\ 2y-x & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$

Equating the corresponding elements, we get

$\Rightarrow \quad 3x + y = 1, - y = 2, 2y - x = - 5$

$\Rightarrow \quad y = - 2$

Putting it in $3x + y = 1$ or $2y - x = - 5$

we get $3x - 2 = 1$ or $- 4 - x = - 5$

$\Rightarrow \quad x = 1$ or $- x = - 1 \quad \Rightarrow \quad x = 1$

Hence, $x = 1$ and $y = -2$

Q.5. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, then find the value of *y*.

Ans. Equating the corresponding elements, we get

$y + 2x = 7$ and $-x = -2$

$\Rightarrow \quad x = 2$ and $y + 2 \times 2 = 7$

$\Rightarrow \quad y = 7 - 4 = 3$

Q.6. If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & 1 & -1 \\ -3 & 4 & 4 \\ 1 & 5 & 2 \end{bmatrix}$, then find $a_{22} + b_{21}$.

Ans. $a_{22} = 4$, $b_{21} = -3$

$\therefore \quad a_{22} + b_{21} = 4 - 3 = 1$

Q.7. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, *A* is an identity matrix?

Ans.

If A is identity matrix, then $A = I_2$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On equating, we get

$\Rightarrow \quad \cos\alpha = 1, \quad \sin\alpha = 0 \quad \Rightarrow \alpha = 0$

Q.8. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$**, then find the value of *k*.**

Ans.

Given: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} (1)(3)+(2)(2) & (1)(1)+(2)(5) \\ (3)(3)+(4)(2) & (3)(1)+(4)(5) \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

Equating the corresponding elements, we get

$$k = 17$$

Q.9. Write a square matrix of order 2, which is both symmetric and skew symmetric.

Ans. Square matrix of order 2, which is both symmetric and skew symmetric is

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.10. From the following matrix equation, find the value of *x* :

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$

Ans.

Given matrix equation

$$\begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$$

Equating the corresponding elements, we get,

$$x + y = 3 \quad \text{and} \quad 3y = 6$$

i.e., $y = 2$ and $x = 1$

$\therefore$ $x = 1, y = 2.$

Q.11. From the following matrix equation, find the value of *x* :

Ans.

$$\begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

Q.12. If $\begin{bmatrix} 7y & 5 \\ 2x - 3y & -3 \end{bmatrix} = \begin{bmatrix} -21 & 5 \\ 11 & -3 \end{bmatrix}$**, then find the value of *x*.**

Ans. Using equality of two matrices

$7y = -21 \quad \Rightarrow \quad y = -3$

$2x - 3y = 11 \quad \Rightarrow \quad 2x + 9 = 11$

$\Rightarrow \quad x = 1$

$\therefore \quad x = 1, y = -3$

Q.13. Write the order of the product matrix.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4]$$

Ans. Order is 3 × 3 because it is product of two matrices having order 3 × 1 and 1 × 3.

Q.14. For a 2 × 2 matrix, $A = [a_{ij}]$, whose elements are given by $a_{ij} = \frac{i}{j}$, write the value of a_{12}.

Ans.

$$\because \qquad a_{ij} = \frac{i}{j}$$

$$\Rightarrow \qquad a_{12} = \frac{1}{2} \qquad [\text{Here } i = 1 \text{ and } j = 2]$$

Q.15. Simplify:

$$\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

Q.16. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$**, then find $A^T - B^T$.**

Ans.

$$\text{Given: } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\therefore \qquad B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\text{Now } A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Q.17. Find the value of $x + y$ from the following equation:

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Ans.

Given:

$$\Rightarrow \quad \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Equating the corresponding elements, we get

$$2 + y = 5 \quad \text{and} \quad 2x + 2 = 8$$

$$\Rightarrow \quad y = 3 \quad \text{and} \quad x = 3$$

$$\therefore \quad x + y = 3 + 3 = 6.$$

Q.18. For what value of *x*, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ **a skew-symmetric matrix?**

Ans. A will be skew symmetric matrix if $A = -A'$

$$\Rightarrow \quad \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix} = - \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -x \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

Equating the corresponding elements, we get $x = 2$

Q.19. If *A* is a 3 × 3 matrix, whose elements are given by $a_{ij} = \frac{1}{3}|-3i + j|$, **then write the value of** a_{23}

Ans. $a_{23} = \frac{1}{3}|-3 \times 2 + 3| = \frac{1}{3}|-6 + 3| = \frac{1}{3} \times 3 = 1$

Q.20. If *A* is a square matrix and |*A*| = 2, then write the value of |*AA*'|, where *A*' is the transpose of matrix *A*.

Ans. $|AA'| = |A|.|A'| = |A|.|A| = |A|^2 = 2^2 = 4$.

[**Note:** $|AB| = |A|.|B|$ and $|A| = |A^T|$, where *A* and *B* are square matrices.]

Q.21. If *A* is a 3 × 3 invertible matrix, then what will be the value of *k* if det $(A^{-1}) = (\det A)^k$.

Ans.

Given, det $(A^{-1}) = (\det A)k$

$\Rightarrow \quad |A^{-1}| = |A|k \quad \Rightarrow \quad \frac{1}{|A|} = |A|^k$

$\Rightarrow \quad |A|^{-1} = |A|k \quad \Rightarrow \quad k = -1$

Q.22. If 4 $\sin^{-1} x + \cos^{-1} x = \pi$, then find the value of x.

Ans.

Given $4\sin^{-1}x + \cos^{-1}x = \pi$

$\Rightarrow \quad 4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$

$\Rightarrow \quad 3\sin^{-1}x = \pi - \frac{\pi}{2} \quad \Rightarrow \quad 3\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{6} \quad \Rightarrow \quad x = \sin\frac{\pi}{6} = \frac{1}{2}$

Very Short Answer Questions (OIQ)

[1 Mark]

Q.1. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then write A^n.

Ans. Given $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \quad \therefore \quad A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

Q.2. If $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then find the least positive integral value of *k*.**

Ans.

Least positive integral value of k is 7. Since we have

$$\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix}^7 = \begin{bmatrix} \cos 2\pi & -\sin 2\pi \\ \sin 2\pi & \cos 2\pi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, then write the value of x.

Ans.

Given $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} . \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2\times 1+3\times(-2) & 2\times(-3)+3\times 4 \\ 5\times 1+7\times(-2) & 5\times(-3)+7\times 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

Equating the corresponding elements, we get

$x = 13$

Q.2. Find the value of *x* + *y* from the following equation:

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Ans.

Given, $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Equating the corresponding elements, we get

$2x+3=7$ and $2y-4=14 \quad \Rightarrow \quad x=\frac{7-3}{2}$ and $y=\frac{14+4}{2}$

$\Rightarrow \quad x = 2$ and $y = 9$

$\therefore \quad x + y = 2 + 9 = 11$

Q.3. If matrix $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **and $A^2 = kA$, then write the value of *k*.**

Ans.

Given: $A^2 = kA$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \Rightarrow k = 2$$

Q.4. If $\begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$, then find the value of *x*.

Ans.

Given,

$$\Rightarrow \begin{bmatrix} 3 & 4 \\ 2 & x \end{bmatrix} \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix} \qquad \Rightarrow \begin{bmatrix} 3(x)+4(1) \\ (2)(x)+(x)(1) \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x+4 \\ 3x \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix}$$

Equating the corresponding elements, we get

$3x + 4 = 19$ and $3x = 15$

$\Rightarrow$ $3x = 19 - 4,$ $3x = 15$

$\Rightarrow$ $3x = 15,$ $x = 5$

$\therefore$ $x = 5$

Q.5. If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of λ.

Ans.

Here, $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

Given, $A^2 = \lambda A$

$\Rightarrow \quad \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$\Rightarrow \quad 6 \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix} \quad \Rightarrow \quad \lambda = 6$

Q.6. If *A* is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

Ans.

We have $A^2 = I$

Now, $(A - I)^3 + (A + I)^3 - 7A = A^3 - 3A^2I + 3AI^2 - I^3 + A^3 + 3A^2I + 3AI^2 + I^3 - 7A$

$= 2A^3 + 6AI^2 - 7A$

$= 2A^3 + 6AI - 7A \quad [\because I^2 = I]$

$= 2A^2 . A + 6A - 7A \quad [\because AI = A]$

$= 2I.A + 6A - 7A \quad [\because A^2 = I]$

$= 2A + 6A - 7A = A \quad [\because IA = A]$

Q.7. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ **is given to be symmetric, find values of *a* and *b*.**

Ans.

We have $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

$\because$ A is symmetric matrix

$\Rightarrow$ $A^T = A$

$\Rightarrow$ $\begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$

Equating both sides, we get

$2b = 3$ and $3a = -2$ $\Rightarrow$ $b = \frac{3}{2}$ and $a = -\frac{2}{3}$.

Q.8. If $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ **and BA = (bij), find b21 + b32.**

Ans.

We have, $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$\therefore$ $BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2 \times 3}$

$[b_{ij}] = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}_{3\times3} \Rightarrow [b_{ij}] = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3\times3}$

Now, $b_{21} = -16$; $b_{32} = -2$

$\therefore$ $b_{21} + b_{32} = -16 - 2 = -18$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Solve for x, $[1 \quad x]\begin{bmatrix}2 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=[0]$.

Ans.

Given: $[1 \quad x]\begin{bmatrix}2 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=[0]$

$\Rightarrow \quad [1 \quad x]\begin{bmatrix}2 & -1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=[0] \qquad \Rightarrow \qquad [2+x \quad -1+2x]\begin{bmatrix}1\\3\end{bmatrix}=[0]$

$\Rightarrow \quad [2+x-3+6x]=[0]$

$\Rightarrow \quad -1+7x=0$

$\Rightarrow \quad x=\frac{1}{7}$

Q.2. If $[2 \quad 1 \quad 3]\begin{bmatrix}-1 & 0 & -1\\-1 & 1 & 0\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}=A$, then find the value of A.

Ans.

Given, $A=[2 \quad 1 \quad 3]\begin{bmatrix}-1 & 0 & -1\\-1 & 1 & 0\\0 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$=[-2-1+0 \quad 0+1+3 \quad -2+0+3]\begin{bmatrix}1\\0\\-1\end{bmatrix}=[-3 \quad 4 \quad 1]\begin{bmatrix}1\\0\\-1\end{bmatrix}$

$A=[-3+0-1]=[-4]$

Q.3. Find the value of x and y if

$$\begin{bmatrix}x+10 & y^2+2y\\0 & -4\end{bmatrix}=\begin{bmatrix}3x+4 & 3\\0 & y^2-5y\end{bmatrix}$$

Ans.

$x + 10 = 3x + 4$ and $y^2 + 2y = 3$

$\Rightarrow \quad 3x - x = 10 - 4 \qquad \Rightarrow \quad y^2 + 2y - 3 = 0$

$\Rightarrow \quad 2x = 6 \qquad \Rightarrow \quad y^2 + 3y - y - 3 = 0$

$\Rightarrow \quad x = 3 \qquad \Rightarrow \quad y(y + 3) - 1(y + 3) = 0$

$\Rightarrow \quad (y + 3)\ (y - 1) = 0$

$\Rightarrow \quad y = 1, - 3 \qquad ...(i)$

Also, $y^2 - 5y = - 4$

$\Rightarrow \quad y^2 - 5y + 4 = 0 \qquad \Rightarrow \quad y^2 - 4y - y + 4 = 0$

$\Rightarrow \quad y(y - 4) - 1(y - 4) = 0 \qquad \Rightarrow \quad (y - 4)\ (y - 1) = 0$

$\Rightarrow \quad y = 4, 1 \qquad ...(ii)$

From (i) and (ii) $y = 1$

i.e., $x = 3$ and $y = 1$

Q.4. If $\begin{bmatrix} a+b & 2 \\ 5 & \text{ab} \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Ans.

$$\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

According to definition of equality of matrix

$\Rightarrow \quad a + b = 6 \quad$ and $\quad ab = 8 \quad \Rightarrow \quad b = \frac{8}{a}$

$\Rightarrow \quad a + \frac{8}{a} = 6 \quad \Rightarrow \quad \frac{a^2+8}{a} = 6$

$\Rightarrow \quad a^2 + 8 = 6a \quad \Rightarrow \quad a^2 - 6a + 8 = 0$

$\Rightarrow \quad a^2 - 4a - 2a + 8 = 0$

$\Rightarrow \quad a(a - 4) - 2(a - 4) = 0 \quad \Rightarrow \quad (a - 2)(a - 4) = 0$

$\Rightarrow \quad a = 2, 4 \quad \therefore \quad b = 4, 2$

i.e., if $a = 2$ then $b = 4$ and if $a = 4$ then $b = 2$

Long Answer Questions-I (PYQ)

[4 Mark]

Q.1. Find the value of x, if $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0.$

Ans.

Given, $\begin{bmatrix}1 & x & 1\end{bmatrix}_{1\times 3}\begin{bmatrix}1 & 3 & 2\\ 2 & 5 & 1\\ 15 & 3 & 2\end{bmatrix}_{3\times 3}\begin{bmatrix}1\\ 2\\ x\end{bmatrix}_{3\times 1} = 0$

$\Rightarrow \quad \begin{bmatrix}1+2x+15 & 3+5x+3 & 2+x+2\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$

$\Rightarrow \quad \begin{bmatrix}16+2x & 6+5x & 4+x\end{bmatrix}\begin{bmatrix}1\\ 2\\ x\end{bmatrix} = 0$

$\Rightarrow \quad [(16+2x).1+(6+5x).2+(4+x).x] = 0$

$\Rightarrow \quad (16+2x)+(12+10x)+(4x+x2) = 0$

$\Rightarrow \quad x^2+16x+28 = 0 \quad \Rightarrow \quad (x+14)(x+2) = 0$

$\Rightarrow \quad x+14 = 0 \quad$ or $\quad x+2 = 0$

Hence, $\quad x = -14 \quad$ or $\quad x = -2$

Q.2. For the following matrices *A* and *B*, verify that (*AB*)' = *B*'*A*'.

$A = \begin{bmatrix}1\\ -4\\ 3\end{bmatrix},\ B = (-1,\ 2,\ 1)$

Ans.

Given: $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, \quad B = (-1,\ 2,\ 1)$

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} [-1 \ \ 2 \ \ 1] = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$B'A' = (-1 \ \ 2 \ \ 1)' \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} [1 \ \ -4 \ \ 3] = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$\therefore \qquad (AB)' = B'A'.$

Q.3. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then find the values of *a* and *b*.

Ans.

Here, $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$$\therefore \qquad A + B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix}$$

$$\Rightarrow \qquad (A+B)^2 = \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} \cdot \begin{bmatrix} 1+a & 0 \\ 2+b & -2 \end{bmatrix} = \begin{bmatrix} 1+a^2+2a & 0 \\ 2+2a+b+ab-4-2b & -2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+2a+1 & 0 \\ 2a-b+ab-2 & 4 \end{bmatrix}$$

$$\text{Again} \quad A^2 + B^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \cdot \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} a^2+b & a-1 \\ ab-b & b+1 \end{bmatrix} = \begin{bmatrix} a^2+b-1 & a-1 \\ ab-b & b \end{bmatrix}$$

Given, $(A + B)^2 = A^2 + B^2$

Given, $(A+B)^2 = A^2 + B^2$

$$\begin{bmatrix} a^2+2a+1 & 0 \\ 2a-b+\text{ab}-2 & 4 \end{bmatrix} = \begin{bmatrix} a^2+b-1 & a-1 \\ \text{ab}-b & b \end{bmatrix}$$

Equating the corresponding elements, we get

$a^2 + 2a + 1 = a^2 + b - 1 \quad \Rightarrow \quad 2a - b = -2 \qquad ...(i)$

$a - 1 = 0 \quad \Rightarrow \quad a = 1 \qquad ...(ii)$

$2a - b + ab - 2 = ab - b \quad \Rightarrow \quad 2a - 2 = 0 \qquad ...(iii)$

$b = 4 \qquad ...(iv)$

$a = 1$, $b = 4$ satisfy all four equations (i), (ii), (iii) and (iv)

Hence, $a = 1$, $b = 4$.

Q.4. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ **and** $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$**. Find a matrix *D* such that *CD* – *AB* = *O*.**

Ans.

Since A, B, C are all square matrices of order 2, and $CD - AB$ is well defined, D must be a square matrix of order 2.

Let $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $CD - AB = 0$ gives

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = O$$

or $$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

or $$\begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equating the corresponding elements of matrices, we get

$$2a + 5c - 3 = 0 \quad ...(i)$$

$$3a + 8c - 43 = 0 \quad ...(ii)$$

$$2b + 5d = 0 \quad ...(iii)$$

and $$3b + 8d - 22 = 0 \quad ...(iv)$$

Solving (i) and (ii), we get $a = -191$, $c = 77$. Solving (iii) and (iv), we get $b = -110$, $d = 44$.

Therefore $$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$$

Q.5. Express the following matrix as the sum of a symmetric and skew symmetric matrix, and verify your result.

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Ans.

Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

A can be expressed as

$$A = \tfrac{1}{2}(A + A') + \tfrac{1}{2}(A - A'), \quad ...(i) \quad \left[\because \tfrac{1}{2}\,(A + A') + \tfrac{1}{2}\,(A - A') = \tfrac{2A}{2} = A\right]$$

where, $A + A'$ and $A - A'$ are symmetric and skew symmetric matrices respectively.

Now, $$A + A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

Putting these values in (i) we get

$$A = \tfrac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \tfrac{1}{2}\begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$$

Verification:

$$\Rightarrow \quad \begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{1}{2} - \frac{5}{2} & -\frac{5}{2} - \frac{3}{2} \\ \frac{1}{2} + \frac{5}{2} & -2+0 & -2-3 \\ -\frac{5}{2} + \frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

Q.6. Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ **satisfies the equation** $x^2 - 6x + 17 = 0$**. Hence, find** A^{-1}

Ans.

We have, $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4-9 & -6-12 \\ 6+12 & -9+16 \end{bmatrix} = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix}$$

$$6A = 6\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} \text{ and } 17I = 17\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$\therefore \quad A^2 - 6A + 17I_2 = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} -5-12+17 & -18+18+0 \\ 18-18+0 & 7-24+17 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence, matrix A satisfies the equation, $x^2 - 6x + 17 = 0$

Now, $A^2 - 6A + 17I_2 = 0 \quad \Rightarrow \quad A^2 - 6A = -17I_2$

Multiplying both sides by A^{-1}, we have

$$A - 6I_2 = -17A^{-1}$$

$$\therefore \quad A^{-1} = \frac{1}{17}(6I_2 - A) = \frac{1}{17}\left\{\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}\right\} = \frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}.$$

Q.7. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ **and** I **is the identity matrix of order 2, then show that** $A^2 = 4A - 3I$**. Hence find** A^{-1}**.**

Ans.

Here, $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} . \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad ...(i)$$

Also, $4A - 3I = 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \quad ...(ii)$

From (i) and (ii), we get $A^2 = 4A - 3I$

Now, we have $A^2 = 4A - 3I$

Pre-multiplying both sides by A^{-1}

$$A^{-1} . A^2 = A^{-1} . (4A - 3I)$$

$$\Rightarrow \quad (A^{-1} . A) . A = 4 A^{-1} . A - 3 A^{-1} . I$$

$$\Rightarrow \quad IA = 4I - 3A^{-1}$$

$$\Rightarrow \quad A = 4I - 3A^{-1}$$

$$\Rightarrow \quad 3A^{-1} = 4I - A$$

$$\Rightarrow \quad A^{-1} = \frac{1}{3}\left(4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\right) = \frac{1}{3}\left(\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\right) \quad \Rightarrow$$
$$\frac{1}{3}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

Q.8. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$**, express A as a sum of two matrices such that one is symmetric and other is skew symmetric.**

Ans.

A can be expressed as

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A'), \qquad ...(i) \qquad \left[\because \frac{1}{2}(A+A') + \frac{1}{2}(A - A') = \frac{1}{2}(A + A' + A - A') = \frac{1}{2} \times 2A = A\right]$$

Where $A + A'$ and $A - A'$ are symmetric and skew symmetric matrices respectively.

Now, $$A + A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix}$$

$$A - A' = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 6 \\ 5 & 3 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

Putting the values of $(A + A')$ and $(A - A')$ in (i), we get

$$A = \frac{1}{2}\begin{bmatrix} 6 & 6 & 5 \\ 6 & 2 & 9 \\ 5 & 9 & 14 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -3 \\ -5 & 3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 1 & 9/2 \\ 5/2 & 9/2 & 7 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 5/2 \\ 1 & 0 & -3/2 \\ -5/2 & 3/2 & 0 \end{bmatrix}$$

Long Answer Questions-I (OIQ)

[4 Mark]

Q.1. If $A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix}$**, show that $A - A^T$ is a skew symmetric matrix where A^T is the transpose of matrix A.**

Ans.

Given: $A = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix} \quad \therefore \quad A^T = \begin{bmatrix} 3 & 7 \\ -4 & 8 \end{bmatrix}$

$$A - A^T = \begin{bmatrix} 3 & -4 \\ 7 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} 0 & -11 \\ 11 & 0 \end{bmatrix}$$

Also, $(A - A^T)^T = \begin{bmatrix} 0 & -11 \\ 11 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 11 \\ -11 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -11 \\ 11 & 0 \end{bmatrix} = -(A - A^T)$

$\Rightarrow \quad (A - A^T)^T$ is a skew symmetric matrix.

Q.2. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

Ans.

We have, $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$$4A = 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix}$$

Hence, $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Now, $\quad A^2 - 4A + I = O$

Therefore, $\quad A.A - 4A = -I$

or $\quad A.A(A^{-1}) - 4AA^{-1} = -IA^{-1}$ $\quad$ (Post multiplying by A^{-1} because $|A| \neq 0$)

or $\quad A(AA^{-1}) - 4I = -A^{-1}$

or $\quad A - 4I = -A^{-1}$ $\quad$ $[A.A^{-1} = I$ and $IA = AI = A]$

or $\quad A^{-1} = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Hence, $\quad A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Q.3. Solve the following:

Q. Prove that the sum of two skew-symmetric matrices is a skew-symmetric matrix.

Ans. Let A and B be two skew-symmetric matrices.

Then, $A' = -A$ and $B' = -B$.

$\therefore \quad (A + B)' = (A' + B') = (-A) + (-B) = -(A + B)$

Hence, $(A + B)$ is again a skew-symmetric.

Q. Express the following matrix as the sum of a symmetric and a skew-symmetric matrix.

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

Ans.

Let $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$

Let $P = \frac{A+A'}{2} = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$ and $P' = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = P,$

Hence, $\frac{A+A'}{2}$ is a symmetric matrix.

Now, $Q = \frac{A-A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$

Also, $Q' = \frac{1}{2}\begin{bmatrix} 0 & -9 & -9 \\ 9 & 0 & 3 \\ 9 & -3 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix} = -Q,$

Hence, $\frac{A-A'}{2}$ is a skew-symmetric matrix.

$$\therefore \quad P + Q = \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 6 & 10 \\ -12 & 16 & 6 \\ -8 & 12 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} = A$$

Hence, $A = \left(\frac{A+A'}{2}\right) + \left(\frac{A-A'}{2}\right)$

= Symmetric matrix + Skew-symmetric matrix.

Q.4. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **Then show that $A^2 - 4A + 71 = 0$. Using this result calculate A^5.**

Ans.

Here, $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$$A^2 = A \times A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$$

Now, $A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \text{ (zero matrix)}$$

$\Rightarrow \quad A^2 - 4A + 7I = 0 \quad \Rightarrow \quad A^2 = 4A - 7I$

$\Rightarrow \quad A.A^2 = 4A.A - 7A.I$ [Pre multiplying by A]

$\Rightarrow \quad A^3 = 4A^2 - 7A$ [$AI = A$]

$\Rightarrow \quad A^3 = 4(4A - 7I) - 7A$ [Putting the value of A^2]

$\Rightarrow \quad A^3 = 16A - 28I - 7A$

$\Rightarrow \quad A^3 = 9A - 28I$

$\Rightarrow \quad A.A^3 = 9A.A - 28A.I$ [Pre multiplying by A]

$\Rightarrow \quad A^4 = 9A^2 - 28A$

$\Rightarrow \quad A^4 = 9(4A - 7I) - 28A$ [Putting the value of A^2]

$\Rightarrow \quad A^4 = 8A - 63I$

$\Rightarrow \quad A.A^4 = 8A^2 - 63A$ [Pre multiplying by A]

$\Rightarrow \quad A^5 = 8(4A - 7I) - 63A = -31A - 56I$

$$= -31 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

Q.5. If $A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N.$

Ans.

We shall prove the result by using the principle of mathematical induction

When $n = 1$, we have

$$A^1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Thus, the result is true for $n = 1$.

Let the result be true for $n = m$.

Then $A^m = \begin{bmatrix} \cos m\theta & \sin m\theta \\ -\sin m\theta & \cos m\theta \end{bmatrix}$

$$\therefore \quad A^{m+1} = A.A^m = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos m\theta & \sin m\theta \\ -\sin m\theta & \cos m\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos m\theta - \sin\theta.\sin m\theta & \cos\theta\sin m\theta + \sin\theta\cos m\theta \\ -\sin\theta\cos m\theta - \cos\theta.\sin m\theta & -\sin\theta\sin m\theta + \cos\theta\cos m\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + m\theta) & \sin(\theta + m\theta) \\ -\sin(\theta + m\theta) & \cos(\theta + m\theta) \end{bmatrix} = \begin{bmatrix} \cos(m+1)\theta & \sin(m+1)\theta \\ -\sin(m+1)\theta & \cos(m+1)\theta \end{bmatrix}$$

Thus, the result is true for $n = (m + 1)$, whenever it is true for $n = m$.

Hence, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$ for all $n \in N$.

Q.6. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and skew-symmetric matrix.

Ans.

Let A be any square matrix. Then,

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = P + Q, \qquad \text{(say)},$$

where, $P = \frac{1}{2}(A + A^T), Q = \frac{1}{2}(A - A^T)$.

Now, $P^T = \left(\frac{1}{2}(A + A^T)\right)^T$ $\qquad [\because \ (KT)^T = K.A^T]$

$\Rightarrow \quad P^T = \frac{1}{2}[A^T + (A^T)^T]$ $\qquad [\because \ (A + B)^T = A^T + B^T]$

$\Rightarrow \quad P^T = \frac{1}{2}(A^T + A)$ $\qquad [\because \ (A^T)^T = A]$

$\Rightarrow \quad P^T = \frac{1}{2}(A + A^T) = P$

$\therefore$ P is symmetric matrix.

Also, $Q^T = \frac{1}{2}(A - A^T)^T = \frac{1}{2}[A^T - (A^T)^T] = \frac{1}{2}[A^T - A]$

$\Rightarrow \quad Q^T = -\frac{1}{2}[A - A^T] = -Q$

$\therefore$ Q is skew-symmetric matrix.

Thus, $A = P + Q$, where P is a symmetric matrix and Q is a skew-symmetric matrix.

Hence, A is expressible as the sum of a symmetric and a skew-symmetric matrix.

Uniqueness: If possible, let $A = R + S$, where R is symmetric and S is skew-symmetric, then,

$$A^T = (R + S)^T = R^T + S^T$$

$\Rightarrow \quad A^T = R - S$ $\qquad [\because \ R^T = R \text{ and } S^T = -S]$

Now, $A = R + S$ and $A^T = R - S$

$\Rightarrow \quad R = \frac{1}{2}[A + A^T] = P, \ S = \frac{1}{2}(A - A^T) = Q$

Hence, A is uniquely expressible as the sum of a symmetric and a skew-symmetric matrix.