Short Answer Questions

[2 marks]

Q.1. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300. Formulate an LPP. for finding how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.

Ans.

Let x and y be the number of necklaces and bracelets manufactured by small firm per day. If P be the profit, then objective function is given by

$P = 100x + 300y$ which is to be maximised under the constrains

$x + y \leq 24$ ...(*i*)

$12x+y\leq16$ ...(*ii*)

$x \geq 1, y \geq 1$ ...(*iii*)

Q.2. Two tailors, *A* and *B*, earn ₹300 and ₹400 per day respectively. *A* can stitch 6 shirts and 4 pairs of trousers while *B*can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

Ans.

Let *A* and *B* work for x and y days respectively.

Let Z be the labour cost.

$Z = 300x + 400y$

Subject to constraints

$6x + 10y \geq 60$

$4x + 4y \geq 32$

$x, y \geq 0$

Q.3. A company produces two types of goods *A* and *B*, that require gold and silver. Each unit of type *A* requires 3 g of silver and 1 g of gold while that of type *B* requires 1 g of silver and 2 g of gold. The company can produce a maximum of 9 g of silver and 8 g of gold. If each unit of type *A* brings a profit of ₹40 and that of type *B* ₹50, formulate LPP to maximize profit.

Ans.

Let x and y be the number of goods *A* and goods *B* respectively. If P be the profit then

$P = 40x + 50y$ which is to be maximised under constraints

$3x + y \leq 9$

$x + 2y \leq 8$

$x \geq 0, y \geq 0$

Q.4. A firm has to transport atleast 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹400 and each small van is ₹200. Not more than ₹3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

Ans.

Let the number of large vans and small vans be x and y respectively.

here transportation cost Z be objective be function,

$Z = 400x + 200y$ which is to be minimized under constraints

$200x + 80y \geq 1200 \quad \Rightarrow \quad 5x + 2y \geq 30$

$400x + 200y \leq 3000 \quad \Rightarrow \quad 2x + y \leq 15$

$x \leq y$, $x \geq 0$, $y \geq 0$

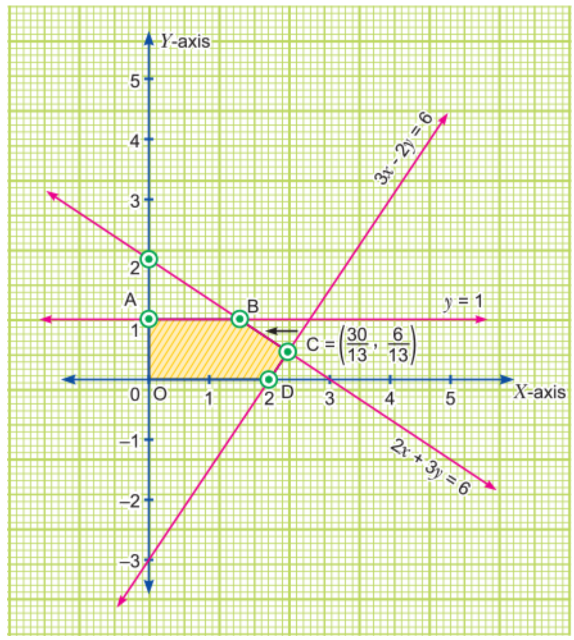
Long Answer Questions-I

[4 marks]

Q.1. Maximise Z = 8*x* + 9*y* subject to the constraints given below :

$2x + 3y \le 6$; $3x - 2y \le 6$; $y \le 1$; $x, y \ge 0$

Ans.



Given constraints are

$$2x + 3y \le 6$$

$$3x - 2y \le 6$$

$$y \le 1$$

$$x, y \ge 0$$

For graph of $2x + 3y \le 6$

We draw the graph of $2x + 3y = 6$

x	0	3
y	2	0

$2 \times 0 + 3 \times 0 \le 6 \Rightarrow (0,0)$ satisfy the constraints.

Hence, feasible region lie towards origin side of line.

For graph of $3x - 2y \le 6$

We draw the graph of line $3x - 2y = 6$.

x	0	2
y	–3	0

$3 \times 0 - 2 \times 0 \leq 6 \quad \Rightarrow \quad$ Origin (0, 0) satisfy $3x - 2y = 6$.

Hence, feasible region lie towards origin side of line.

For graph of $y \leq 1$

We draw the graph of line $y = 1$, which is parallel to x-axis and meet y-axis at 1.

$0 \leq 1 \Rightarrow$ feasible region lie towards origin side of $y = 1$.

Also, $x \geq 0$, $y \geq 0$ says feasible region is in Ist quadrant

Therefore, *OABCDO* is the required feasible region, having corner point $O(0, 0)$, $A(0, 1)$,

$B\left(\frac{3}{2}, 1\right)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$ $D(2, 0)$.

Here, feasible region is bounded. Now the value of objective function $Z = 8x + 9y$ is obtained as.

Corner Point	$Z = 8x + 9y$
O (0, 0)	0
A (0, 1)	9
$B(\frac{3}{2}, 1)$	21
$C(\frac{30}{13}, \frac{6}{13})$	22.6
$D(2, 0)$	16

Z is maximum when $x = \frac{30}{13}$ and $y = \frac{6}{13}$.

Q.2. Minimize and maximize $Z = 5x + 2y$ subject to the following constraints:

$x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x \geq 0$, $y \geq 0$

Ans.

Here, objective function is

$$Z = 5x + 2y \qquad ...(i)$$

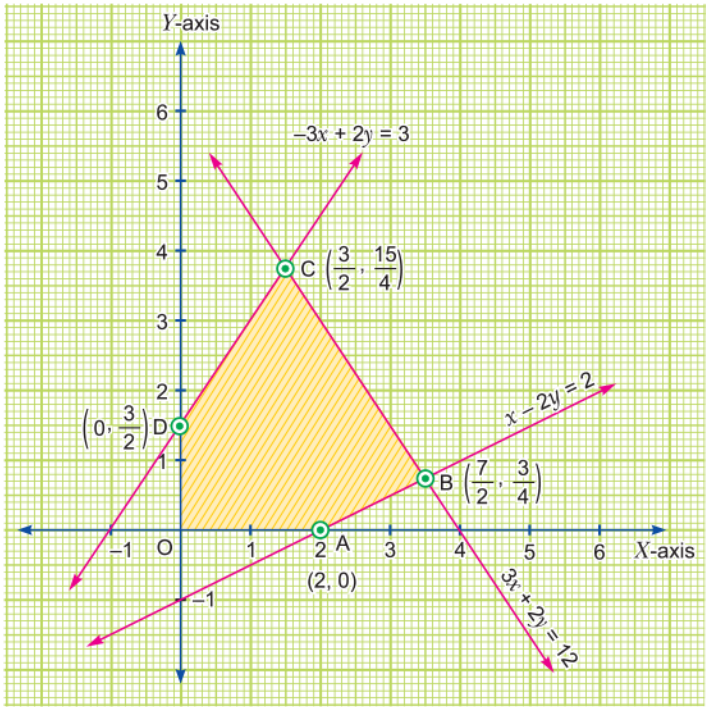
Subject to the constraints :

$$x - 2y \leq 2 \qquad ...(ii)$$

$$3x + 2y \leq 12 \qquad ...(iii)$$

$$-3x + 2y \leq 3 \qquad ...(iv)$$

$$x \geq 0,\ y \geq 0 \qquad ...(v)$$



Graph for $x - 2y \leq 2$

We draw graph of $x - 2y = 2$ as

x	0	2
y	– 1	0

$0 - 2 \times 0 \leq 2$

[By putting $x = y = 0$ in the equation]

i.e., (0, 0) satisfy (*ii*) ⇒ feasible region lie origin side of line $x - 2y = 2$.

Graph for $3x + 2y \leq 12$

We draw the graph of $3x + 2y = 12$.

x	0	4
y	6	0

$3 \times 0 + 2 \times 0 \leq 12$ [By putting $x = y = 0$ in the given equation]

i.e., (0, 0) satisfy (*iii*) ⇒ feasible region lie origin side of line $3x + 2y = 12$.

Graph for $-3x + 2y \le 3$

We draw the graph of $-3x + 2y \le 3$

x	-1	0
y	0	1.5

$-3 \times 0 + 2 \times 0 \le 3$[By putting $x = y = 0$]

i.e., (0, 0) satisfy (*iv*) ⇒ feasible region lies origin side of line $-3x + 2y = 3$.

$x \ge 0, y \ge 0$ ⇒ feasible region is in Its quadrant.

Now, we get shaded region having corner points *O, A, B, C* and *D* as feasible region. The co-ordinates of *O,*

A, B, C and *D* are $O(0, 0)$, $A(2, 0)$ $B\left(\frac{7}{2}, \frac{3}{4}\right)$, $C\left(\frac{3}{2}, \frac{15}{4}\right)$ and D $\left(0, \frac{3}{2}\right)$, respectively.

Now, we evaluate Z at the corner point as.

Corner Point	$Z = 5x + 2y$	
$O\,(0, 0)$	0	Minimum
$A\,(2, 0)$	10	
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	19	Maximum
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	15	
$D\left(0, \frac{3}{2}\right)$	3	

Hence,Z is minimum at $x = 0$, $y = 0$ and minimum value = 0

Z is maximum at $x = \frac{7}{2}$, $y = \frac{3}{4}$ and maximum value = 19

Q.3. Determine graphically the minimum value of the objective function

$Z = -50x + 20y$ ...(*i*)

Subject to the constraints

$2x - y \ge -5$ ...(*ii*)

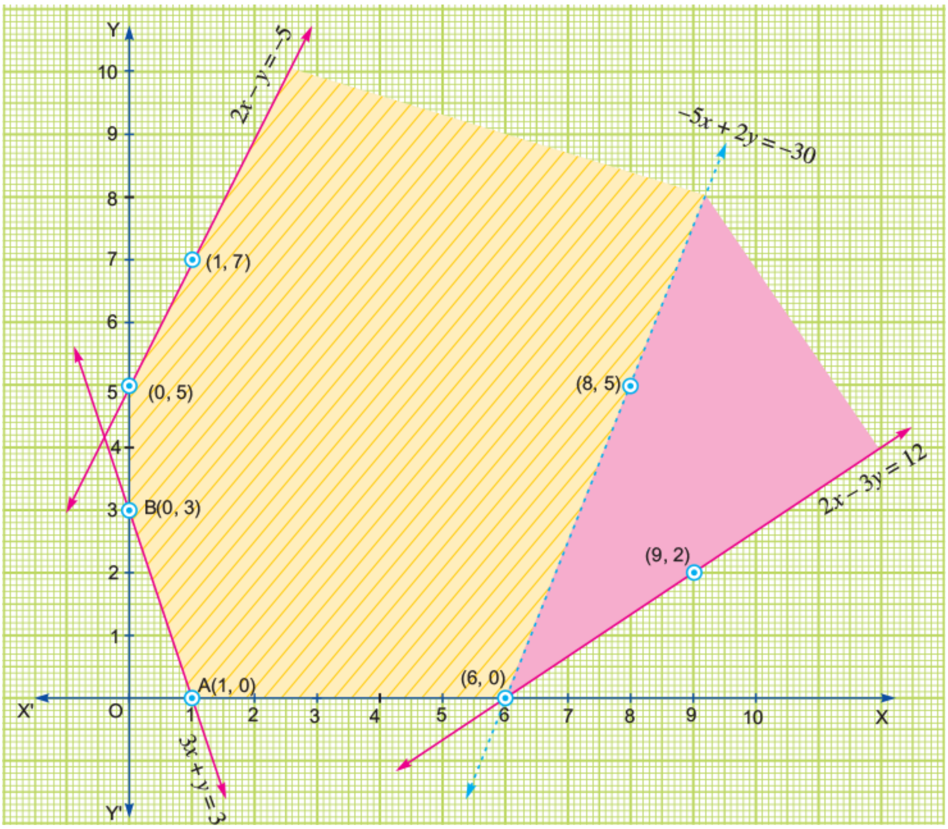
$3x + y \ge 3$ ...(*iii*)

$2x - 3y \leq 12$ ...(*iv*)

$x \geq 0, y \geq 0$ ...(*v*)

Ans.

First of all, let us graph the feasible region of the system of inequalities (*ii*) to (*v*). The feasible region (Shaded) is shown in the figure. Observe that the feasible region is unbounded.



We now evaluate Z at the corner points.

Corner Point	$Z = -50x + 20y$
(0, 5)	100
(0, 3)	60
(1, 0)	–50
(6, 0)	–300 ← Smallest

From this table, we find that –300 is the smallest value of *Z* at the corner point (6, 0). Can we say that minimum value of *Z* is (–)300? Note that if the region would have been bounded, this smallest value of *Z*is the minimum value of *Z*. But here we see that the feasible region is unbounded. Therefore, –300 may or may not be the minimum value of *Z*. To decide this issue, we graph the inequality.

$$-50x + 20y < -300$$

i.e., $-5x + 2y < -30$

and check whether the resulting open half plane has points in common with feasible region or not. If it has common points, then –300 will not be the minimum value of Z otherwise, –300 will be the minimum value of Z.

As shown in the figure, it has common points. Therefore, $Z = -50x + 20y$ has no minimum value subject to the given constraints.

Long Answer Questions-II (PYQ)

[6 marks]

Q.1. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

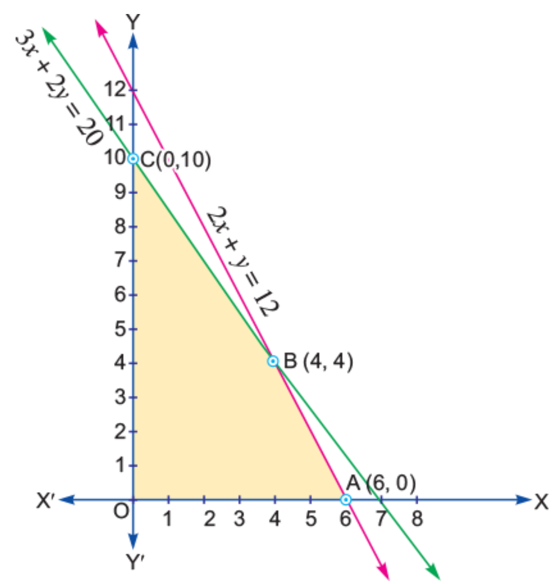
Ans.

Let the manufacturer produces x pedestal lamps and y wooden shades; then time taken by x pedestal lamps and y wooden shades on grinding/cutting machines = $(2x + y)$ hours and time taken on the sprayer = $(3x + 2y)$ hours.

Since, grinding/cutting machine is available for at the most 12 hours.

$\therefore \quad 2x + y \le 12$

and sprayer is available for at most 20 hours.



Thus, we have

$3x + 2y \le 20$

Now, profit on the sale of x lamps and y shades is,

$Z = 5x + 3y$.

So, our problem is to find x and y so as to

Maximise $Z = 5x + 3y$ …(*i*)

Subject to the constraints:

$3x + 2y \le 20$ …(*ii*)

$2x + y \le 12$ …(*iii*)

$x \ge 0$ …(*iv*)

$y \geq 0$ ...(v)

The feasible region (shaded) *OABC* determined by the linear inequalities (*ii*) to (*v*) is shown in the figure. The feasible region is bounded.

Let us evaluate the objective function at each corner point as shown below:

Corner Point	$Z = -50x + 20y$
(0, 5)	100
(0, 3)	60
(1, 0)	–50
(6, 0)	–300 ← Smallest

We find that maximum value of Z is ₹ 32 at *B*(4, 4). Hence, manufacturer should produce 4 lamps and 4 shades to get maximum profit of ₹ 32.

Q.2. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is ₹20 and ₹10 respectively, find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make it as an LPP and solve graphically.

Ans.

Let the number of tennis rackets and cricket bats manufactured by factory be x and y respectively.

Here, profit on x rackets and y bats is the objective function Z.

$Z = 20x + 10y$ ...(i)

We have to maximise Z subject to the constraints:

$1.5x + 3y \leq 42$ ...(ii) [Constraint for machine hour]

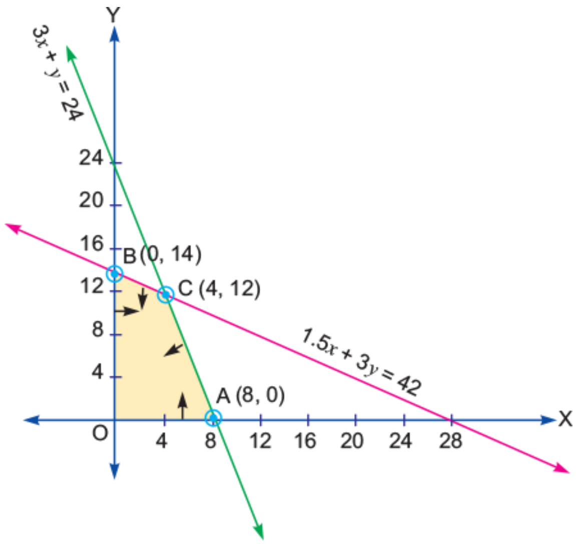
$3x + y \leq 24$ ...(iii) [Constraint for craft man's hour]

$x, y \geq 0$ ...(iv) [Non-negative constraints]

Graph of $x = 0$ and $y = 0$ is the y-axis and x-axis respectively.

∴ Graph of $x \geq$, $y \geq 0$ is the Ist quadrant.

Graph of $1.5x + 3y = 42$



x	0	28
y	14	0

∴ Graph for $1.\ 5x + 3y \leq 42$ is the part of Ist quadrant which contains the origin.

Graph for $3x + y \leq 24$

Graph of $3x + y = 24$

x	0	8
y	24	0

∴ Graph of $3x + y \leq 24$ is the part of Ist quadrant in which origin lie

Hence, shaded area *OACB* is the feasible region.

For coordinate of *C* equation $1.\ 5x + 3y = 42$ and $3x + y = 24$ are solved as

$$5x + 3y = 42 \qquad \ldots(v)$$

$$3x + y = 24 \qquad \ldots(vi)$$

$2 \times (v) - (vi) \quad \Rightarrow \quad 3x + 6y = 84$

$$-3x \pm y = -24$$

$$5y = 60$$

$\Rightarrow\ y = 12 \qquad\qquad \Rightarrow\ x = 4$ (Substituting $y = 12$ in (*iv*))

Now, value of objective function Z at each corner of feasible region is

Corner Point	$Z = 20x + 10y$
O (0, 0)	0
A (8, 0)	$20 \times 8 + 10 \times 0 = 160$
B (0, 14)	$20 \times 0 + 10 \times 14 = 140$
C (4, 12)	$20 \times 4 + 10 \times 12 = 200$ ← Maximum

Therefore, maximum profit is ₹ 200, when factory make 4 tennis rackets and 12 cricket bats.

Q.3. A dealer wishes to purchase a number of fans and sewing machines. He has only ₹ 5,760 to invest and has space for at the most 20 items. A fan costs him ₹ 360 and a sewing machine ₹ 240. He expects to sell a fan at a profit of ₹22 and a sewing machine for a profit of ₹ 18. Assuming that he can sell all the items that he buys, how should he invest his money to maximise his profit? Solve it graphically.

OR

A dealer in a rural area wishes to purchase some sewing machines. He has only ₹ 57,600 to invest and has space for at most 20 items. An electronic machine costs him ₹ 3,600 and a manually operated machine costs ₹ 2,400. He can sell an electronic machine at a profit of ₹ 220 and a manually operated machine at a profit of ₹ 180. Assuming that he can sell all the machines that he buys, how should he invest his money in order to maximise his profit? Make it as an LPP and solve it graphically.

Ans.

Let the dealer purchases x fans and y sewing machines, then cost of x fans and y sewing machines is given by

$360x + 240y$

$\therefore \qquad 360x + 240y \leq 5,760$

$\Rightarrow \qquad 3x + 2y \leq 48$

As, he has space for at most 20 items,

$\therefore \qquad x + y \leq 20$

Now, profit earned by the dealer on selling x fans and y sewing machines is = $22x + 18y$

Hence, our LPP is

Maximise $Z = 22x + 18y$ ...(*i*)

Subject to the constraints:

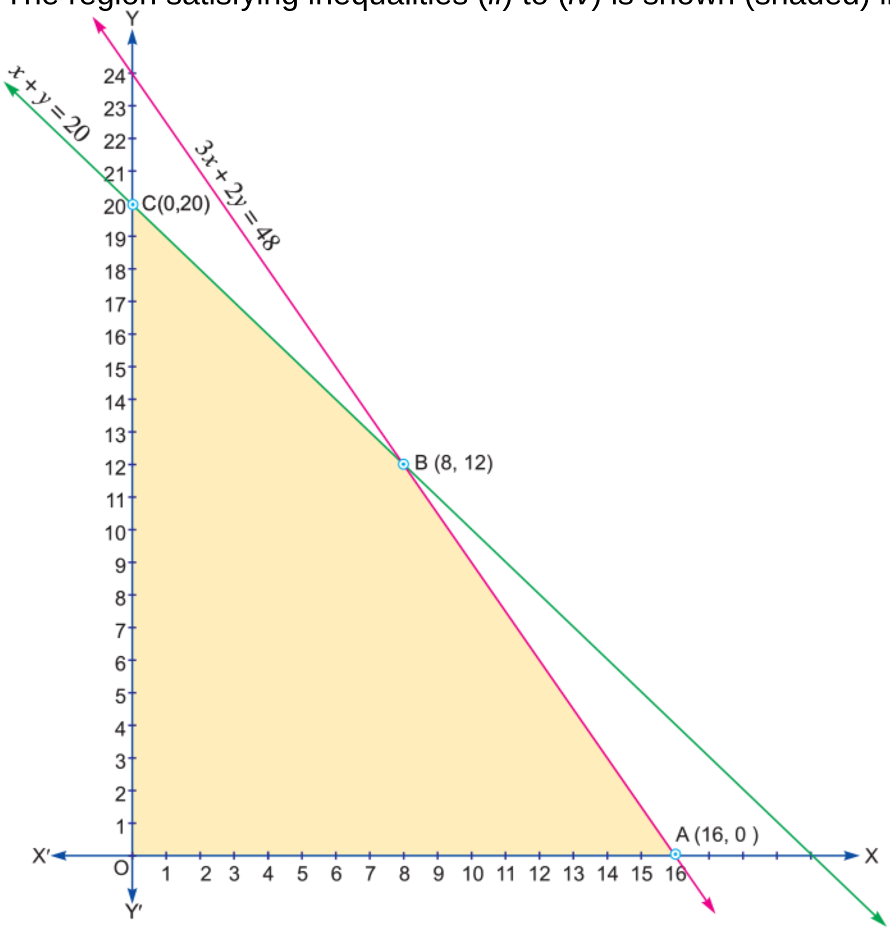
$3x + 2y \le 48$ ...(*ii*)

$x + y \le 20$ ...(*iii*)

$x, y \ge 0$ ...(*iv*)

Let us evaluate, $Z = 22x + 18y$ at each corner point.

The region satisfying inequalities (*ii*) to (*iv*) is shown (shaded) in the figure.



Corner Point	$Z = 22x + 18y$
O (0, 0)	0
A (16, 0)	352
B (8, 12)	392 ← Maximum
C (0, 20)	360

Thus, maximum value of Z is 392 at B (8, 12).

Hence the profit is maximum *i.e.*, ₹ 392 when he buys 8 fans and 12 sewing machines.

OR

Solve yourself as above solution. Here $Z = 220x + 180y$ is objective function.

Q.4. The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2. B_1costs ₹ 5 per kg and B_2 costs ₹ 8 per kg. Strength considerations dictate that the brick should contain not more than 4 kg of B_1 and minimum 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of brick satisfying the above conditions. Formulate this situation as an LPP and solve it graphically.

Ans.

Let x kg of B_1 and y kg of B_2 are taken for making brick.

Here, $Z = 5x + 8y$ is the cost which is objective function and is to be maximised subjected to following constraints.

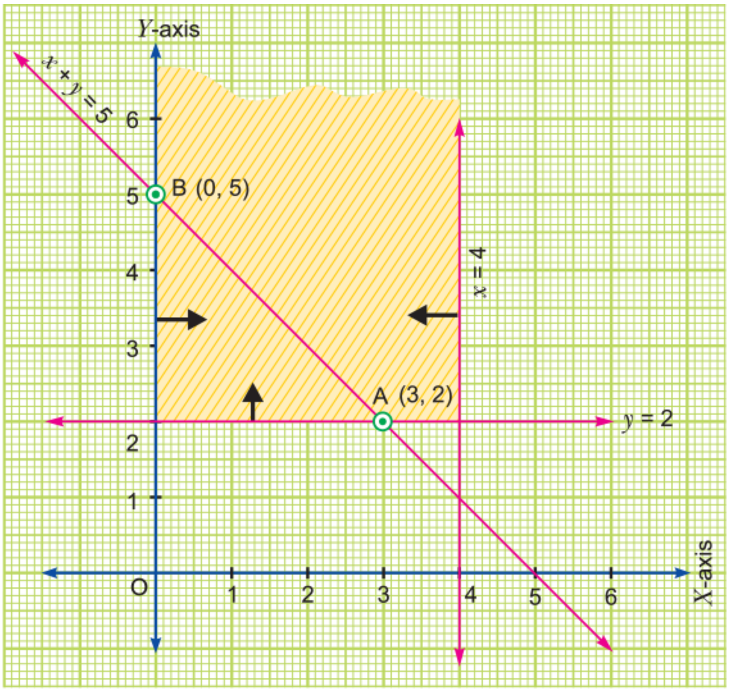
$x + y = 5$ … (*i*)

$x \leq 4$ … (*ii*)

$y \geq 2$ … (*iii*)

$x \geq 0, y \geq 0$ … (*iv*)

In this case, constraint (*i*) is a line passing through the feasible region determined by constraints (*ii*), (*iii*) and (*iv*).



Therefore, maximum or minimum value of objective function 'Z' exist on end points of line (constraint) (i) in feasible region *i.e.*, at A or B.

At A (3, 2) $Z = 5 \times 3 + 8 \times 2 = 15 + 16 = 31$

At B (0, 5) $Z = 5 \times 0 + 8 \times 5 = 0 + 40 = 40$

Hence, cost of brick is minimum when 3 kg of B_1 and 2 kg of B_2 are taken.

Q.5. A diet is to contain at least 80 units of Vitamin *A* and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs ₹ 4 per unit and F_2 costs ₹ 6 per unit. One unit of food F_1 contains 3 units of Vitamin *A* and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin *A* and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Ans.

Let x units of food F_1 and y units of food F_2 are required to be mixed.

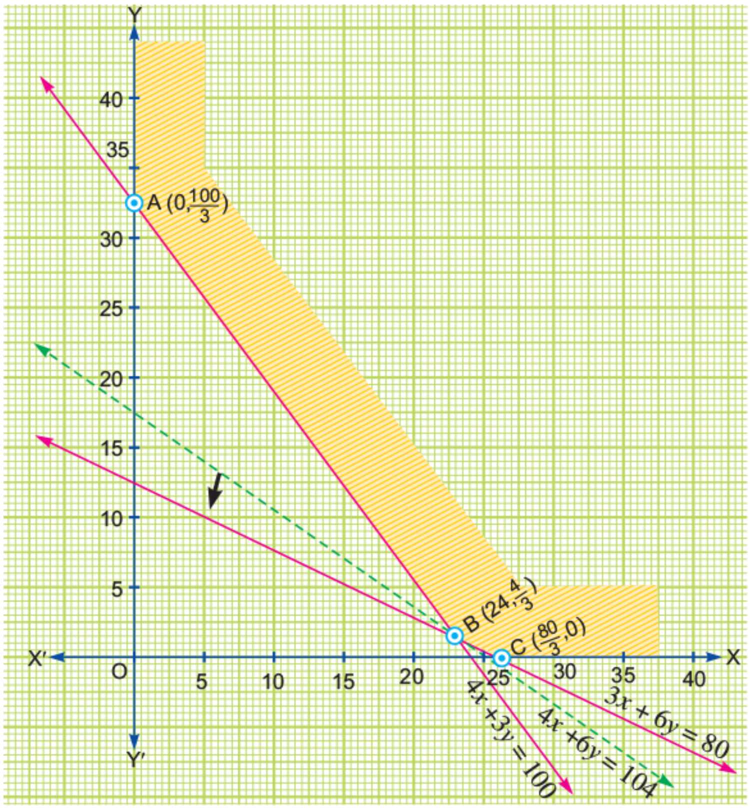
Cost = $Z = 4x + 6y$ ... (i) is to be minimised subject to following constraints.

$3x + 6y \geq 80$ ...(ii)

$4x + 3y \geq 100$ ...(iii)

$x \geq 0$, $y \geq 0$ ...(*iv*)

To solve the LPP graphically, the graph is plotted as shown.



The shaded regions in the graph is the feasible solution of the problem. The corner points are

$$A\ \left(0,\ \tfrac{100}{3}\right),\ B\ \left(24, \tfrac{4}{3}\right)\ \text{and}\ C\ \left(\tfrac{80}{3}, 0\right).$$

The value of *Z* at corner point is given as.

Corner Point	$Z = 4x + 6y$	
$A\left(0, \frac{100}{3}\right)$	$4 \times 0 + 6 \times \frac{100}{3} = 200$	
$B\left(24, \frac{4}{3}\right)$	$4 \times 24 + 6 \times \frac{4}{3} = 104$	Minimum
$C\left(\frac{80}{3}, 0\right)$	$4 \times \frac{80}{3} + 0 = 106.67$	

Since, feasible region is unbounded therefore a graph of $4x + 6y < 104$ is drawn which is shown in figure by dotted line.

Also, since there is no point common in feasible region and region $4x + 6y < 104$.

Hence, for minimum cost ₹ 104, 24 units of food F_1 and 43 units of food F_2 is required.

Q.6. A small firm manufacturers gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹300 and that on a chain is ₹190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

Ans.

Total number of rings and chains manufactured per day = 24

Time taken in manufacturing ring = 1 hour

Time taken in manufacturing chain = 30 minutes

Time available per day = 16 hours

Maximum profit on ring = ₹ 300

Maximum profit on chain = ₹ 190

Let number of gold rings manufactured per day = x and chains manufactured per day = y

LPP is

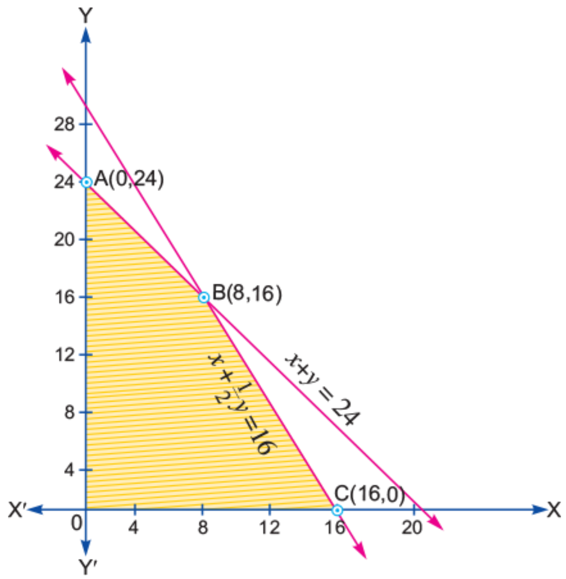
Maximize $Z = 300x + 190y$ ...(*i*)

Subject to constraints $x \geq 0, y \geq 0$ ...(*ii*)

$x + y \leq 24$ ...(*iii*)

$x +$ 12y≤16 ...(*iv*)

Possible points for maximum Z are



A (0, 24), *B* (8, 16) and *C* (16, 0).

Corner Point	$Z = 300x + 190y$
A (0, 24)	4560
B (8, 16)	5440 ← Maximum
C (16, 0)	4800

Z is maximum at (8, 16).

Hence, 8 gold rings and 16 chains must be manufactured per day.

Q.7. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs ₹10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs ₹4. How many mix packets mix from S and T should the company purchase to honour the contract requirement and yet minimize cost? Make a LPP and solve graphically.

Ans.

Let *x* and *y* units of packet of mixes be purchased from *S* and *T* respectively. If *Z* is total cost then

Z = 10*x* + 4*y* ...(*i*)

is objective function, which we have to minimize.

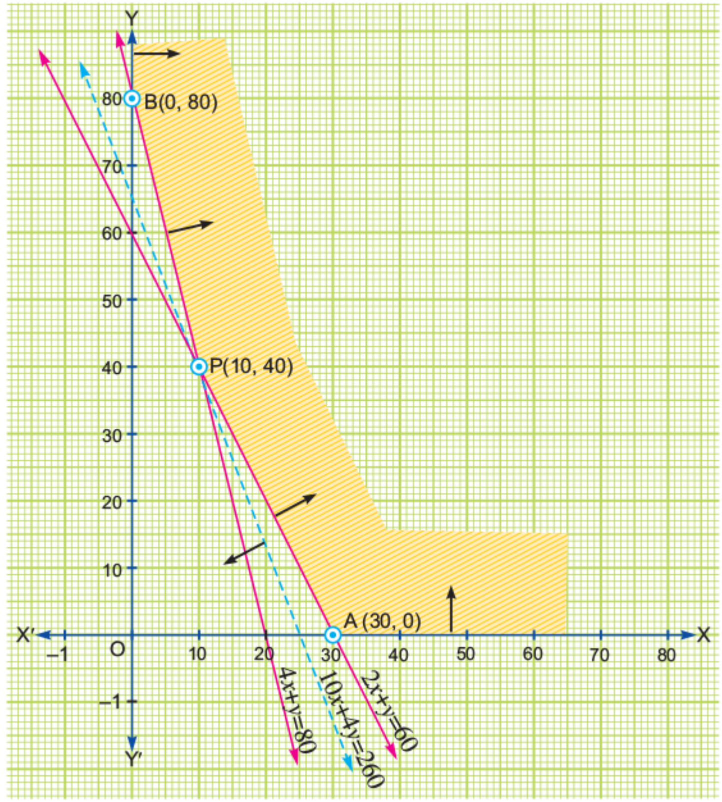
Here, constraints are:

$4x + y \geq 80$ ...(*ii*)

$2x + y \geq 60$ ...(*iii*)

Also, $x, y \geq 0$ ...(*iv*)

On plotting graph of above constraints or inequalities (*ii*), (*iii*) and (*iv*), we get shaded region having corner point *A*, *P*, *B* as feasible region.



For coordinate of *P*.

Point of intersection of

$2x + y = 60$ ...(*v*)

and $4x + y = 80$ ...(*vi*)

(*v*) – (*vi*)

$\Rightarrow \quad 2x + y - 4x - y = 60 - 80$

$\Rightarrow \quad -2x = -20$

$\Rightarrow \quad x = 10$

$\Rightarrow \quad y = 40$

coordinate of $P \equiv (10, 40)$

Now the value of *Z* is evaluated at corner point in the following table

Corner Point	$Z = 10x + 4y$
A (30, 0)	300
P (10, 40)	260 ← Minimum
B (0, 80)	320

Since, feasible region is unbounded. Therefore we have to draw the graph of the inequality.

$10x + 4y < 260$ ...(*vii*)

Since, the graph of inequality (*vii*) does not have any point common.

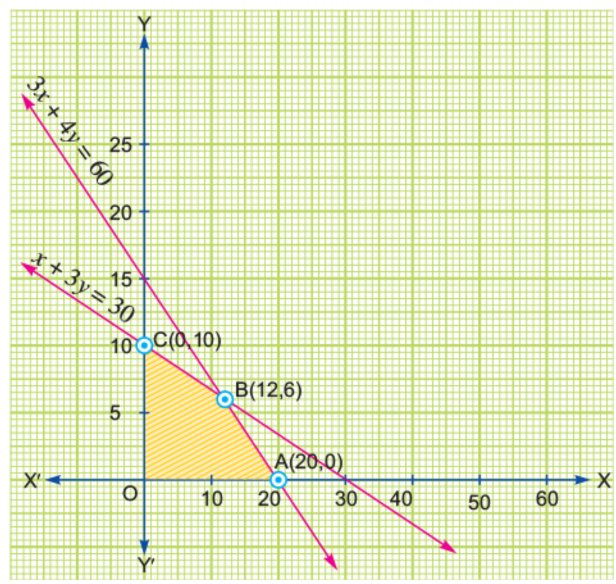
So, the minimum value of Z is 260 at (10, 40).

i.e., minimum cost of each bottle is ₹ 260 if the company purchases 10 packets of mixes from *S* and 40 packets of mixes from supplier *T*.

Q.8. A manufacturing company makes two types of teaching aids *A* and *B* of mathematics for class XII. Each type of *A* requires 9 labour hours for fabricating and 1 labour hour for finishing. Each type of *B* requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30, respectively. The company makes a profit of ₹ 80 on each piece of type *A* and ₹ 120 on each piece of type *B*. How many pieces of type *A* and *B* should be manufactured per week to get a maximum profit? What is the maximum profit per week?

Ans.

Let x and y be the number of pieces of type A and B manufactured per week respectively. If Z be the profit then,



Objective function, $Z = 80x + 120y$ ...(*i*)

We have to maximize Z, subject to the constraints

$9x + 12y \leq 180$

$3x + 4y \leq 60$ ...(*ii*)

$x + 3y \leq 30$ ...(*iii*)

$x \geq 0, y \geq 0$ ...(*iv*)

The graph of constraints are drawn and feasible region *OABC* is obtained, which is bounded having corner points O (0, 0), A(20, 0), B (12, 6) and C (0, 10)

Now the value of objective function is obtained at corner points as

Corner point	$Z = 80x + 120y$	
O (0, 0)	0	
A (20, 0)	1600	
B (12, 6)	1680	← Maximum
C (0, 10)	1200	

Hence, the company will get the maximum profit of ₹1,680 by making 12 pieces of type *A* and 6 pieces of type *B* of teaching aid.

Q.9. A cooperative society of farmers has 50 hectares of land to grow two crops *A* and *B*. The profits from crops *A* and *B* per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops *A* and *B* at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the protection of fish and other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically.

Ans.

Let x and y hectares of land be allocated to crop *A* and *B* respectively. If Z is the profit then

$Z = 10500x + 9000y$ …(*i*)

We have to maximize Z subject to the constraints:

$x + y \leq 50$ …(*ii*)

$20x + 10y \leq 800 \Rightarrow 2x + y \leq 80$ …(*iii*)

$x \geq 0, y \geq 0$ …(*iv*)

The graph of system of inequalities (*ii*) to (*iv*) are drawn, which gives feasible region *OABC* with corner points *O* (0, 0), *A* (40, 0), *B* (30, 20) and *C* (0, 50).

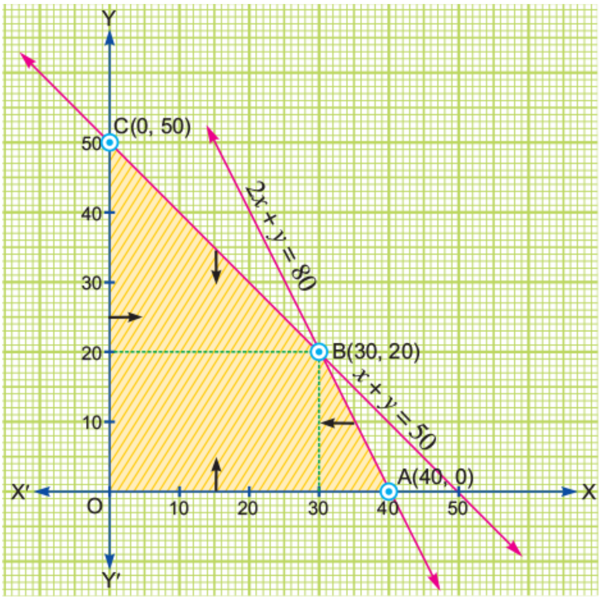


Table for $x + y = 50$

$\boldsymbol{x}$	0	50
$\boldsymbol{y}$	50	0

Table for $2x + y = 80$

$\boldsymbol{x}$	0	40
$\boldsymbol{y}$	80	0

Feasible region is bounded.

Now,

Corner point	$Z = 10500x + 9000y$	
O (0, 0)	0	
A (40, 0)	420000	
B (30, 20)	495000	← Maximum
C (0, 50)	450000	

Hence, the co-operative society of farmers will get the maximum profit of ₹ 4,95,000 by allocating 30 hectares for crop *A* and 20 hectares for crop *B*.

Q.10. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods *A* and *B*. To produce one unit of *A*, 2 workers and 3 units of capital are

required while 3 workers and 1 unit of capital is required to produce one unit of *B*. If *A* and *B* are priced at ₹ 100 and ₹120 per unit respectively, how should he use his resources to maximise the total revenue? Form the above as an LPP and solve graphically.

Ans.

Let x, y unit of goods *A* and *B* are produced respectively.

Let Z be total revenue

Here, $Z = 100x + 120y$ ....(*i*)

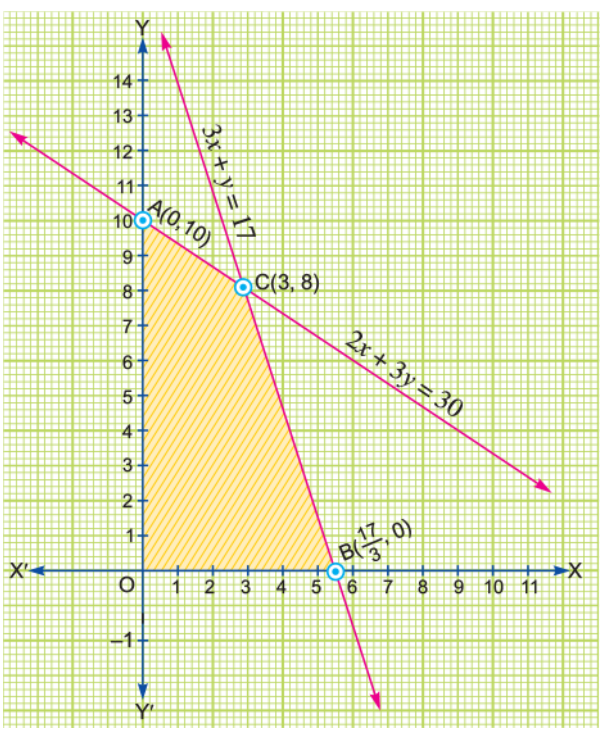
Subjects to constraints:

Also $2x + 3y \leq 30$ ....(*ii*)

$3x + y \leq 17$ ....(*iii*)

$x, \; y \geq 0$ ....(*iv*)

On plotting graph of above constants or inequalities (*ii*), (*iii*) and (*iv*). We get shaded region as feasible region having corner points *A, O, B* and *C*.



For coordinate of '*C*'

Two equations (*ii*) and (*iii*) are solved and we get coordinate of *C* = (3, 8)

Now, the value of *Z* is evaluated at corner point as:

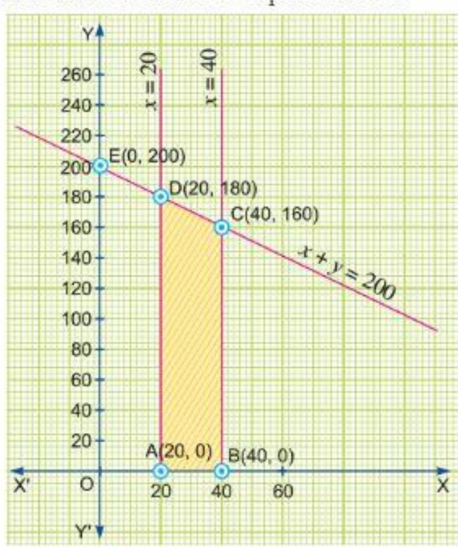
Corner point	$Z = 100x + 120y$
O (0, 0)	0
A (0, 10)	1200
$B\left(\frac{17}{3},0\right)$	$\frac{1700}{3}$
C (3, 8)	1260 ← Maximum

Therefore, maximum revenue is ₹1, 260 when 3 workers and 8 units capital are used for production.

Q.11. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of ₹ 400 is made on each economy ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the airline's fund. The airline reserves at least 20 seats for executive class. However, at least four times as many passengers prefer to travel by economy class than by the executive class. Determine, how many tickets of each type must be sold in order to maximise the net profit of the airline. Make the above as an LPP and solve graphically.

Ans.

Let there be x tickets of executive class and y tickets of economy class be sold. Let Z be net profit of the airline.



Here, we have to maximise Z.

Now, $Z = 500x \times \frac{80}{100} + 400y \times \frac{75}{100}$

$Z = 400x + 300y \quad ....(i)$

Subject to constraints:

$x \geq 20 \quad ....(ii)$

Also $x + y \leq 200 \quad ....(iii)$

$x + 4x \leq 200 \quad [\because y = 4x]$

$\Rightarrow \quad 5x \leq 200$

$\Rightarrow \quad x \le 40 \qquad \qquad ....(iv)$

Shaded region is feasible region having corner points *A* (20, 0), *B* (40,0) *C* (40, 160), *D* (20,180), *E*(0,200).

Now, value of Z is calculated at corner point as

Corner points	$Z = 400x + 300y$	
A (20, 0)	8,000	
B (40, 0)	16,000	
C (40, 160)	64,000	← Maximum
D (20, 180)	62,000	
E (0, 200)	60,000	

Hence, 40 tickets of executive class and 160 tickets of economy class should be sold to maximise the net profit of the airlines.

Q.12. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows : [*CBSE Delhi 2008*]

Machine	Area occupied	Labour force	Daily output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

Ans.

Let the owner buys x machines of type A and y machine of type B.

Then $\quad 1000x + 1200y \le 9000 \qquad ...(i)$

$12x + 8y \le 72 \qquad ...(ii)$

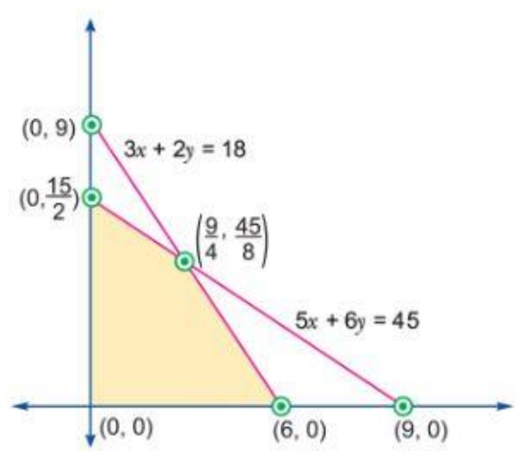
Objective function is to maximize $z = 60x + 40y$

From (i)

$10x + 12y \le 90$

or $\quad 5x + 6y \le 45 \qquad ...(iii)$

$3x + 2y \le 18 \qquad ...(iv) \qquad$ [from (ii)]



We plot the graph of inequations shaded region in the feasible solution (*iii*) and (*iv*).

The shaded region in the figure represents the feasible region which is bounded. Let us now evaluate Z at each corner point.

Z at (0, 0) is $60 \times 0 + 40 \times 0 = 0$

Z at $(0, 0)$ is $60 \times 0 + 40 \times 0 = 0$

Z at $\left(0, \frac{15}{2}\right)$ is $60 \times 0 + 40 \times \frac{15}{2} = 300$

Z at $\left(\frac{9}{4}, \frac{45}{8}\right)$ is $60 \times \frac{9}{4} + 40 \times \frac{45}{8} = 135 + 225 = 360.$

max. Z = 360

Therefore there must be either *x* = 6, *y* = 0 or $x = \frac{9}{4}, y = \frac{45}{8}$ but second case is not possible as *x* and *y* are whole numbers.

Hence there must be 6 machines of type *A* and no machine of type *B* is required for maximum daily output.

Long Answer Questions-II (OIQ)

[6 marks]

Q.1. Anil wants to invest at most ₹ 12,000 in bonds *A* and *B*. According to the rules, he has to invest at least ₹ 2,000 in bond *A* and at least ₹ 4,000 in bond *B*. If the rate of interest on bond *A* is 8% per annum and on bond *B* is 10% per annum, how should he invest his money for maximum interest? Solve the problem graphically.

Ans.

Let Anil invests ₹x in bond A and ₹y in bond B then

Interest on bond

$A = x \times \frac{8}{100} = ₹\ \frac{2x}{25}$

and interest on bond

$B = y \times \frac{10}{100} = ₹\frac{y}{10}$

His total annual interest

$= ₹\ \left(\frac{2x}{25} + \frac{y}{10}\right)$

Thus, our LPP is to

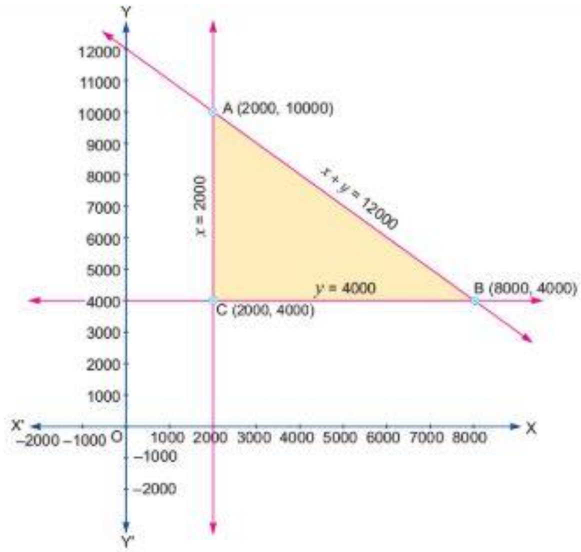
Maximise $Z = \frac{2x}{25} + \frac{y}{10}$ ... (i)

Subject to constraints:

$x \geq 2,000$ ...(ii)

$y \geq 4,000$ ...(iii)

and $x + y \leq 12,000$ ...(iv)



On plotting inequalities (ii) to (iv), we have the required region shown (shaded) in the figure.

Now, we evaluate Z at the corner points A (2000, 10000), B (8000, 4000) and C (2000, 4000).

Corner Point	$Z = \frac{2x}{25} + \frac{y}{10}$
A (2000, 10000)	1160 ← Maximum
B (8000, 4000)	1040
C (2000, 4000)	560

∴ Z is a maximum *i.e.*, ₹1,160 when x = ₹2,000 and y = ₹10,000.

So, Anil should invest ₹2,000 in bond *A* and ₹10,000 in bond *B* to earn maximum profit of ₹1,160.

Q.3. (Transportation problem) There are two factories located one at place *P* and the other at place *Q*. From these locations, a certain commodity is to be delivered

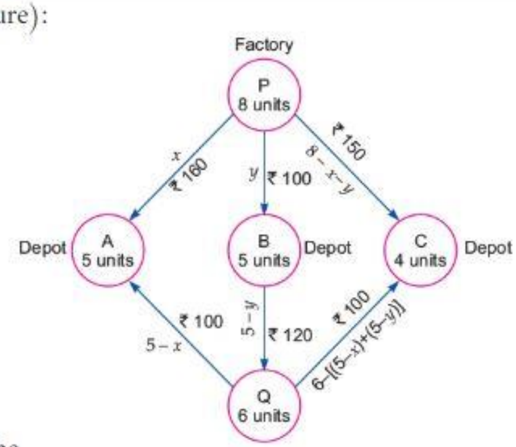
to each of the three depots situated at *A*, *B* and *C*. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at *P* and *Q* are respectively 8 and 6 units. The cost of transportation per unit is given below:

From/To	Cost (in ₹)		
	A	*B*	*C*
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost?

Ans.

The problem can be explained diagrammatically as follows (figure):



Let x units and y units of the commodity be transported from the factory at P to the depots at A and B respectively. Then $(8 - x - y)$ units will be transported to depot at C.

Hence, we have

$$x \geq 0,\ y \geq 0 \quad \text{and} \quad 8 - x - y \geq 0$$

i.e., $$x \geq 0,\ y \geq 0 \quad \text{and} \quad x + y \leq 8$$

Now, the weekly requirement of the depot at A is 5 units of the commodity. Since x units are transported from the factory at P, the remaining $(5 - x)$ units need to be transported from the factory at Q. Obviously, $5 - x \geq 0$, *i.e.*, $x \leq 5$.

Similarly, $(5 - y)$ and $6 - (5 - x + 5 - y) = x + y - 4$ units are to be transported from the factory at Q to the depots at *B* and *C* respectively.

Thus, $5 - y \geq 0$, $x + y - 4 \geq 0$

i.e., $y \leq 5$, $x + y \geq 4$

Total transportation cost Z is given by

$$Z = 160x + 100y + 100(5 - x) + 120(5 - y) + 100(x + y - 4) + 150(8 - x - y)$$

$$= 10(x - 7y + 190)$$

Therefore, the problem reduces to

Minimise $Z = 10\,(x - 7y + 190)$

Subject to the constraints:

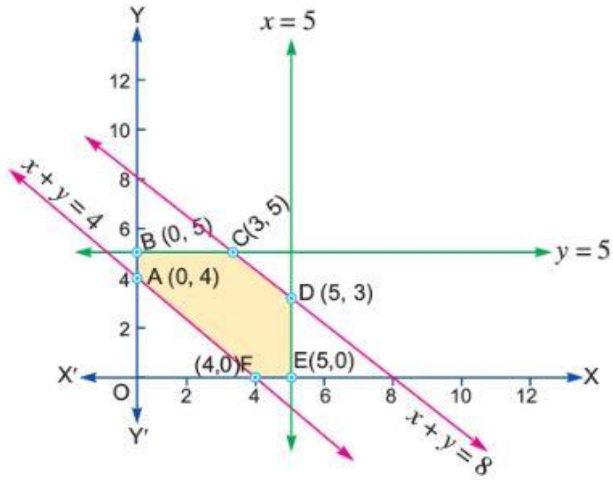
$x \geq 0,\ y \geq 0$ ... (i)

$x + y \leq 8$ ... (ii)

$x \leq 5$ ...(iii)

$y \leq 5$ ...(iv)

and $x + y \geq 4$ ...(v)



The shaded region *ABCDEF* represented by the constraints (*i*) to (*v*) is the feasible region (see figure).

Observe that the feasible region is bounded. The coordinates of the corner points *A, B, C, D, E* and F of the feasible region are (0, 4), (0, 5), (3, 5), (5, 3), (5, 0) and (4, 0) respectively.

Let us evaluate *Z* at these points.

Corner Point	$Z = 10\,(x - 7y + 190)$	
A (0, 4)	1620	
B (0, 5)	1550	← Minimum
C (3, 5)	1580	
D (5, 3)	1740	
E (5, 0)	1950	
F (4, 0)	1940	

From the table, we see that the minimum value of *Z* is 1550 at the point *B* (0, 5).

Hence, the optimal transportation strategy will be to deliver 0, 5 and 3 units from the factory at *P* and 5, 0 and 1 units from the factory at *Q* to the depots at *A*, *B* and *C* respectively. Corresponding to this strategy, the transportation cost would be minimum, *i.e.*, ₹1550.

Q.3. A village has 500 hectares of land to grow two types of plants, *X* and *Y*. The contributions of total amount of oxygen produced by plant *X* and plant *Y* are 60% and 40% per hectare respectively. To control weeds, a liquid herbicide has to be used for *X* and *Y* at rates of 20 litres and 10 litres per hectare, respectively. Further no more than 8000 litres of herbicides should be used in order to protect aquatic animals in a pond which collects drainage from this land. How much land

should be allocated to each crop so as to maximise the total production of oxygen?

Ans.

Let plants X and Y be grown in x and y hectares.

So, $x \geq 0$ and $y \geq 0$

$x + y \leq 500$ ...(*i*)

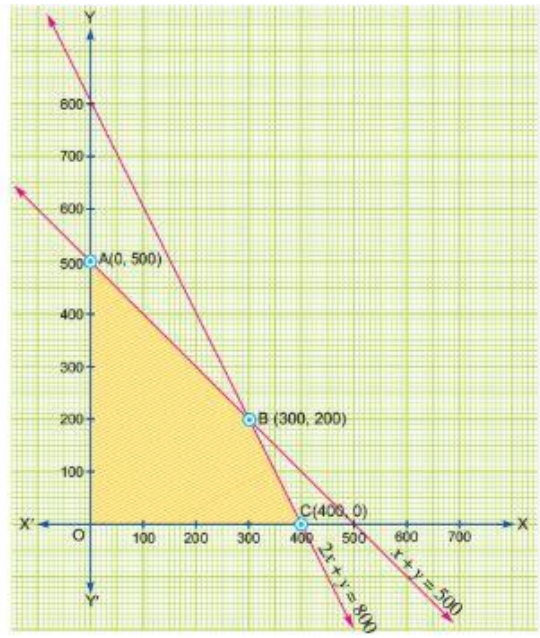
Contribution of oxygen by the plants = 60% of x + 40% of y

$$Z = \frac{6x}{10} + \frac{4y}{10} = 0.6x + 0.4y$$

Also, Amount of liquid herbicides required = $(20x + 10y)$ litres

Given $20x + 10y \leq 8000$

$2x + y \leq 800$ ...(*ii*)



The LPP for given problem is

Maximum, $Z = 0.6x + 0.4y$

Subject to constraints:

$x + y \leq 500$ ...(*iii*)

and $2x + y \leq 800$ ...(*iv*)

$x, y \geq 0$

Sketching a graph for the above LPP, we get the region shown in the figure.

Solving $x + y = 500$ and $2x + y = 800$, we get $x = 300$ and $y = 200$ as B (300, 200)

Corner point	Value of the optimizing function	
O (0, 0)	$Z = 0 + 0 = 0$	
A (0, 500)	$Z = 0.6 \times 0 + 0.4 \times 500 = 200$	
B (300, 200)	$Z = 0.6 \times 300 + 0.4 \times 200 = 180 + 80 = 260$	Maximum
C (400, 0)	$Z = 0.6 \times 400 + 0.4 \times 0 = 240$	

Maximum production of oxygen will be achieved when plant *X* is planted in 300 hectares and plant *Y* is planted in 200 hectares.