RD Sharma Solutions Class 12 Maths Chapter 2 Ex 2.3

Functions Ex 2.3 Q 1(i)

$f(x) = e^x$ and $g(x) = \log_e x$

Now, $f \circ g(x) = f(g(x)) = f(\log_e x) = e^{\log_e x} = x$

$$f \circ g(x) = x$$

$g \circ f(x) = g(f(x)) = g(e^x) = \log_e e^x = x$

$\Rightarrow \quad g \circ f(x) = x$

Functions Ex 2.3 Q 1(ii)

$f(x) = x^2, \quad g(x) = \cos x$

Domain of f and Domain of $g = R$

Range of $f = (0, \infty)$

Range of $g = (-1, 1)$

$\therefore$ Range of $f \subset$ domain of $g \Rightarrow g \circ f$ exist

Range of $g \subset$ domain of $f \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(x^2) = \cos x^2$$

And

$$f \circ g(x) = f(f(x)) = f(\cos x) = \cos^2 x$$

Functions Ex 2.3 Q1(iii)

$f(x) = |x|$ and $g(x) = \sin x$

Range of $f = (0, \infty) \subset$ Domain $g(R) \Rightarrow g \circ f$ exist

Range of $g = [-1, 1] \subset$ Domain of $(R) \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = |\sin x|$$

And

$$g \circ f(x) = g(f(x)) = g(|x|) = \sin|x|$$

Functions Ex 2.3 Q1(iv)

$f(x) = x + 1$ and $g(x) = e^x$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = (0, \infty) \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(x+1) = e^{x+1}$$

And

$$f \circ g(x) = f(g(x)) = f(e^x) = e^x + 1$$

Functions Ex 2.3 Q1(v)

$f(x) = \sin^{-1} x$ and $g(x) = x^2$

Range of $f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = (0, \infty) \subseteq$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(x^2) = \sin^{-1} x^2$$

And

$$g \circ f(x) = g(f(x)) = g(\sin^{-1} x) = (\sin^{-1} x)^2$$

Functions Ex 2.3 Q 1(vi)

$f(x) = x + 1$ and $g(x) = \sin x$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exists

Range of $g = [-1, 1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exists

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = \sin x + 1$$

And

$$g \circ f(x) = g(f(x)) = g(x+1) = \sin(x+1)$$

Functions Ex 2.3 Q1(vii)

$f(x) = x + 1$ and $g(x) = 2x + 3$

Range of $f = R \subseteq$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = R \subseteq$ Domain of $R = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(2x + 3) = (2x + 3) + 1 = 2x + 4$$

And

$$g \circ f(x) = g(f(x)) = g(x + 1) = 2(x + 1) + 3$$

$$\Rightarrow \quad g \circ f(x) = 2x + 5$$

Functions Ex 2.3 Q1(viii)

$f(x) = c, \quad c \in R$ and

$g(x) = \sin x^2$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = [-1, 1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(c) = \sin c^2$$

And

$$f \circ g(x) = f(g(x)) = f(\sin x^2) = c$$

Functions Ex 2.3 Q1(ix)

$f(x) = x^2 + 2$ and $g(x) = 1 - \frac{1}{1 - x}$

Range of $f = (2, \infty) \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = R - [1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f\left(\frac{-x}{1 - x}\right) = \frac{x^2}{(1 - x)^2} + 2$$

And

$$g \circ f(x) = g(f(x)) = g(x^2 + 2) = \frac{-(x^2 + 2)}{1 - (x^2 + 2)}$$

$$\Rightarrow \quad g \circ f(x) = \frac{x^2 + 2}{x^2 + 1}$$

Functions Ex 2.3 Q2

We have, $f(x) = x^2 + x + 1$ and $g(x) = \sin x$

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x)$$

$$\Rightarrow \quad f \circ g(x) = \sin^2 x + \sin x + 1$$

Again, $g \circ f(x) = g(f(x)) = g(x^2 + x + 1)$

$$\Rightarrow \quad g \circ f(x) = \sin(x^2 + x + 1)$$

Clearly

$$f \circ g \neq g \circ f$$

Functions Ex 2.3 Q3

We have $\quad f(x) = |x|$

We assume the domain of $f = R$

Range of $f = (0, \infty)$

$\therefore$ Range of f $\subset$ domain of f

$\therefore$ $f \circ f$ exists.

Now,

$$f \circ f(x) = f(f(x)) = f(|x|) = ||x|| = f(x)$$

$\therefore$ $f \circ f = f$

Functions Ex 2.3 Q4

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + 1$$

$\therefore$ Range of $f = R$ and range of $g = [1, \infty]$

$\therefore$ Range of $f \subseteq$ Domain of $g(R)$ and range of $g \subseteq$ domain of $f(R)$

$\therefore$ both fog and gof exist.

i) $f \circ g(x) = f(g(x)) = f(x^2 + 1)$

$= 2(x^2 + 1) + 5$

$\Rightarrow$ $f \circ g(x) = 2x^2 + 7$

ii) $g \circ f(x) = g(f(x)) = g(2x + 5)$

$= (2x + 5)^2 + 1$

$\Rightarrow$ $g \circ f(x) = 4x^2 + 20x + 26$

iii) $f \circ f(x) = f(f(x)) = f(2x + 5)$

$= 2(2x + 5) + 5$

$f \circ f(x) = 4x + 15$

iv) $f^2(x) = [f(x)]^2 = (2x + 5)^2$

$= 4x^2 + 20x + 25$

$\therefore$ from (iii) & (iv)

$f \circ f \neq f^2$

Functions Ex 2.3 Q5

We have, $f(x) = \sin x$ and $g(x) = 2x$.

Domain of f and g is R

Range of $f = [-1, 1]$

Range of $g = R$

$\therefore$ Range of $f \subset$ Domain g and

Range of $g \subseteq$ Domain f

$\therefore$ fog and gof both exist.

i) $g \circ f(x) = g(f(x)) = g(\sin x) = g \circ f(x) = 2 \sin x$

ii) $f \circ g(x) = f(g(x)) = f(2x) = \sin 2x$

$\therefore$ $g \circ f \neq f \circ g$

Functions Ex 2.3 Q6

$f, g,$ and h are real functions given by $f(x) = \sin x$, $g(x) = 2x$ and $h(x) = \cos x$

To prove: $f \circ g = g \circ (fh)$

L.H.S

$$f \circ g(x) = f(g(x))$$
$$= f(2x) = \sin 2x$$
$$\Rightarrow f \circ g(x) = 2\sin x \cos x \text{ ....(A)}$$

R.H.S

$$g \circ (fh)(x) = go(f(x).h(x))$$
$$= g(\sin x \cos x)$$
$$g \circ (fh)(x) = 2\sin x \cos x \text{ ....(B)}$$

from A & B

$$f \circ g(x) = g \circ (fh)(x)$$

Functions Ex 2.3 Q7

We are given that f is a real function and g is a function given by $g(x) = 2x$

To prove; $g \circ f = f + f$.

L.H.S

$$g \circ f(x) = g(f(x)) = 2f(x)$$
$$= f(x) + f(x) = \text{R.H.S}$$

$\Rightarrow \quad g \circ f = f + f$

Functions Ex 2.3 Q8

$$f(x) = \sqrt{1-x}\ , \quad g(x) = log_e^x$$

Domain of f and g are R.

Range of $f = (-\infty, 1)$

Range of $g = (0, e)$

Clearly Range $f \subset$ Domain $g \Rightarrow g \circ f$ exists

Range $g \subset$ Domain $f \Rightarrow f \circ g$ exists

$$\therefore\ g \circ f(x) = g(f(x)) = g\left(\sqrt{1-x}\right)$$
$$g \circ f(x) = log_e^{\sqrt{1-x}}$$

Again

$$f \circ g(x) = f(g(x)) = f\left(log_e^x\right)$$
$$f \circ g(x) = \sqrt{1 - log_e^x}$$

Functions Ex 2.3 Q9

$f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ and $g:[-1,1]\to R$ defined as $f(x)=\tan x$ and $g(x)=\sqrt{1-x^2}$

Range of f : let $y=f(x) \Rightarrow y=\tan x$

$$\Rightarrow x=\tan^{-1} y$$

Since $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right), y \in (-\infty,\infty)$

$\therefore$ Range of f $\subset$ domain of $g=[-1,1]$

$\therefore$ $g\circ f$ exists.

By similar argument $f\circ g$ exists.

Now,

$$f\circ g(x)=f(g(x))=f\left(\sqrt{1-x^2}\right)$$

$$f\circ g(x)=\tan\sqrt{1-x^2}$$

Again

$$g\circ f(x)=g(f(x))$$
$$=g(\tan x)$$
$$g\circ f(x)=\sqrt{1-\tan^2 x}$$

Functions Ex 2.3 Q10

$$f(x)=\sqrt{x+3} \text{ and } g(x)=x^2+1$$

Now,

Range of $f=[-3,\infty]$ and

Range of $g=(1,\infty)$

Then, Range of $f\subset$ Domain g and

Range of $g\subset$ Domain f

$\therefore$ $f\circ g$ and $g\circ f$ exist.

Now,

$$f\circ g(x)=f(g(x))=f\left(x^2+1\right)$$
$$f\circ g(x)=\sqrt{x^2+4}$$

Again,

$$g\circ f(x)=g(f(x))=g\left(\sqrt{x+3}\right)$$
$$=\left(\sqrt{x+3}\right)^2+1$$
$$g\circ f(x)=x+4$$

Functions Ex 2.3 Q11(i)

We have, $f(x) = \sqrt{x-2}$

Clearly, Domain$(f) = [2, \infty)$ and Range$(f) = [0, \infty)$.

We observe that range(f) is not a subset of domain of f.

$$\therefore \text{Domain of } (fof) = \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\}$$
$$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\}$$
$$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\}$$
$$= \{x : x \in [2, \infty) \text{ and } x - 2 \geq 4\}$$
$$= \{x : x \in [2, \infty) \text{ and } x \geq 6\}$$
$$= [6, \infty)$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2} - 2}$$

$\therefore$ $fof : [6, \infty) \to R$ defined as

$$(fof)(x) = \sqrt{\sqrt{x-2} - 2}$$

Functions Ex 2.3 Q11(ii)

We have, $f(x) = \sqrt{x-2}$

Clearly, Domain$(f) = [2, \infty)$ and Range$(f) = [0, \infty)$.

We observe that range(f) is not a subset of domain of f.

$\therefore$ Domain of $(fof) = \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\}$

$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\}$

$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\}$

$= \{x : x \in [2, \infty) \text{ and } x-2 \geq 4\}$

$= \{x : x \in [2, \infty) \text{ and } x \geq 6\}$

$= [6, \infty)$

Clearly, range of f $= [0, \infty) \not\subset$ Domain of (fof).

$\therefore$ Domain of $((fof)of) = \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (fof)\}$

$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [6, \infty)\}$

$= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 6\}$

$= \{x : x \in [2, \infty) \text{ and } x-2 \geq 36\}$

$= \{x : x \in [2, \infty) \text{ and } x \geq 38\}$

$= [38, \infty)$

Now,

$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

$(fofof)(x) = (fof)(f(x)) = (fof)\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$

$\therefore$ fofof : $[38, \infty) \to R$ defined as

$(fofof)(x) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$

Functions Ex 2.3 Q11(iii)

We have, $f(x)=\sqrt{x-2}$

Clearly, Domain$(f)=[2,\infty)$ and Range$(f)=[0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$\therefore$ Domain of $(fof)=\{x: x\in \text{Domain }(f) \text{ and } f(x)\in \text{Domain }(f)\}$

$$=\{x: x\in[2,\infty) \text{ and } \sqrt{x-2}\in[2,\infty)\}$$

$$=\{x: x\in[2,\infty) \text{ and } \sqrt{x-2}\geq 2\}$$

$$=\{x: x\in[2,\infty) \text{ and } x-2\geq 4\}$$

$$=\{x: x\in[2,\infty) \text{ and } x\geq 6\}$$

$$=[6,\infty)$$

Clearly, range of $f=[0,\infty)\not\subset$ Domain of (fof).

$\therefore$ Domain of $((fof)of)=\{x: x\in \text{Domain }(f) \text{ and } f(x)\in \text{Domain }(fof)\}$

$$=\{x: x\in[2,\infty) \text{ and } \sqrt{x-2}\in[6,\infty)\}$$

$$=\{x: x\in[2,\infty) \text{ and } \sqrt{x-2}\geq 6\}$$

$$=\{x: x\in[2,\infty) \text{ and } x-2\geq 36\}$$

$$=\{x: x\in[2,\infty) \text{ and } x\geq 38\}$$

$$=[38,\infty)$$

Now,

$$(fof)(x)=f(f(x))=f\left(\sqrt{x-2}\right)=\sqrt{\sqrt{x-2}-2}$$

$$(fofof)(x)=(fof)(f(x))=(fof)\left(\sqrt{x-2}\right)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore$ $fofof:[38,\infty)\to R$ defined as

$$(fofof)(x)=\sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$$(fofof)(38)=\sqrt{\sqrt{\sqrt{38-2}-2}-2}=\sqrt{\sqrt{\sqrt{36}-2}-2}=\sqrt{\sqrt{6-2}-2}=\sqrt{\sqrt{4}-2}=\sqrt{2-2}=0$$

Functions Ex 2.3 Q11(iv)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2, \infty)$ and $\text{Range}(f) = [0, \infty)$.

We observe that range(f) is not a subset of domain of f.

$$\begin{aligned}\therefore \text{Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\}\\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\}\\ &= \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\}\\ &= \{x : x \in [2, \infty) \text{ and } x - 2 \geq 4\}\\ &= \{x : x \in [2, \infty) \text{ and } x \geq 6\}\\ &= [6, \infty)\end{aligned}$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2} - 2}$$

$\therefore$ $fof : [6, \infty) \to R$ defined as

$$(fof)(x) = \sqrt{\sqrt{x-2} - 2}$$

$$f^2(x) = [f(x)]^2 = \left[\sqrt{x-2}\right]^2 = x - 2$$

$\therefore$ $f^2 : [2, \infty) \to R$ defined as

$$f^2(x) = x - 2$$

$\therefore$ $fof \neq f^2$

Functions Ex 2.3 Q12

$$f(x) \begin{cases} 1 + x & 0 \leq x \leq 2 \\ 3 - x & 2 \leq x \leq 3 \end{cases}$$

$\therefore$ Range of $f = [0, 3] \subseteq$ Domain of f.

$$\therefore\ f \circ f(x) = f(f(x)) = f \begin{cases} 1 + x & 0 \leq x \leq 2 \\ 3 - x & 2 < x \leq 3 \end{cases}$$

$$f \circ f(x) = \begin{cases} 2 + x & 0 \leq x \leq 1 \\ 2 - x & 1 < x \leq 2 \\ 4 - x & 2 < x \leq 3 \end{cases}$$