RD Sharma Solutions Class 12 Maths Chapter 1 Ex 1.1

Relations Ex 1.1 Q1(i)

A be the set of human beings.

$R = \{(x, y): \text{ x and y work at the same place}\}$

Reflexive:

$\therefore$ x and x works together

$\therefore$ $(x,x) \in R$

$\Rightarrow$ R is reflexive

Symmetric: If x and y work at the same place, which implies,
y and x work at the same place

$\therefore$ $(y,x) \in R$

$\Rightarrow$ R is symmetric

Transitive: If x and y work at the same place
then x and y work at the same place and y and z work at the same place

$\Rightarrow$ $(x,z) \in R$ and

Hence,

$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(ii)

A be the set of human beings.

$R = \{(x,y) :$ x and y lives in the same locality $\}$

Reflexive: since x and x lives in the same locality

$\Rightarrow \quad (x,x) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(x,y) \in R$

$\Rightarrow \quad x$ and y lives in the same locality

$\Rightarrow \quad y$ and x lives in the same locality

$\Rightarrow \quad (y,x) \in R$

Transitive: Let $(x, y) \in$ R and $(y, z) \in$ R

$(x, y) \in R$

$\Rightarrow$ x and y lives in the same locality

and $(y, z) \in R$

$\Rightarrow$ y and z lives in the same locality

$\Rightarrow$ x and z lives in the same locality

$\Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive

Relations Ex 1.1 Q1(iii)

$R = \{(x, y) : x \text{ is wife of } y\}$

Reflexive: since x can not be wife of x

$\therefore \quad (x,x) \notin R$

$\Rightarrow \quad R$ is not reflexive

Symmetric: Let $(x,y) \in R$

$\Rightarrow \quad x$ is wife of y
$\Rightarrow \quad y$ is husband of x
$\Rightarrow \quad (y,x) \notin R$

$\Rightarrow \quad R$ is not symmetric

Transitive: Let $(x,y) \in R$ and $(y,z) \in R$

$\Rightarrow \quad x$ is wife of y and y is husband of z
which is a contradiction
$\Rightarrow \quad (x,z) \notin R$

$\Rightarrow \quad R$ is not transitive

Relations Ex 1.1 Q1(iv)

A be the set of human beings
R = {(x, y) : x is father of y}
Reflexive: since x can not be father of x
$\therefore$ (x, x) $\notin$ R

$\Rightarrow$ R is not reflexive

Symmetric: Let (x, y) $\in$ R

$\Rightarrow$ x is father of y
$\Rightarrow$ y can not be father of x

$\Rightarrow$ (y, x) $\notin$ R

$\Rightarrow$ R is not symmetric

Transitive: Let (x, y) $\in$ R and (y, z) $\in$ R

$\Rightarrow$ x is father of y and y is father of z
$\Rightarrow$ x is grandfather of z

$\Rightarrow$ (x, z) $\notin$ R

$\Rightarrow$ R is not transitive

Relations Ex 1.1 Q2

We have, $A = \{a, b, c\}$

$R_1 = \{(a,a)(a,b)(a,c)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

R_1 is reflexive as $(a,a) \in R_1, (b,b) \in R_1$ & $(c,c) \in R_1$

R_1 is not symmetric as $(a,b) \in R_1$ but $(b,a) \in R_1$

R_1 is not transitive as $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

$R_2 = \{(a,a)\}$

R_2 is not reflexive as $(b,b) \notin R_2$

R_2 is symmetric and transitive.

$R_3 = \{(b,c)\}$

R_3 is not reflexive as $(b,b) \notin R_3$

R_3 is not symmetric

R_3 is not transitive.

$R_4 = \{(a,b)(b,c)(c,a)\}$

R_4 is not reflexive on set A as $(a,a) \notin R_4$

R_4 is not symmetric as $(a,b) \in R_4$ but $(b,a) \notin R_4$

R_4 is not transitive as $(a,b) \in R_4$ and $(b,c) \in R_4$ but $(a,c) \notin R_4$

Relations Ex 1.1 Q3

$R_1 = \left\{(x, y), x, y \in Q_0, x = \frac{1}{y}\right\}$

Reflexivity: Let, $x \in Q_0$

$\Rightarrow \quad x \neq \frac{1}{x}$

$\Rightarrow \quad (x, x) \in R_1$

$\therefore \quad R_1$ is not reflexive

Symmetric: Let, $(x,y) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad (y, x) \in R_1$

$\therefore \quad R_1$ is symmetric symmetric

Transitive: Let, $(x, y) \in R_1$ and $(y, z) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$ and $y = \frac{1}{z}$

$\Rightarrow \quad x = z$

$\Rightarrow \quad (x, z) \notin R_1$

$\therefore \quad R_1$ is not trasitive

Relations Ex 1.1 Q3(ii)

Reflexivity: Let, $a \in z$

$\Rightarrow \quad |a - a| = 0 \leq 5$

$\therefore \quad (a, a) \in R_2 \Rightarrow \quad R_2$ is reflexive

Symmetricity: Let, $(a, b) \in R_2$

$\Rightarrow \quad |a - a| \leq 5$

$\Rightarrow \quad |b - a| \leq 5$

$\Rightarrow \quad |b, a| \in R_2 \quad \Rightarrow \quad R_2$ is symmetric

Transitivity: Let, $(a,b) \in R_2$ and $(b,c) \in R_2$

$\Rightarrow \quad |a - b| \leq 5$ and $|b - c| \leq 5$

$\not\Rightarrow \quad |a - c| \leq 5$

$\Rightarrow \quad R_2$ is not transitive

$$\left[\begin{array}{l} \therefore \text{ if } a = 15, b = 11, c = 7 \\ \qquad \Rightarrow \quad |15 - 11| \leq 5 \text{ and } |11 - 7| \leq 5 \\ \qquad\qquad \text{but } |15 - 7| \geq 5 \end{array}\right]$$

Relations Ex 1.1 Q4

(i) We have, $A = \{1, 2, 3\}$ and

$R_1 = \{(1,1)(1,3)(3,1)(2,2)(2,1)(3,3)\}$

$\therefore \quad (1,1), (2,2)$ and $(3,3) \in R_1$

$\therefore \quad R_1$ is not Reflexive

Now,

$\therefore \quad (2,1) \in R_1$ but $(1,2) \notin R_1$

$\therefore \quad R_1$ is not Symmetric

Again,

$\therefore \quad (2,1) \in R_1$ and $(1,3) \in R_1$ but $(2,3) \notin R_1$

$\therefore \quad R_1$ is not Transitive

(ii) $R_2 = \{(2,2), (3,1), (1,3)\}$

$\therefore \quad (1,1) \notin R_2$

$\Rightarrow \quad R_2$ is not reflexive

Now, $(1,3) \in R_2$

$\Rightarrow \quad (3,1) \in R_2$

$\Rightarrow \quad R_2$ is symmetric

Again, $(3,1) \in R_2$ and $(1,3) \in R_2$ but $(3,3) \notin R_1$

$\therefore \quad R_2$ is not transitive

(iii) $R_3 = \{(1,3)(3,3)\}$

$\therefore \quad (1,1) \notin R_3$

$\Rightarrow \quad R_3$ is not reflexive

Now, $(1,3) \in R_3$ but $(3,1) \in R_3$

$\Rightarrow \quad R_3$ is not symmetric

Again, It is clear that R_3 is transitive

Relations Ex 1.1 Q5.

(i) aRb if a-b ⟩ 0

Let R be the set of real numbers.

Reflexivity: Let $a \in R$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad (a, a) \notin R$

$\therefore \quad R$ is not reflexive

Symmetric: Let aRb

$\Rightarrow \quad a - a > 0$

$\Rightarrow \quad b - a < 0$

$\therefore \quad b \not{R} a$

$\therefore \quad R$ is not Symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad a - a >$ and b $- c > 0$

$\Rightarrow \quad a - c > 0$

$\Rightarrow \quad aRc$

$\therefore \quad R$ is Transitive

Relations Ex 1.1 Q5(ii)

We have, $a R b$ iff $1+ab>0$

Let R be the set of real numbers

Reflexive: Let $a \in R$

$\Rightarrow \quad 1+a^2>0$

$\Rightarrow \quad a R a$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let aRb

$\Rightarrow \quad 1+ab>0$

$\Rightarrow \quad 1+ba>0$

$\Rightarrow \quad bRa$

$\Rightarrow \quad R$ is symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad 1+ab>0$ and $1+bc>0$

$\not\Rightarrow \quad 1+ac>0$

$\Rightarrow \quad R$ is not transitive

Relations Ex 1.1 Q5(iii)

We have, aRb if $|a| \le b$

Reflexivity: Let $a \in R$

$\Rightarrow \quad |a| \not\le a \qquad [\because \quad |-2| = 2 > -2]$

$\Rightarrow \quad R$ is not reflexive

Symmetric: Let aRb

$\Rightarrow \quad |a| \le b$

$\not\Rightarrow \quad |b| \le a \qquad \left[\because \quad \begin{array}{l} \text{Let } a=4,\ b=6 \\ |4| \le 8 \text{ but } |8| > 4 \end{array}\right]$

$\Rightarrow \quad R$ is not symmetric

Transitive: Let aRb and bRc

$\Rightarrow \quad |a| \le b$ and $|b| \le c$

$\Rightarrow \quad |a| \le |b| \le c$

$\Rightarrow \quad |a| \le c$

$\Rightarrow \quad a R c$

$\Rightarrow \quad R$ is transitive

Relations Ex 1.1 Q6.

Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as:

$R = \{(a, b): b = a + 1\}$

Therefore, $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

We find $(a, a) \notin R$, where $a \in A$.

For instance, $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

Therefore, R is not reflexive.

It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.

Therefore, R is not symmetric.

Now, $(1, 2), (2, 3) \in \mathbf{R}$

But, $(1, 3) \notin R$

Therefore, R is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q7.

$R = \{(a, b): a \le b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Therefore, R is not reflexive.

Now, $(1, 2) \in R$ (as $1 < 2^3 = 8$)

But, $(2, 1) \notin R$ (as $2^3 > 1$)

Therefore, R is not symmetric.

We have

$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q8

Let A be a set.

Then $I_A = \{(a, a) ; a \in A\}$ is the identity relation on A.

Hence, every identity relation on a set is reflexive by definition.

Converse:

Let $A = \{(a, b, c)\}$ be a set.

Let $R = \{(a, a)(b, b)(c, c)(a, b)\}$ be a relation defined on A.

Clearly R is reflexive on set A, but it is not identity relation on set A as $(a, b) \in R$

Hence, a reflexive relation need not be identity relation.

Relations Ex 1.1 Q9

We have, $A = \{1, 2, 3, 4\}$

(i) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)\}$ is a relation on set A which is reflexive, transitive but not symmetric

(ii) $R = \{(2,3)(3,2)\}$ is a relation on set A which is symmetric but neither reflexive nor transitive

(iii) $R = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,1)\}$ is a relation on set A which is reflexive, symmetric and transitive

Relations Ex 1.1 Q10

We have, $R - \{(x, y); x, y \in N, 2x + y = 41\}$

Then Domain of R is $x \in N$, such that

$$2x + y = 41$$

$$\Rightarrow \quad x = \frac{41 - y}{2}$$

Since $y \in N$, largest value that x can take corresponds to the smallest value that y can take.

$$\therefore \quad x = \{1, 2, 3, \dots\dots\dots.20\}$$

Range of R is $y \in N$ such that

$$2x + y = 41$$

$$\Rightarrow \quad y = 41 - 2x$$

Since, $x = \{1, 2, 3, \dots\dots\dots.20\}$

$$\therefore \quad y = \{39, 37, 35, 33, \dots\dots\dots\dots.7, 5, 3, 1\}$$

Since, $(2,2) \notin R, R$ is not reflexive.

Also, since $(1,39) \in R$ but $(39,1) \notin R$, R is not symmetric.

Finally, since, $(15,11) \in R$ and $(11,19) \in R$ but $(15,19) \notin R$

$\therefore$ R is not trasitive.

Relations Ex 1.1 Q11

No, it is not necessary that a relation which is symmetric and transitive is reflexive as well.

For Example,

Let $A = \{a, b, c\}$ be a set and

$R_2 = \{(a, a)\}$ is a relation defined on A.

Clearly,

R_2 is symmetric and transitive but not reflexive.

Relations Ex 1.1 Q12

It is given that an integer m is said to be relative to another integer n if m is a multiple of n.

In other words

$$R = \{(m,n); \quad m = kn, k \in z\}$$

Reflexivity: Let, m $\in z$

$\Rightarrow \quad m = 1.m$

$\Rightarrow \quad (m,m) \in R$

$\therefore \quad R$ is reflexive

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a = kb \quad$ and $\quad b = k'c$

$\Rightarrow \quad a = kk'c \qquad [\because \quad kk' \in z]$

$\Rightarrow \quad a = lc \qquad [\because \quad l = kk' \in z]$

$\Rightarrow \quad (a,c) \in R$

$\therefore \quad R$ is transitive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a = kb$

$\Rightarrow \quad b = \frac{1}{k}a \qquad$ but $\frac{1}{k} \notin z$ if $k \in z$

$\therefore \quad (b,a) \notin R$

$\therefore \quad R$ is not symmetric

Relations Ex 1.1 Q13

We have,

relation R = " $\geq$ " on the set R of all real numbers

Reflexivity: Let $a \in R$

$\Rightarrow \quad a \geq a$

$\Rightarrow \quad$ " $\geq$ " is reflexive

Symmetric: Let a,b $\in R$

such that $a \geq b \not\Rightarrow b \geq a$

$\therefore \quad$ " $\geq$ " not symmetric

Transitivity: Let a,b,c $\in R$

and $a \geq b$ & $b \geq c$

$\Rightarrow \quad a \geq c$

$\Rightarrow \quad$ " $\geq$ " is transitive

Relations Ex 1.1 Q14

(i)Let $A = \{4, 6, 8\}$.

Define a relation R on A as:

$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$

Relation R is reflexive since for every $a \in A$, $(a, a) \in R$ i.e., $(4, 4), (6, 6), (8, 8)\} \in R$.

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is not transitive since $(4, 6), (6, 8) \in R$, but $(4, 8) \notin R$.

Hence, relation R is reflexive and symmetric but not transitive.

(ii) Define a relation R in **R** as:

$R = \{a, b): a^3 \geq b^3\}$

Clearly $(a, a) \in R$ as $a^3 = a^3$. $a = a$.

Therefore, R is reflexive.

Now, $(2, 1) \in R$ (as $2^3 \geq 1^3$)

But, $(1, 2) \notin R$ (as $1^3 < 2^3$)

Therefore, R is not symmetric.

Now, Let $(a, b), (b, c) \in R$.

$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$

$\Rightarrow a^3 \geq c^3$

$\Rightarrow (a, c) \in R$

Therefore, R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

Hence, relation R is transitive but not reflexive and symmetric.

(iv)Let $A = \{5, 6, 7\}$.

Define a relation R on A as $R = \{(5, 6), (6, 5)\}$.

Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin R$.

Now, as $(5, 6) \in R$ and also $(6, 5) \in R$, R is symmetric.

$\Rightarrow (5, 6), (6, 5) \in R$, but $(5, 5) \notin R$

Therefore, R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(v)Consider a relation R in **R** defined as:

$R = \{(a, b): a < b\}$

For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.

Therefore, R is not reflexive.

Now, $(1, 2) \in R$ (as $1 < 2$)

But, 2 is not less than 1.

Therefore, $(2, 1) \notin R$

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in R$

Therefore, R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

Relations Ex 1.1 Q15

We have,

$A = \{1,2,3\}$ and $R\{(1,2)(2,3)\}$

Now,

To make R reflexive, we will add $(1,1)(2,2)$ and $(3,3)$ to get

$\therefore \quad R' = \{(1,2)(2,3)(1,1,)(2,2)(3,3)\}$ is reflexive

Again to make R' symmetric we shall add $(3,2)$ and $(2,1)$

$\therefore \quad R'' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)\}$ is reflexive and symmetric

Now,

To make R'' transitive we shall add $(1,3)$ and $(3,1)$

$\therefore \quad R''' = \{(1,2)(2,3)(1,1)(2,2)(3,3)(3,2)(2,1)(1,3)(3,1)\}$

$\therefore \quad R'''$ is reflexive, symmetric and transitive

Relations Ex 1.1 Q16

We have, $A = \{1,2,3\}$ and $R = \{(1,2)(1,1)(2,3)\}$

To make R transitive we shall add $(1,3)$ only.

$\therefore \quad R' = \{(1,2)(1,1)(2,3)(1,3)\}$

Relations Ex 1.1 Q17

A relation R in A is said to be reflexive if aRa for all a∈A

R is said to be transitive if aRb and bRc ⇒ aRc
for all a, b, c ∈ A.
Hence for R to be reflexive (b, b) and (c, c) must be there in the set R.
Also for R to be transitive (a, c) must be in R because (a, b) ∈ R and (b, c) ∈ R so (a, c) must be in R.
So at least 3 ordered pairs must be added for R to be reflexive and transitive.

Relations Ex 1.1 Q18

A relation R in A is said to be reflexive if aRa for all a∈A, R is symmetric if aRb ⇒ bRa, for all a, b ∈ A and it is said to be transitive if aRb and bRc ⇒ aRc for all a, b, c ∈ A.

- x > y, x, y ∈ N

(x, y) ∈ {(2, 1), (3, 1).......(3, 2), (4, 2)....}
This is not reflexive as (1, 1), (2, 2)....are absent.
This is not symmetric as (2,1) is present but (1,2) is absent.
This is transitive as (3, 2) ∈ R and (2,1) ∈ R also (3,1) ∈ R ,similarly this property satisfies all cases.

- x + y = 10, x, y ∈ N

(x, y)∈ {(1, 9), (9, 1), (2, 8), (8, 2), (3, 7), (7, 3), (4, 6), (6, 4), (5, 5)}
This is not reflexive as (1, 1),(2, 2)..... are absent.
This only follows the condition of symmetric set as (1, 9)∈R also (9, 1)∈R similarly other cases are also satisfy the condition.
This is not transitive because {(1, 9),(9, 1)}∈R but (1, 1) is absent.

- xy is square of an integer, x, y ∈ N

(x, y) ∈ {(1, 1), (2, 2), (4, 1), (1, 4), (3, 3), (9, 1), (1, 9), (4, 4), (2, 8), (8, 2), (16, 1), (1, 16)...........}
This is reflexive as (1,1),(2,2)..... are present.
This is also symmetric because if aRb ⇒ bRa, for all a,b∈N.
This is transitive also because if aRb and bRc ⇒ aRc for all a, b, c ∈ N.

- x + 4y = 10, x, y ∈ N

(x, y) ∈ {(6, 1), (2, 2)}
This is not reflexive as (1, 1), (2, 2).....are absent.
This is not symmetric because (6,1) ∈ R but (1,6) is absent.
This is not transitive as there are only two elements in the set having no element common.