RD Sharma
Solutions
Class 12 Maths
Chapter 1
Ex 1.2

Relations Ex 1.2 Q1 We have, $R = \{(a,b): a-b \text{ is divisible by 3; a,b, } \in Z\}$ To prove: R is an equivalence relation Proff: Reflexivity: Let a∈ Z ⇒ a-a=0 ⇒ a-a is divisible by 3 \Rightarrow $(a,a) \in R$ R is reflexive \Rightarrow Symmetric: Let $a,b \in Z$ and $(a,b) \in R$ ⇒ a - b is divisible by 3 $\Rightarrow a-b=3p$ For some $p \in \mathbb{Z}$ \Rightarrow $b-a=3\times(-p)$ ⇒ b-a∈R R is symmetric \Rightarrow Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$ a-b=3p and b-c=3q For some $p,q\in Z$ \Rightarrow a-c=3(p+q)⇒ a-c is divisible by 3 ⇒ $(a,c) \in R$ ⇒ R is transitive \Rightarrow

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

TWe have,

 $R = \{(a,b): a-b \text{ is divisible by } 2; a,b, \in Z\}$ To prove: R is an equivalence relation

Proff: Reflexivity: Let a \in Z

a-a=0

a - a is divisible by 🤈 $(a,a) \in R$ \Rightarrow

R is reflexive \Rightarrow

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

d⇒ a − b is divisible by 2

 $\Rightarrow b - a = 2 \times (-p)$ $b-a\in R$

 \Rightarrow

 \Rightarrow

R is symmetric

a-c=2(p+q)

R is transitive

 $(a,c) \in R$

a-c is divisible by 2

 $\Rightarrow a-b=2p$ For some $p \in \mathbb{Z}$

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

a-b=2p and b-c=q For some $p,q\in Z$

$R = \{(a,b): (a-b) \text{ is divisible by 5} \} \text{ on } Z.$ We want to prove that R is an equivalence relation on Z. Now, Reflexivity: Let a∈ Z

Relations Ex 1.2 Q3

We have.

$$\Rightarrow$$
 a-a is divisible by 5.

Symmetric: Let
$$(a,b) \in R$$

$$\Rightarrow a-b=5P \quad \text{For some } P \in Z$$

$$\Rightarrow b - a = 5 \times (-P)$$

$$\Rightarrow b - a = 5 \times (dy = b)$$

 \Rightarrow

 \Rightarrow

$$(b,a) \in R$$
, so R if

$$(b,a) \in R$$
, so R is

$$(b,a) \in R$$
, so R is symmetric

Hence, R is equivalence relation on Z

Transitive: Let
$$(a,b) \in R$$
 and $(b,c) \in R$

 \Rightarrow a-c is divisible by 5.

R is transitive.

 $\Rightarrow a-c=5(p+q)$

a-b=5p and b-c=5q For some p, $q \in \mathbb{Z}$

Thus, R being reflexive, symmetric and transitive on Z.







Now. Reflexivity: Let a $\in Z$ $\Rightarrow a-a=0\times n$ ⇒ a – a is divisible by n ⇒ (a,a) ∈ R R is reflexive \Rightarrow

⇒
$$a-b=np$$
 For some $p ∈ Z$
⇒ $b-a=n(-p)$
⇒ $b-a$ is divisible by n

Symmetric: Let $(a,b) \in R$

 $R = \{(a,b): a-b \text{ is divisible by n}\} \text{ on } Z.$

$$\Rightarrow \qquad (b,a) \in R$$

$$\Rightarrow \qquad R \text{ is symmetric}$$

Transitive: Let
$$(a,b) \in R$$
 and $(b,c) \in R$

$$\Rightarrow \quad a-b=xp \text{ and } b-c=xq \quad \text{For some p,q} \in Z$$

$$\Rightarrow a-b=xp \text{ and } b-c=$$

$$\Rightarrow a-c=n(p+q)$$

$$\Rightarrow a-c \text{ is divisible by n}$$

$$\Rightarrow \quad a-c \text{ is divisibl}$$
$$\Rightarrow \quad (a,c) \in R$$

$$\Rightarrow \quad a-c \text{ is divis}$$

$$\Rightarrow \quad (a,c) \in R$$

 \Rightarrow

Relations Ex 1.2 Q4

R is transitive

Hence, R is an equivalence relation on Z

Thus, R being reflexive, symmetric and transitive on Z.



Relations Chapter 1 Ex 1.2 Q5 We have, Z be set of integers and

 $R = \{(a,b): a,b \in \mathbb{Z} \text{ and } a+b \text{ is even } \}$ be a relation on \mathbb{Z} .

We want to prove that R is an equivalence relation on Z.

Reflexivity: Let
$$a \in Z$$

Now,

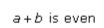
 \Rightarrow

 \Rightarrow

 \Rightarrow

 $(a,a) \in R$

R is reflexive



⇒ b+a is even

 $(b,a) \in R$,

R is symmetric

a+c is even

R is transitive

Hence, R is an equivalence relation on Z

 $\{a,c\} \in R$

a+b is even and b+c is even

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some $a,b,c \in Z$

Symmetric: Let $a,b \in \mathbb{Z}$ and $(a,b) \in \mathbb{R}$

[if a is even \Rightarrow a+a is even] if a is odd \Rightarrow a+a is even]

[if b is odd,then a and c must be odd $\Rightarrow a+c$ is even,

If b is even, then a and c must be even $\Rightarrow a+c$ is even

Let Z be set of integers

 $R = \{(m,n): m-n \text{ is divisible by } 13\}$ be a relation on Z.

Reflexivity: Let
$$m \in Z$$

Now,

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow SO

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

 \Rightarrow

$$m-m=0$$

m – m is divisible by 13

 \Rightarrow $(m,m) \in R$,

 $n-m=13\times(-p)$

R is symmetric

m-q=13(p+s)

 $(m,q) \in R$

R is transitive

m-q is divisible by 13

Hence, R is an equivalence relation on Z

 $(n-m) \in R$,

R is reflexive

m-n=13.p For some $p \in Z$

n-m is divisible by 13

Transitivity: Let $(m,n) \in R$ and $(n,q) \in R$ For some $m,n,q \in Z$

m-n=13p and n-q=13s For some p,s \in Z

Symmetric: Let $m, n \in \mathbb{Z}$ and $(m, n) \in \mathbb{R}$

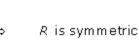
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(x, y) R (u, v) \Leftrightarrow xv = yu
TPT Reflexive \therefore xy = yx
                       \therefore (x, y) R (x, y)
TPT Symmetric Let (x, y) R (u, v)
TPT (u, v) R(x, y)
Given xv = yu
\Rightarrow yu = xv
\Rightarrow uy = vx
\therefore (u, v) R (x, y)
Transitive Let (x, y) R (u, v) and (u, v) R (p, q) ......(i)
TPT (x, y) R (p, q)
TPT \quad xq = yp
from (1) xv = yu \& uq = vp
xvuq = yuvp
xq = yp
       R is transitive
since R is reflexive symmetric & transitive all means it is an equivalence relation
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Relations Ex 1.2 Q8 We have, $A = \{x \in z : 0 \le x \le 12\}$ be a set and $R = \{(a,b): a = b\}$ be a relation on A Now, Reflexivity: Let a ∈ A a = a \Rightarrow $(a,a) \in R$ R is reflexive \Rightarrow

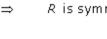
$$\Rightarrow \qquad a = b$$

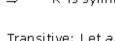
$$\Rightarrow \qquad b = a$$

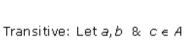
$$\Rightarrow \qquad (b, a) \in R$$







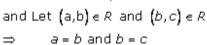








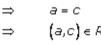
Symmetric: Let $a,b \in A$ and $(a,b) \in R$







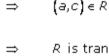




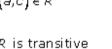


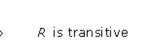


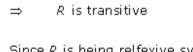


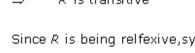


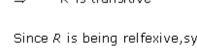
 $R = \{(1,1):1=1\}$

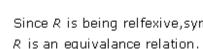












Since
$$\mathcal R$$
 is being relfexive, symmetric and transitive, so $\mathcal R$ is an equivalance relation.

Also, we need to find the set of all elements related to 1.

Thus, the set of all elements related to 1 is 1.

Since the relation is given by, R={(a,b):a=b}, and 1 is an element of A,



(i) We have, L is the set of lines. $R = \{(L_1, L_2): L_1 \text{ is parallel to } L_2\}$ be a relation on LNow,

Since a line is always parallel to itself.
$$(L_1, L_2) \in R$$

 \Rightarrow

 \Rightarrow

⇒

Reflexivity: Let $L_1 \in L$

Relations Ex 1.2 Q9

R is reflexive

Symmetric: Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

 \Rightarrow L_1 is parallel to L_2

 \Rightarrow L_2 is parallel to L_1

 \Rightarrow $(L_1, L_2) \in R$

 \Rightarrow

 \Rightarrow $(L_1, L_3) \in R$ ⇒ R is transitive

y = 2x + c For all $c \in R$

R is symmetric

Transitive: Let L_1, L_2 and $L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

 L_1 is parallel to L_2 and L_2 is parallel to L_3

 L_1 is parallel to L_3

Where R is the set of real numbers.

(ii) The set of lines parallel to the line y = 2x + 4 is

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

 $R = \{(P_1, P_2): P_1 \text{ and } P_2 \text{ have same the number of sides}\}$

R is reflexive since $(P_t, P_t) \in R$ as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$.

⇒ P₁ and P₂have the same number of sides.

 \Rightarrow P₂ and P₁ have the same number of sides.

 \Rightarrow (P₂ P₁) \in R

∴R is symmetric.

Now,

Let (P_1, P_2) , $(P_2, P_3) \in R$.

⇒ P₁ and P₂ have the same number of sides. Also, P₂ and P3 have the same number of sides.

⇒ P₁ and P3 have the same number of sides.

 \Rightarrow (P₁, P3) \in R

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in \triangle related to triangle T is the set of all triangles.

Let A be set of points on plane.

Let $R = \{(P,Q): OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Reflexivity: Let $p \in A$

Now,

 \Rightarrow

Since $OP = OP \Rightarrow (P,P) \in R$

R is reflexive

Symmetric: Let $(P,Q) \in R$ for $P,Q \in A$

Then OP = OQ

 \Rightarrow OQ = OP

 \Rightarrow $(Q,P) \in R$

R is symmetric \Rightarrow

Transitive: Let $(P,Q) \in R$ and $(Q,S) \in R$

OP = OQ and OQ = OS

 \Rightarrow OP = OS \Rightarrow $(P,S) \in R$

R is transitive \Rightarrow

Thus, R is an equivalence relation on A

elations Ex 1.2 Q12

Given $A=\{1,2,3,4,5,6,7\}$ and $R=\{(a,b):both a and b are either odd or even number\}$

Therefore,

 $R = \{(1,1),(1,3),(1,5),(1,6),(3,3),(3,5),(3,7),(5,5),(5,7),(7,7),(7,5),(7,3),(5,3),(6,1),(5,1),(3,1),(6,1)$

(2,2),(2,4),(2,6),(4,4),(4,6),(6,6),(6,4),(6,2),(4,2)

Form the relation Rit is seen that Ris symmetric, reflecive and transitive also. Therefore Ris an equivalent relation.

From the relation R it is seen that {1,3,5,7} are related with each other only and {2,4,6} are related with each other

$$S = \{(a,b): a^2 + b^2 = 1\}$$

Now,

Reflexivity: Let
$$a = \frac{1}{4}e$$

Reflexivity: Let
$$a = \frac{1}{2} \in \mathbb{R}$$

Then,
$$a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$$

Then,
$$a^{-} + a^{-} = \frac{-}{4} + \frac{-}{4} = \frac{-}{4}$$

$$\Rightarrow (a, a) \notin S$$

S is not reflexive.

Hence, S in not an equivalence relation on R

We have, $\,\,$ Z be set of integers and $\,$ Z $_{0}$ be the set of non-zero integers.

$$R = \{(a,b)(c,d): ad = bc\}$$
 be a relation on $z \times z_0$

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$$\Rightarrow$$
 $((a,b),(a,b)) \in R$

Symmetric: Let $((a,b),(c,d)) \in R$

$$\Rightarrow$$
 ad = bc

$$\Rightarrow$$
 $((c,d),(a,b)) \in R$

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$$\Rightarrow$$
 ad = bc and cf = de

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \text{ and } \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{\theta}{f}$$

$$\Rightarrow$$
 af = be

We have, $\,\,$ Z be set of integers and $\,$ Z $_{0}$ be the set of non-zero integers.

$$R = \{(a,b)(c,d): ad = bc\}$$
 be a relation on Z and Z_0 .

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$$\Rightarrow$$
 $((a,b),(a,b)) \in R$

Symmetric: Let $((a,b),(c,d)) \in R$

$$\Rightarrow$$
 ad = bc

$$\Rightarrow$$
 $((c,d),(a,b)) \in R$

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Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$$\Rightarrow$$
 ad = bc and cf = de

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \text{ and } \frac{c}{d} = \frac{e}{f}$$

$$\Rightarrow \frac{a}{b} = \frac{e}{f}$$

$$\Rightarrow$$
 af = be

$$\Rightarrow$$
 $(a,b)(e,f) \in R$

⇒ R is transitive

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15.

R and S are two symmetric relations on set A

(i) To prove: $R \wedge S$ is symmetric

Let
$$(a,b) \in R \cap S$$

$$\Rightarrow$$
 $(a,b) \in R$ and $(a,b) \in S$

$$\Rightarrow$$
 $(b,a) \in R$ and $(b,a) \in S$ $[\because R \text{ and } S \text{ are symmetric}]$

$$\Rightarrow$$
 $(b,a) \in R \land S$

$$\Rightarrow$$
 $R \land S$ is symmetric

To prove: $R \cup S$ is symmetric.

Let
$$(a,b) \in R \cup S$$

$$\Rightarrow$$
 $(a,b) \in R$ or $(a,b) \in S$

$$\Rightarrow$$
 $(b,a) \in R$ or $(b,a) \in S$ $[\because R \text{ and } S \text{ are symmetric}]$

$$\Rightarrow$$
 $(b,a) \in R \cup S$

$$\Rightarrow$$
 $R \cup S$ is symmetric

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.

This means that there is an $a \in R \cup S$ such that $(a, a) \notin R \cup S$

Since $a \in R \cup S$,

If $a \in R$, then $(a, a) \in R$ $[\because R \text{ is reflexive}]$

Hence, $R \cup S$ is reflexive

We will prove this by means of an example.

Let $A = \{a, b, c\}$ be a set and

Relations Ex 1.2 Q16.

but $\{a,c\} \notin R \cup S$

Clearly R and S are transitive relation on A

Here, $(a,b) \in R \cup S$ and $(b,c) \in R \cup S$

 $R \cup S$ is not transitive

$$R = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$$
 and



 $S = \{(a, a)(b, b)(c, c)(b, c)(c, b)\}$ are two relations on A

Now, $R \cup S = \{(a,a)(b,b)(c,c)(a,b)(b,a)(b,c)(c,b)\}$