

RD Sharma
Solutions
Class 12 Maths
Chapter 2
Ex 2.2

Functions Ex2.2 Q1(i)

Since, $f: R \rightarrow R$ and $g: R \rightarrow R$

$\therefore f \circ g: R \rightarrow R$ and $g \circ f: R \rightarrow R$

Now, $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

$$g \circ f(x) = g(2x + 3) = (2x + 3)^2 + 5$$

$$\Rightarrow g \circ f(x) = 4x^2 + 12x + 14$$

$$f \circ g(x) = f(g(x)) = f(x^2 + 5) = 2(x^2 + 5) + 3$$

$$\Rightarrow f \circ g(x) = 2x^2 + 13$$

Functions Ex2.2 Q1(ii)

$$f(x) = 2x + x^2 \quad \text{and} \quad g(x) = x^3$$

$$g \circ f(x) = g(f(x)) = g(2x + x^2)$$

$$g \circ f(x) = (2x + x^2)^3$$

$$f \circ g(x) = f(g(x)) = f(x^3)$$

$$\therefore f \circ g(x) = 2x^3 + x^6$$

Functions Ex2.2 Q1(iii)

$$f(x) = x^2 + 8 \text{ and } g(x) = 3x^3 + 1$$

$$\text{Thus, } g \circ f(x) = g[f(x)]$$

$$\Rightarrow g \circ f(x) = g[x^2 + 8]$$

$$\Rightarrow g \circ f(x) = 3[x^2 + 8]^3 + 1$$

$$\text{Similarly, } f \circ g(x) = f[g(x)]$$

$$\Rightarrow f \circ g(x) = f[3x^3 + 1]$$

$$\Rightarrow f \circ g(x) = [3x^3 + 1]^2 + 8$$

$$\Rightarrow f \circ g(x) = [9x^6 + 1 + 6x^3] + 8$$

$$\Rightarrow f \circ g(x) = 9x^6 + 6x^3 + 9$$

Functions Ex2.2 Q1(iv)

$$f(x) = x \text{ and } g(x) = |x|$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x)$$

$$\therefore g \circ f(x) = |x|$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(|x|)$$

$$\therefore f \circ g(x) = |x|$$

Functions Ex2.2 Q1(v)

$$f(x) = x^2 + 2x - 3 \text{ and } g(x) = 3x - 4$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x^2 + 2x - 3)$$

$$\therefore g \circ f(x) = 3(x^2 + 2x - 3) - 4$$

$$\Rightarrow g \circ f(x) = 3x^2 + 6x - 13$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(3x - 4)$$

$$\begin{aligned} \therefore f \circ g(x) &= (3x - 4)^2 + 2(3x - 4) - 3 \\ &= 9x^2 + 16 - 24x + 6x - 8 - 3 \end{aligned}$$

$$\therefore f \circ g(x) = 9x^2 - 18x + 5$$

Functions Ex2.2 Q1(vi)

$$f(x) = 8x^3 \text{ and } g(x) = x^{1/3}$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(8x^3)$$

$$= (8x^3)^{1/3}$$

$$\therefore g \circ f(x) = 2x$$

$$\text{and, } f \circ g(x) = f(g(x)) = f(x^{1/3})$$

$$= 8(x^{1/3})^3$$

$$\therefore f \circ g(x) = 8x$$

Functions Ex2.2 Q2

Let $f = \{(3, 1), (9, 3), (12, 4)\}$ and

$$g = \{(1, 3), (3, 3), (4, 9), (5, 9)\}$$

Now,

$$\text{range of } f = \{1, 3, 4\}$$

$$\text{domain of } f = \{3, 9, 12\}$$

$$\text{range of } g = \{3, 9\}$$

$$\text{domain of } g = \{1, 3, 4, 5\}$$

since, range of $f \subset$ domain of g

$\therefore g \circ f$ is well defined.

Again, range of $g \subseteq$ domain of f

$\therefore f \circ g$ is well defined.

Now $g \circ f = \{(3, 3), (9, 3), (12, 9)\}$

$$f \circ g = \{(1, 1), (3, 1), (4, 3), (5, 3)\}$$

Functions Ex2.2 Q3

We have,

$$f = \{(1, -1), (4, -2), (9, -3), (16, 4)\} \text{ and}$$

$$g = \{(-1, -2), (-2, -4), (-3, -6), (4, 8)\}$$

Now,

$$\text{Domain of } f = \{1, 4, 9, 16\}$$

$$\text{Range of } f = \{-1, -2, -3, 4\}$$

$$\text{Domain of } g = \{-1, -2, -3, 4\}$$

$$\text{Range of } g = \{-2, -4, -6, 8\}$$

Clearly range of $f =$ domain of g

$\therefore g \circ f$ is defined.

but, range of $g \neq$ domain of f

$\therefore f \circ g$ is not defined.

Now,

$$g \circ f(1) = g(-1) = -2$$

$$g \circ f(4) = g(-2) = -4$$

$$g \circ f(9) = g(-3) = -6$$

$$g \circ f(16) = g(4) = 8$$

$\therefore g \circ f = \{(1, -2), (4, -4), (9, -6), (16, 8)\}$

Functions Ex2.2 Q4

$A = \{a, b, c\}$, $B = \{u, v, w\}$ and

$f = A \rightarrow B$ and $g : B \rightarrow A$ defined by

$f = \{(a, v), (b, u), (c, w)\}$ and

$g = \{(u, b), (v, a), (w, c)\}$

For both f and g , different elements of domain have different images

$\therefore f$ and g are one-one

Again for each element in co-domain of f and g , there is a pre image in domain

$\therefore f$ and g are onto

Thus, f and g are bijections.

Now,

$$g \circ f = \{(a, a), (b, b), (c, c)\} \text{ and}$$

$$f \circ g = \{(u, u), (v, v), (w, w)\}$$

Functions Ex2.2 Q5

We have, $f : R \rightarrow R$ given by $f(x) = x^2 + 8$ and

$g : R \rightarrow R$ given by $g(x) = 3x^3 + 1$

$$\begin{aligned} \therefore f \circ g(x) &= f(g(x)) = f(3x^3 + 1) \\ &= (3x^3 + 1)^2 + 8 \end{aligned}$$

$$\therefore f \circ g(2) = (3 \times 8 + 1)^2 + 8 = 625 + 8 = 633$$

Again

$$\begin{aligned} g \circ f(x) &= g(f(x)) = g(x^2 + 8) \\ &= 3(x^2 + 8)^3 + 1 \end{aligned}$$

$$\therefore g \circ f(1) = 3(1 + 8)^3 + 1 = 2188$$

Functions Ex2.2 Q6

We have, $f : R^+ \rightarrow R^+$ given by

$$f(x) = x^2$$

$g : R^+ \rightarrow R^+$ given by

$$g(x) = \sqrt{x}$$

$$\therefore f \circ g(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

Also,

$$g \circ f(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

Thus,

$$f \circ g(x) = g \circ f(x)$$

Functions Ex2.2 Q7

We have, $f : R \rightarrow R$ and $g : R \rightarrow R$ are two functions defined by

$$f(x) = x^2 \text{ and } g(x) = x + 1$$

Now,

$$f \circ g(x) = f(g(x)) = f(x + 1) = (x + 1)^2$$

$$\therefore f \circ g(x) = x^2 + 2x + 1 \dots\dots\dots (i)$$

$$g \circ f(x) = g(f(x)) = g(x^2) = x^2 + 1 \dots\dots\dots (ii)$$

from (i) & (ii)

$$f \circ g \neq g \circ f$$

Functions Ex2.2 Q8

Let $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined as

$$f(x) = x + 1 \text{ and } g(x) = x - 1$$

Now,

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(x - 1) = x - 1 + 1 \\ &= x = I_R \dots\dots\dots (i) \end{aligned}$$

Again,

$$\begin{aligned} f \circ g(x) &= f(g(x)) = g(x + 1) = x + 1 - 1 \\ &= x = I_R \dots\dots\dots (ii) \end{aligned}$$

from (i) & (ii)

$$f \circ g = g \circ f = I_R$$

Functions Ex2.2 Q9

We have, $f : N \rightarrow Z_0$, $g : Z_0 \rightarrow Q$ and

$$h : Q \rightarrow R$$

$$\text{Also, } f(x) = 2x, \quad g(x) = \frac{1}{x} \text{ and } h(x) = e^x$$

Now, $f : N \rightarrow Z_0$ and $h \circ g : Z_0 \rightarrow R$

$$\therefore (h \circ g) \circ f : N \rightarrow R$$

also, $g \circ f : N \rightarrow Q$ and $h : Q \rightarrow R$

$$\therefore h \circ (g \circ f) : N \rightarrow R$$

Thus, $(h \circ g) \circ f$ and $h \circ (g \circ f)$ exist and are function from N to set R .

$$\begin{aligned} \text{Finally. } (h \circ g) \circ f(x) &= (h \circ g)(f(x)) = (h \circ g)(2x) \\ &= h\left(\frac{1}{2x}\right) \\ &= e^{\frac{1}{2x}} \end{aligned}$$

$$\begin{aligned} \text{now, } h \circ (g \circ f)(x) &= h(g(2x)) = h\left(\frac{1}{2x}\right) \\ &= e^{\frac{1}{2x}} \end{aligned}$$

Hence, associativity verified.

Functions Ex2.2 Q10

We have,

$$\begin{aligned} h \circ (g \circ f)(x) &= h(g \circ f(x)) = h(g(f(x))) \\ &= h(g(2x)) = h(3(2x) + 4) \\ &= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N} \end{aligned}$$

$$\begin{aligned} ((h \circ g) \circ f)(x) &= (h \circ g)(f(x)) = (h \circ g)(2x) \\ &= h(g(2x)) = h(3(2x) + 4) \\ &= h(6x + 4) = \sin(6x + 4) \quad \forall x \in \mathbf{N} \end{aligned}$$

This shows, $h \circ (g \circ f) = (h \circ g) \circ f$

Functions Ex2.2 Q11

Define $f: \mathbf{N} \rightarrow \mathbf{N}$ by, $f(x) = x + 1$

And, $g: \mathbf{N} \rightarrow \mathbf{N}$ by,

$$g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

We first show that f is not onto.

For this, consider element 1 in co-domain \mathbf{N} . It is clear that this element is not an image of any of the elements in domain \mathbf{N} .

Therefore, f is not onto.

Now, $g \circ f: \mathbf{N} \rightarrow \mathbf{N}$ is defined by,

Functions Ex2.2 Q12

Define $f: \mathbf{N} \rightarrow \mathbf{Z}$ as $f(x) = x$ and $g: \mathbf{Z} \rightarrow \mathbf{Z}$ as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$$g(-1) = |-1| = 1$$

$$g(1) = |1| = 1$$

Therefore, $g(-1) = g(1)$, but $-1 \neq 1$.

Therefore, g is not injective.

Now, $g \circ f: \mathbf{N} \rightarrow \mathbf{Z}$ is defined as $g \circ f(x) = g(f(x)) = g(x) = |x|$.

Let $x, y \in \mathbf{N}$ such that $g \circ f(x) = g \circ f(y)$.

$$\Rightarrow |x| = |y|$$

Since x and $y \in \mathbf{N}$, both are positive.

$$\therefore |x| = |y| \Rightarrow x = y$$

Hence, $g \circ f$ is injective

Functions Ex2.2 Q13

We have, $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one functions

Now we have to prove $g \circ f: A \rightarrow C$ in one-one

let $x, y \in A$ such that

$$g \circ f(x) = g \circ f(y)$$

$$\Rightarrow g(f(x)) = g(f(y))$$

$$\Rightarrow f(x) = f(y) \quad [\because g \text{ in one-one}]$$

$$\Rightarrow x = y \quad [\because f \text{ in one-one}]$$

$\therefore g \circ f$ is one-one function

Functions Ex2.2 Q14

We have, $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto functions.

Now, we need to prove: $g \circ f: A \rightarrow C$ is onto.

Let $y \in C$, then

$$g \circ f(x) = y$$

$$\Rightarrow g(f(x)) = y \dots\dots\dots (i)$$

Since g is onto, for each element in C , then exists a preimage in B .

$$\therefore g(x) = y \dots\dots\dots (ii)$$

From (i) & (ii)

$$f(x) = \alpha.$$

Since f is onto, for each element in B there exists a preimage in A

$$\therefore f(x) = \alpha \dots\dots\dots (iii)$$

From (ii) and (iii) we can conclude that for each $y \in C$, there exists a preimage in A such that $g \circ f(x) = y$

$$\therefore g \circ f \text{ is onto}$$