RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.2

Binary Operations Ex 3.2 Q1

We have,

$a * b = l.c.m.(a, b)$ for all $a, b \in N$

(i)

Now,

$2 * 4 = l.c.m\ (2,4) = 4$

$3 * 5 = l.c.m\ (3,5) = 15$

$1 * 6 = l.c.m\ (1,6) = 6$

(ii)

Commutativity:

Let $a, b \in N$ then,

$a * b = l.c.m(a, b)$

$= l.c.m(b, a)$

$= b * a$

$\Rightarrow \quad a * b = b * a$

$\therefore$ $*$ is commutative on N.

Associativity:

Let $a, b, c \in N$ then,

$(a * b) * c = l.c.m(a, b) * c$

$= l.c.m(a, b, c) \quad ---(i)$

and, $a * (b * c) = a * l.c.m(b, c)$

$= l.c.m(a, b, c) \quad ---(ii)$

From (i) and (ii)

$(a * b) * c = a * (b * c)$

$\therefore$ $*$ is associative on N.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in \mathbf{N}$

Also, $(a * b) * c = (1 * c) = 1$

and $a * (b * c) = (a * 1) = 1 \qquad \forall a, b, c \in \mathbf{N}$

Hence, $\mathbf{N}$ is both associative and commutative.

(ii) $a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a,$

which shows $*$ is commutative.

Further, $(a * b) * c = \left(\frac{a+b}{2}\right) * c = \frac{\left(\frac{a+b}{2}\right)+c}{2} = \frac{a+b+2c}{4}$

$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{a+\left(\frac{b+c}{2}\right)}{2} = \frac{2a+b+c}{2} \neq \frac{a+b+2c}{4}$

Hence, $*$ is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$a*b=b$ for all $a,b\in A$

Commutativity: Let $a,b\in A$, then

$a*b=b\neq a=b*a$

$\Rightarrow$ $a*b\neq b*a$

$\therefore$ '$*$' is not commutative on A.

Associativity: Let $a,b,c\in A$, then

$(a*b)*c=b*c=c$ ---(i)

and, $a*(b*c)=a*c=c$ ---(ii)

From (i) and (ii)

$(a*b)*c=a*(b*c)$

$\Rightarrow$ '$*$' is associative on A.

Binary Operations Ex 3.2 Q4(i)

'$*$' is a binary operator on Z defined by $a*b=a+b+ab$ for all $a,b\in Z$.

Commutativity of '$*$':

Let $a,b\in Z$, then

$a*b=a+b+ab=b+a+ba=b*a$

$\therefore$ $a*b=b*a$

Associative of '$*$':

Let $a,b\in Z$, then

$(a*b)*c=(a+b+ab)*c=a+b+ab+c+ac+bc+abc$

$=a+b+c+ab+bc+ac+abc$ ---(i)

Again, $a*(b*c)=a*(b+c+bc)$

$=a+b+c+bc+ab+ac+abc$ ---(ii)

From (i) & (ii), we get

$(a*b)*c=a*(b*c)$

$\therefore$ $*$ is commutative and associative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a, b \in N$, then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$\therefore \quad a * b = b * a$

$\therefore \quad *$ is commutative on N

Associative:

Let $a, b, c \in N$, then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab}.c} \qquad ---(i)$$

and, $a * (b * c) = a * 2^{bc} = 2^{a.2^{bc}} \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a - b \neq b - a = b * a$$

$\therefore \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a - b) * c = a - b - c \qquad ---(i)$$

and, $a * (b * c) = a * (b - c) = a - b + c \qquad ---(ii)$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then

$$a * b = a^2 + b^2 = b^2 + a^2 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on Q.

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = \left(a^2 + b^2\right) * c = \left(a^2 + b^2\right)^2 + c^2 \qquad ---(i)$$

and, $\quad a * (b * c) = a * \left(a^2 + b^2\right) = a^2 + \left(b^2 + c^2\right)^2 \qquad ---(ii)$

From (i) & (ii),

$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q4(v)

Binary operation '$\circ$' defined on Q, given by $a \circ b = \frac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\therefore \quad \circ$ is commutative on Q.

Associativity:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \qquad ---(i)$$

$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \quad$ '$\circ$' is associative on Q.

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let $a, b \in Q$, then

$$a * b = ab^2 \neq ba^2 = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\therefore$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = ab^2 * c = ab^2c^2 \qquad ---(i)$$

& $$a * (b * c) = a * bc^2 = a\left(bc^2\right)^2 \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a + ab \qquad ---(i)$$

$$b * a = b + ab \qquad ---(ii)$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \qquad ---(i)$$

$$a * (b * c) = a * (b + bc)$$

$$= a + ab + abc \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$a * b = a + b - 7$$
$$= b + a - 7$$
$$= b * a$$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad *$ is commutative on R

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = (a + b - 7) * c$$
$$= a + b - 7 + c - 7$$
$$= a + b + c - 17 \qquad ---(i)$$

and, $a * (b * c) = a * (b + c - 7)$
$$= a + b + c - 7 - 7$$
$$= a + b + c - 17 \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \quad *$ is associative on R

Binary Operations Ex 3.2 Q4(ix)

Commutativity:

Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on $R - \{-1\}$

Associativity:

Let $a, b, c \in R - \{-1\}$, then

$$(a * b) * c = \left(\frac{a}{b+1}\right) * c$$

$$= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \qquad ---(i)$$

& $\quad a * (b * c) = a * \left(\frac{b}{c+1}\right)$

$$= \frac{a}{\frac{b}{c+1}+1} = \frac{a(c+1)}{b+c+1} \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on $R - \{-1\}$

Binary Operations Ex 3.2 Q4(x)

Commutativity:

Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad *$ is commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (ab + 1) * c$$

$$= abc + c + 1 \qquad ---(i)$$

$$a * (b * c) = a * (bc + 1)$$

$$= abc + a + 1 \qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \quad *$ is not associative on Q.

Binary Operations Ex 3.2 Q4(xi)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow \quad a * b \neq b * a$$

$\Rightarrow$ '$*$' is not commutative on N

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$\Rightarrow$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xii)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow \quad a * b \neq b * a$$

$\Rightarrow$ '$*$' is not commutative on N

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = \left(a^b\right)^c = a^{bc} \qquad ---(i)$$

$$a * (b * c) = a * b^c = (a)^{b^c} \qquad ---(ii)$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$\Rightarrow$ '$*$' is not associative on N.

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:

Let $a, b \in Z$ then,

$$a * b = a - b \neq b - a = b * a$$

$\Rightarrow$ $a * b \neq b * a$

$\Rightarrow$ $*$ is not commutative on Z

Associativity:

Let $a, b, c \in Z$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \quad ---(i)$$

& $a * (b * c) = a * (b - c) = (a - b + c) \quad ---(ii)$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:

Let $a, b \in Q$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$\Rightarrow$ $a * b = b * a$

$\therefore$ $*$ is commutative on Q

Associativity:

Let $a, b, c \in Q$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \quad ---(i)$$

and, $a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \quad ---(ii)$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q4(xv)

Commutativity:

Let $a, b \in Q$ then,

$$a * b = (a-b)^2 = (b-a)^2 = b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore$ '$*$' is commutative on Q.

Associativity:

Let $a, b, c \in Q$ then,

$$(a * b) * c = (a-b)^2 * c = \left[(a-b)^2 - c\right]^2 \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b-c)^2 = \left[a - (b-c)^2\right]^2 \qquad ---(ii)$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore$ $*$ is not associative on Q.

Binary Operations Ex 3.2 Q5

The binary operator o defined on $Q - \{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q - \{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$\Rightarrow \quad a \circ b = b \circ a$

$\Rightarrow$ '$\circ$' is commutative on $Q - \{-1\}$.

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$

$$b * a = 3b + 7a$$

$\therefore \quad a * b \neq b * a$

Hence, '$*$' is not commutative on Z.

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$a*b = ab+1$ for all $a,b \in Z$

Associativity: Let $a,b,c \in Z$, then

$(a*b)*c = (ab+1)*c$

$= abc+c+1$ $\quad$ $---(i)$

and, $a*(b*c) = a*(bc+1)$

$= abc+a+1$ $\quad$ $---(ii)$

From (i) & (ii)

$\therefore$ $(a*b)*c \neq a*(b*c)$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$a*b = a+b+ab$ for all $a,b \in S = R-\{-1\}$

Now, $\forall\ a,b \in S$

$a*b = a+b+ab \in S$

$\because$ if $a+b+ab = -1$

$\Rightarrow$ $a+b(1+a)+1 = 0$

$\Rightarrow$ $(a+1)(b+1) = 0$

$\Rightarrow$ $a = -1$ or $b = -1$

but $a \neq -1$ and $b \neq -1$ (given)

$\therefore$ $a+b+ab \neq -1$

$\Rightarrow$ $a*b \in S$ for $ab \in S$

$\Rightarrow$ '$*$' is a binary operator on S

Commutativity: Let $a,b \in S$

$\Rightarrow$ $a*b = a+b+ab = b+a+ba = b*a$

$\Rightarrow$ $a*b = b*a$

and, $a*(b*c) = a*(b+c+bc)$

$= a+b+c+bc+ab+ac+abc \quad ---(ii)$

From (i) and (ii)

$(a*b)*c = a*(b*c)$

$\therefore$ '$*$' is associative on S.

Now, $(2*x)*3 = 7$

$\Rightarrow \quad (2+x+2x)*3 = 7$

$\Rightarrow \quad 2+x+2x+3+6+3x+6x = 7$

$\Rightarrow \quad 11+12x = 7$

$\Rightarrow \quad 12x = -4$

$\Rightarrow \quad x = \frac{-4}{12} \qquad \Rightarrow x = \frac{-1}{3}$

Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as

$a*b = \frac{a-b}{2}$ for all $a, b \in Q$.

Now,

Associativity: Let $a, b, c \in Q$, then

$(a*b)*c = \frac{a-b}{2}*c = \frac{\frac{a-b}{2}-c}{2}$

$= \frac{a-b-2c}{4} \quad ---(i)$

and, $a*(b*c) = a*\frac{b-c}{2} = \frac{a-\frac{b-c}{2}}{2}$

$= \frac{2a-b+c}{4} = \quad ---(ii)$

From (i) & (ii)

$(a*b)*c \neq a*(b*c)$

Hence, '$*$' is not associative on Q.

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as

$a * b = a + 3b - 4$ for all $a, b \in Z$

Now,

Commutativity: Let $a, b \in Z$, then

$a * b = a + 3b - 4 \neq b + 3a - 4 = b * a$

$\Rightarrow \quad a * b \neq b * a$

$\Rightarrow$ '$*$' is not commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$(a * b) * c = (a + 3b - 4) * c = a + 3b - 4 + 3c - 4$

$= a + 3b + 3c - 8 \quad ---(i)$

and, $a * (b * c) = a * (b + 3c - 4) = a + 3(b + 3c - 4) - 4$

$= a + 3b + 9c - 16 \quad ---(ii)$

From (i) & (ii)

$(a * b) * c \neq a * (b * c)$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$a * b = \frac{ab}{5}$ for all $a, b \in Q$

Now,

Associativity: Let $a, b, c \in Q$, then

$(a * b) * c = \frac{ab}{5} * c = \frac{abc}{25} \quad ---(i)$

and, $a * (b * c) = a * \frac{bc}{5} = \frac{abc}{25} \quad ---(ii)$

From (i) & (ii)

$\therefore \quad (a * b) * c = a * (b * c)$

$\Rightarrow$ $*$ is associative on Q.

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a*b=\frac{ab}{7} \text{ for all } a,b\in Q$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{ab}{7}*c=\frac{abc}{49} \quad ---(i)$$

and, $$a*(b*c)=a*\frac{bc}{7}=\frac{abc}{49} \quad ---(ii)$$

From (i) & (ii)

$$(a*b)*c=a*(b*c)$$

$\Rightarrow$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a*b=\frac{a+b}{2} \text{ for all } a,b\in Q.$$

Now,

Associativity: Let $a,b,c\in Q$, then

$$(a*b)*c=\frac{a+b}{2}*c=\frac{\frac{a+b}{2}+c}{2}$$

$$=\frac{a+b+2c}{4} \quad ---(i)$$

and, $$a*(b*c)=a*\frac{b+c}{2}$$

$$=\frac{a+\frac{b+c}{2}}{2}$$

$$=\frac{2a+b+c}{4}= \quad ---(ii)$$

From (i) & (ii)

$$(a*b)*c\neq a*(b*c)$$

Hence, '$*$' is not associative on Q.