RD Sharma Solutions Class 12 Maths Chapter 4 Ex 4.1

Inverse Trigonometric Functions Ex 4.1 Q1.

Let $\tan^{-1}\left(-\sqrt{3}\right)=y$. Then, $\tan y=-\sqrt{3}=-\tan\frac{\pi}{3}=\tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Concept Insight:

The range for $\tan^{-1}$ is same as $\sin^{-1}$ except that it is an open interval, as $\tan(-\pi/2)$ and $\tan(\pi/2)$ are not defined. So the method of finding principal value is same as $\sin^{-1}$ given in the first problem. Also note that $\tan(-x) = -\tan x$.

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=y$. Then, $\cos y=-\frac{1}{\sqrt{2}}=-\cos\left(\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{3\pi}{4}\right).=-\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y$. Then, $\operatorname{cosec} y=-\sqrt{2}=-\operatorname{cosec}\left(\frac{\pi}{4}\right)=\operatorname{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

We know that for any $x\in[-1,1]$, $\cos^{-1}x$ represents angle in $[0,\pi]$

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ = an angle in $[0,\pi]$ whose cosine is $\left(-\frac{\sqrt{3}}{2}\right)$

$$=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

$$\therefore\quad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{5\pi}{6}$$

We know that, for any $x\in R$, $\tan^{-1}x$ represents an angle in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ whose tangent is x.

So,

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ whose tangest is $\frac{1}{\sqrt{3}}$

$$=\frac{\pi}{6}$$

$$\therefore\ \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}.$$

We know that, for $x \in R$, $sec^{-1}x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$sec^{-1}\left(-\sqrt{2}\right)$ = An angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is $\left(-\sqrt{2}\right)$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$sec^{-1}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$.

We know that, for any $x \in R$, $cot^{-1}x$ represents an angle in $(0, \pi)$

$cot^{-1}\left(-\sqrt{3}\right)$ = An angle in $(0, \pi)$ whose contangent is $\left(-\sqrt{3}\right)$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$\therefore cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$.

We know that, for any $x \in R$, $\sec^{-1}x$ represents an angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

$\sec^{-1}(2)$ = An angle is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is 2

$$= \frac{\pi}{3}$$

$\therefore \sec^{-1}(2) = \frac{\pi}{3}$.

We know that, for any $x \in R$. $\operatorname{cosec}^{-1}x$ is an angle in $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle is $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$$= \frac{\pi}{3}$$

$\therefore \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$.

Inverse Trigonometric Functions Ex 4.1 Q2.

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

$$\therefore \sin^{-1}\frac{1}{2}=\frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]=\tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$=\tan^{-1}1=\frac{\pi}{4}$$

Concept Insight:

Solve the innermost bracket first, so first find the principal value of $\sin^{-1}(1/2)$

Let $\tan^{-1}(1)=x$. Then, $\tan x=1=\tan\frac{\pi}{4}$.

$$\therefore \tan^{-1}(1)=\frac{\pi}{4}$$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$. Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$.

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z$. Then, $\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

$$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$

$$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$$

$\tan^{-1}\left(\sqrt{3}\right)$ = Angle in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ whose tangent is $\sqrt{3}$

$$= \frac{\pi}{3}$$

$\sec^{-1}(-2)$ = An angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is (-2)

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$cosec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$$= \frac{\pi}{3}$$

Hence,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + cosec^{-1}\frac{2}{\sqrt{3}}$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= 0$$

$$\therefore\ \tan^{-1}\sqrt{3} - \sec^{-1}\left(-\sqrt{2}\right) + cosec^{-1}\left(\frac{2}{\sqrt{3}}\right) = 0$$