RD Sharma Solutions Class 12 Maths Chapter 5 Ex 5.3

Algebra of Matrices Ex 5.3 Q1

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$$= \begin{bmatrix} (a)(a)+(b)(b) & (a)(-b)+(b)(a) \\ (-b)(a)+(a)(b) & (-b)(-b)+(a)(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} (1)(1)+(-2)(-3) & (1)(2)+(-2)(2) & (1)(3)+(-2)(-1) \\ (2)(1)+(3)(-3) & (2)(2)+(3)(2) & (2)(3)+(3)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} (2)(1)+(3)(0)+(4)(3) & (2)(-3)+(3)(2)+(4)(0) & (2)(5)+(3)(4)+(4)(5) \\ (3)(1)+(4)(0)+(5)(3) & (3)(-3)+(4)(2)+(5)(0) & (3)(5)+(4)(4)+(5)(5) \\ (4)(1)+(5)(0)+(6)(3) & (4)(-3)+(5)(2)+(6)(0) & (4)(5)+(5)(4)+(6)(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0` & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q2(i)

Given, $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q2(ii)

Given, $A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+01 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii), $AB \neq BC$

Algebra of Matrices Ex 5.3 Q2(iii)

Given, $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \quad ---(ii)$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q3(i)

$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Since order of A is 2×2 and order of B is 2×3,
So AB is possible but BA is not possible order of AB is 2×3.

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(2) & (1)(2)+(-2)(3) & (1)(3)+(-2)(1) \\ (2)(1)+(3)(2) & (2)(2)+(3)(3) & (2)(3)+(3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exits

Algebra of Matrices Ex 5.3 Q3(ii)

Here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Order of A = 3×2 and order of B = 2×3 So,

AB and BA Both exits and order of AB = 3×3 and order of BA = 2×2

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(4)+(2)(0) & (3)(5)+(2)(1) & (3)(6)+(2)(2) \\ (-1)(4)+(0)(0) & (-1)(5)+(0)(0) & (-1)(6)+(0)(2) \\ (-1)(4)+(1)(0) & (-1)(5)+(1)(1) & (-1)(6)+(1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+0 & -6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(3)+(5)(-1)+(6)(-1) & (4)(2)+(5)(0)+(6)(1) \\ (0)(3)+(1)(-1)+(2)(-1) & (0)(2)+(1)(0)+(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iii)

Here,

$$A=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}, B=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$

Order of A = 1×4 and order of B=4×1 So,

AB and BA both exist and order of AB = 1×1 and order of BA=4×4, So

$$AB=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$

$$=\begin{bmatrix}(1)(0)+(-1)(1)+(2)(3)+(3)(2)\end{bmatrix}$$

$$=\begin{bmatrix}0-1+6+6\end{bmatrix}$$

$$AB=\begin{bmatrix}11\end{bmatrix}$$

$$BA=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}$$

$$BA=\begin{bmatrix}(0)(1) & (0)(-1) & (0)(2) & (0)(3)\\(1)(1) & (1)(-1) & (1)(2) & (1)(3)\\(3)(1) & (3)(-1) & (3)(2) & (3)(3)\\(2)(1) & (2)(-1) & (3)(2) & (2)(3)\end{bmatrix}$$

$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Hence,

$$AB=\begin{bmatrix}11\end{bmatrix}$$

$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q3(iv)

$$\begin{bmatrix}a & b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+\begin{bmatrix}a & b & c & d\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

$$=\begin{bmatrix}ac+bd\end{bmatrix}+\begin{bmatrix}a^2+b^2+c^2+d^2\end{bmatrix}$$

$$=\begin{bmatrix}ac+bd=a^2+b^2+c^2+d^2\end{bmatrix}$$

Hence,

$$\begin{bmatrix}a & b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix}+\begin{bmatrix}a & b & c & d\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

$$=\begin{bmatrix}ac+bd+a^2+b^2+c^2+d^2\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q4(i)

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(i)$$

$$BA = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii),

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q4(ii)

$$A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \qquad ---(i)$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-22+9 & -4+10-5 & -9+0+1 \\ 30-44+10 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii)

$$AB \neq BA$$

Algebra of Matrices Ex 5.3 Q5(i)

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(ii)

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 0+0+3 & 2+0+6 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10+12+60 \end{bmatrix}$$

$$= \begin{bmatrix} 82 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 82 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q5(iii)

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}-\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

Hence,

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}-\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} & 0 & -1 & 1 \\ = & 2 & 0 & -2 \\ & 5 & -2 & -3 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q6

Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I_2 \qquad \text{---(i)}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = I_2 \qquad \text{---(ii)}$$

$$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^2 = I_2 \qquad \text{---(iii)}$$

Hence,

From equation (i), (ii) and (iii),

$$A^2 = B^2 = C^2 = I_2$$

Algebra of Matrices Ex 5.3 Q7

Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$$3A^2 - 2B + I$$

$$= 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0+1 & -12+8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Hence,

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q8

Given, $A = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$

$$(A - 2I)(A - 3I)$$

$$= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)\left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} 4-2 & 2-0 \\ -1-0 & 1-2 \end{bmatrix}\right)\left(\begin{bmatrix} 4-3 & 2-0 \\ -1-0 & 1-3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$(A - 2I)(A - 3I) = 0$$

Algebra of Matrices Ex 5.3 Q9

Given, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+1\\ 0+0 & 0+1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$$A^3=A^2\cdot A$$

$$=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+0 & 1+2\\ 0+0 & 0+1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

Hence,

$$A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q10

Given, $A=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$

$$A^2=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$$

$$=\begin{bmatrix}a^2b^2-a^2b^2 & ab^3-ab^3\\ -a^3b+a^3b & -a^2b^2+a^2b^2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$=0$$

Hence,

$$A^2=0$$

Algebra of Matrices Ex 5.3 Q11

Given, $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$A^2 = A.A$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin^2 + \cos 2\theta \sin^2\theta \\ -\cos 2\theta \sin^2\theta - \sin^2\theta \cos^2\theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta & 2\sin^2\theta \cos^2\theta \\ -2\sin^2 \cos 2\theta & \cos 4\theta \end{bmatrix}$

$\{\text{since } \cos^2\theta - \sin^2\theta = \cos 2\theta\}$

$= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$\{\text{since } \sin^2\theta = 2\sin\theta\cos\theta\}$

Hence,

$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q12

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -2-3+5 & 6+9-15 & 10+15+25 \\ 1+4-5 & -3-12+15 & -5-20+25 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$AB = O_{3\times 3}$ ---(i)

$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+12-15 & 5+15-20 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$BA = O_{3\times 3}$ ---(ii)

From equation (i) and (ii),

$AB = BA = O_{3\times 3}$

Algebra of Matrices Ex 5.3 Q13

Given, $A = \begin{bmatrix} 0 & c & -b \\ -c & o & a \\ b & -a & 0 \end{bmatrix}, B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+abc-abc & 0+b^2c-b^2c & 0+bc^2-bc^2 \\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3\times3} \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$AB = BA = O_{3\times3}$$

Algebra of Matrices Ex 5.3 Q14

Given, $A= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$AB = A$$

$$BA = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$BA = B$$

Algebra of Matrices Ex 5.3 Q15

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \quad ---(i)$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ---(ii)$$

Subtracting equation(ii) from equation(i),

$$A^2 - B^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q16(i)

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & +0+0+3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \qquad ---(i)$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+6 & -4+2-3 & 4+4+3 \\ 1+0+4 & -1+1-2 & 1+2+2 \\ 3+0+2 & -3+0-1 & 3+0+1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 8 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 10-15+0 & 20+0+0 & -10+5+11 \end{bmatrix}$$

$$= \begin{bmatrix} 5-6+0 & 10+0+0 & -5-2+5 \\ 5-12+0 & 10+0+0 & -5-4+4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{---(i)}$$

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1-3+0 & 2+0+0 & -1-1+1 \\ 0+3+0 & 0+0+0 & 0+1+2 \\ 2-3+0 & 4+0+0 & -2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & -1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8+6-3 & 8+0+12 & -4+6-6 \\ -2+3-2 & 2+0+8 & -1+3-4 \\ -6+0-1 & 6+0+4 & -3+0-2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{--(ii)}$$

From equation (i) and (ii),

$$(AB)C = A(BC)$$

Algebra of Matrices Ex 5.3 Q17(i)

Given, $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$AB = AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+-1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{---(ii)}$$

Using equation (i) and (ii),

$$A(B+C) = AB + AC$$

Algebra of Matrices Ex 5.3 Q17(ii)

Given, $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{---(i)}$$

$$AB + AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$A(B+C) = AB + AC$$

Algebra of Matrices Ex 5.3 Q18

Given,

$$A=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}, B=\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}$$

$$C=\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\left[\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}-\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}\right]$$

$$=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}0-1&5-5&-4-2\\-2+1&1-1&3-0\\-1-0&0+1&2-1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}-1&0&-6\\-1&0&3\\-1&1&1\end{bmatrix}$$

$$=\begin{bmatrix}-1+0+2&0+0-2&-6+0-2\\-3+1+0&0+0+0&-18-3+0\\2-1-1&0+0+1&12+3+1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}1&-2&-8\\-2&0&-21\\0&1&16\end{bmatrix} \quad ---(i)$$

$$AB-AC=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}-\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}$$

$$=\begin{bmatrix}0+0+2&5+0+0&-4+0-4\\0+2+0&15-1+0&-12-3+0\\0-2-1&-10+1+0&8+3+2\end{bmatrix}-\begin{bmatrix}1+0+0&5+0+2&2+0-2\\3+1+0&15-1+0&6+0+0\\0-2-1&-10+1+1&-4+0+1\end{bmatrix}$$

$$=\begin{bmatrix}2&5&-8\\2&14&-15\\-3&-9&13\end{bmatrix}-\begin{bmatrix}1&7&0\\4&14&6\\-3&-10&-3\end{bmatrix}$$

$$=\begin{bmatrix}2-1&5-7&-8-0\\2-4&14-14&-14-6\\-3+3&-9+10&13+3\end{bmatrix}$$

$$AB-AC=\begin{bmatrix}1&-2&-8\\-2&0&-21\\0&1&16\end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii),

$$A(B-C)=AB-AC$$

Algebra of Matrices Ex 5.3 Q19

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 & \\ 2 & 0 & 2 & \\ 0 & 3 & 2 & 0 \\ 4 & 0 & 4 & \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0-3+0 & 0+2+0 \\ 4+0+8 & -2+0+6 \\ 0-9+8 & 0+6+6 \\ 8+0+16 & -4+0+12 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0+6 & -3-6 & 3+8 & -6-8 & 6+0 \\ 0+12 & 12-12 & -12+16 & 24-16 & -24+0 \\ 0+36 & -1-36 & 1+48 & -2-48 & 2+0 \\ 0+24 & 24-24 & -24+34 & 48-32 & -48+0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

Here, $a_{43} = 8, a_{22} = 0$

Algebra of Matrices Ex 5.3 Q20

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \quad ---(i)$$

$$pI + qA + rA^2$$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix}$$

$$pI + qA + rA^2$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q21

Given, W is a complex cube root of unity,

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+w & w+w^2 & w^2+1 \\ w+w^2 & w^2+1 & 1+w \\ w^2+w & 1+w^2 & w+1 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w^2 & -1 & -w \\ -1 & -w & -w^2 \\ -1 & -w & -w^2 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \qquad \begin{Bmatrix} \textit{Since } 1+w+w^2 = 0 \\ \textit{and } w^3 = 1 \end{Bmatrix} \qquad \text{---(i)}$$

$$= \begin{bmatrix} -w^2 - w - w^3 \\ -1 - w^2 - w^4 \\ -1 - w^2 - w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w\left(1+w+w^2\right) \\ -1 - w^2 - w^3 w \\ -1 - w^2 - w^3 w \end{bmatrix}$$

$$= \begin{bmatrix} -w.0 \\ -1 - w^2 - w \\ -1 - w^2 - w \end{bmatrix} \qquad \{\text{using reason (i)}\}$$

$$= \begin{bmatrix} 0 \\ -\left(1+w+w^2\right) \\ -\left(1+w+w^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -(0) \\ -(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q22

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$A^2 = A, A$$
$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 4+3-5 & -6-12+15 & -10-15+20 \\ -2-4+5 & 3+16-15 & 5+20-20 \\ 2+3-4 & -3-12+12 & -5-15+16 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$
$$= A$$

Hence,
$$A^2 = A$$

Algebra of Matrices Ex 5.3 Q23

Given, $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$A^2 = A.A$$
$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= I_3$$

Hence,
$$A^2 = I_3$$

Algebra of Matrices Ex 5.3 Q24(i)

Given,

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 1+0+2x & 0+2+x & 2+1+0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+1 & 2+x & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+1+2+x+3 \end{bmatrix} = 0$$

$$\Rightarrow \quad 3x+6 = 0$$

$$\Rightarrow \quad x = -\frac{6}{3}$$

$$\Rightarrow \quad x = -2$$

Algebra of Matrices Ex 5.3 Q24(ii)

Given that $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

By multiplication of matrices, we have,

$$\begin{bmatrix} 2\times 1+3\times(-2) & 2\times(-3)+3\times 4 \\ 5\times 1+7\times(-2) & 5\times(-3)+7\times 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow x = 13$$

Algebra of Matrices Ex 5.3 Q25

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} (2x+4)x+4(x+2)-1(2x+4) \end{bmatrix} = 0$$

$$\Rightarrow \quad 2x^2+4x+4x+8-2x-4 = 0$$

$$\Rightarrow \quad 2x+6x+4 = 0$$

$$\Rightarrow \quad 2x^2+2x+4x+4 = 0$$

$$\Rightarrow \quad 2x(x+1)+4(x+1) = 0$$

$$\Rightarrow \quad (x+1)(2x+4) = 0$$

$$\Rightarrow \quad x+1 = 0 \text{ or } 2x+4 = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Algebra of Matrices Ex 5.3 Q26

Given,

$$\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\ 1\\ 1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}0-2+x & 1-1+x & -1-3+x\end{bmatrix}\begin{bmatrix}0\\ 1\\ 1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}x-2 & x & x-4\end{bmatrix}\begin{bmatrix}0\\ 1\\ 1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}0(x-2)+x.1+1.(x-4)\end{bmatrix} = 0$$

$$\Rightarrow 0+x+x-4 = 0$$

$$\Rightarrow 2x-4 = 0$$

$$\Rightarrow x = 2$$

Hence,

$$x = 2$$

Algebra of Matrices Ex 5.3 Q27

Given, $A = \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}, I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$A^2 - A + 2I$$

$$= \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} - \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} + 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix} - \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix} - \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} + \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}1-3+2 & -2+2+0\\ 4-4+0 & -4+2+2\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - A + 2I = 0$$

Algebra of Matrices Ex 5.3 Q28

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

And

$A^2 = 5A + \lambda I$

$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3+2 & -1+4 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 15+\lambda & 5 \\ -5 & 10+\lambda \end{bmatrix}$

Since, Corresponding entries of equal matrices are equal, So

$8 = 15 + \lambda$

$\lambda = 8 - 15$

$\lambda = -7$

Algebra of Matrices Ex 5.3 Q29

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$A^2 - 5A + 7I_2$

$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

Hence, $A^2 - 5A + 7I_2 = 0$

Algebra of Matrices Ex 5.3 Q30

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$A^2 - 2A + 3I_2$

$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

Hence,

$A^2 - 2A + 3I_2 = 0$

Algebra of Matrices Ex 5.3 Q31

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $A^3 - 4A^2 + A = 0$

Algebra of Matrices Ex 5.3 Q32

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$A^2 - 12A - I$$

$$= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Since $A^2 - 12A - I = 0$

So,

A is a root of the equation $A^2 - 12A - I = 0$

Algebra of Matrices Ex 5.3 Q33

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$A^2 = 5A - 14I$$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$A^2 - 5A - 14I = 0$$

Algebra of Matrices Ex 5.3 Q34

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $A^3 - 4A^2 + A = 0$

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O = \text{R.H.S.}$$

$\therefore A^2 - 5A + 7I = O$

Since $A^2 - 5A + 7I = O$, we have

$A^2 = 5A - 7I$

Therefore, $A^4 = A^2 \times A^2 = (5A - 7I)(5A - 7I)$

$\Rightarrow A^4 = 25A^2 - 35AI - 35IA + 49I$

$\Rightarrow A^4 = 25A^2 - 70A + 49I$

$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I$

$\Rightarrow A^4 = 125A - 175I - 70A + 49I$

$\Rightarrow A^4 = 55A - 126I$

$$\Rightarrow A^4 = 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 - 126 & 55 - 0 \\ -55 - 0 & 110 - 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q35

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k - 2 & -2k \\ 4k & -2k - 2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1.

Algebra of Matrices Ex 5.3 Q36

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$A^2 - 8A + kI = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

corresponding entries of equal matrices are equal, so

$$-7 + k = 0$$

$$k = 7$$

Algebra of Matrices Ex 5.3 Q37

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$$f(A) = A^2 - 2A - 3I$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$f(A) = 0$$

Algebra of Matrices Ex 5.3 Q38

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$2\lambda + \mu = 7$ --- (i)

$\lambda = 4$ --- (ii)

Put λ from equation (ii) in equation (i),

$2(4) + \mu = 7$

$\mu = 7 - 8$

$\mu = -1$

Hence. $\lambda = 4, \mu = -1$

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^3 - 4A^2 + A$

$$= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-7+2 & 45-12+3 \\ 15-4+1 & 26-7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A = \begin{bmatrix} 21 & 36 \\ 12 & 21 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q39

Given,

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$5x = 1$ and $10x - 2 = 0$

$\Rightarrow x = \frac{1}{5}$ and $x = \frac{1}{5}$

Hence, $x = \frac{1}{5}$

Algebra of Matrices Ex 5.3 Q40(i)

Here,

$$\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & 0-3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow \left[(x-2)x - 15\right] = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow x(x-5) + 3(x-5) = 0$$

$$\Rightarrow (x-5)(x+3) = 0$$

$$\Rightarrow x - 5 = 0 \quad \text{or} \quad x + 3 = 0$$

$$\Rightarrow x = 5 \quad \text{or} \quad x = -3$$

So,

$x = 5$ or -3

Algebra of Matrices Ex 5.3 Q40(ii)

We have:

$$[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix}=O$$

$$\Rightarrow [x+0-2 \quad 0-10+0 \quad 2x-5-3]\begin{bmatrix}x\\4\\1\end{bmatrix}=O$$

$$\Rightarrow [x-2 \quad -10 \quad 2x-8]\begin{bmatrix}x\\4\\1\end{bmatrix}=O$$

$$\Rightarrow \left[x(x-2)-40+2x-8\right]=O$$

$$\Rightarrow \left[x^2-2x-40+2x-8\right]=[0]$$

$$\Rightarrow \left[x^2-48\right]=[0]$$

$$\therefore x^2-48=0$$

$$\Rightarrow x^2=48$$

$$\Rightarrow x=\pm 4\sqrt{3}$$

Algebra of Matrices Ex 5.3 Q41

Given, $A=\begin{bmatrix}1 & 2 & 0\\3 & -4 & 5\\0 & -1 & 3\end{bmatrix}$

$$A^2-4A+3I_3$$

$$=\begin{bmatrix}1 & 2 & 0\\3 & -4 & 5\\0 & -1 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\3 & -4 & 5\\0 & -1 & 3\end{bmatrix}-4\begin{bmatrix}1 & 2 & 0\\3 & -4 & 5\\0 & -1 & 3\end{bmatrix}+3\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+6+0 & 2-8+0 & 0+10+0\\3-12+0 & 6+16-5 & 0-20+15\\0-3+0 & 0+4-3 & 0-5+9\end{bmatrix}-\begin{bmatrix}4 & 8 & 0\\12 & -16 & 20\\0 & -4 & 12\end{bmatrix}+\begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}7 & -6 & 10\\-9 & 17 & -5\\-3 & 1 & 4\end{bmatrix}-\begin{bmatrix}4 & 8 & 0\\12 & -16 & 20\\0 & -4 & 12\end{bmatrix}+\begin{bmatrix}3 & 0 & 0\\0 & 3 & 0\\0 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}7-4+3 & -6-8+0 & 10-0+0\\-9-12+0 & 17+16+3 & -5-20+0\\-3-0+0 & 1+4+0 & 4-12+3\end{bmatrix}$$

$$=\begin{bmatrix}6 & -14 & 10\\-21 & 36 & -25\\-3 & 5 & -5\end{bmatrix}$$

Hence,

$$A^2-4A+3I_3=\begin{bmatrix}6 & -14 & 10\\-21 & 36 & -25\\-3 & 5 & -5\end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q42

Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

And $f(x) = x^2 - 2x$

$\Rightarrow f(A) = A^2 - 2A$

$$\Rightarrow f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 0+4+0 & 0+5+4 & 0+0+6 \\ 0+20+0 & 4+25+0 & 8+0+0 \\ 0+8+0 & 0+10+6 & 0+0+9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 4-0 & 9-2 & 6-4 \\ 20-8 & 29-10 & 8-0 \\ 8-0 & 16-4 & 9-6 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q43

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

And $\quad f(x) = x^3 + 4x^2 - x$

$\Rightarrow \quad f(x) = A^3 + 4A^2 - A \qquad$ ---(i)

$A^2 = A \times A$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$A^3 = A^2 \times A$

$$= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$f(A) = A^3 + 4A^2 - A$

$$= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4\begin{bmatrix} 4 & -15 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q44

Given that, $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ and $f(x)=x^3-6x^2+7x+2$

Therefore, $f(A)=A^3-6A^2+7A+2I_3$

First find A^2:

$$A^2=A\times A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

Now, Let us find A^3:

$$A^3=A^2\times A=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

Thus,

$f(A)=A^3-6A^2+7A+2I_3$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$=\begin{bmatrix}21-30+7+2&0&34-48+14+0\\12-12+0&8-24+14+2&23-30+7+0\\34-48+14+0&0&55-78+21+2\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, A is a root of the polynomial.

Algebra of Matrices Ex 5.3 Q45

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$A^2 - 4A - 5I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 4A - 5I = 0$$

Algebra of Matrices Ex 5.3 Q46

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \quad \text{---(i)}$$

$$-2x + 3z = 7 \quad \text{---(ii)}$$

$$5y - 7u = -6 \quad \text{---(iii)}$$

$$-2y + 3u = 2 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$

$$\underline{-10x + 15z = 35}$$

$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$

$$\Rightarrow \quad 5x = 16 + 21$$

$$\Rightarrow \quad 5x = 5$$

$$\Rightarrow \quad x = 1$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$

$$\underline{-10y + 15u = 10}$$

$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$

$$\Rightarrow \quad 5y - 7(-2) = -6$$

$$\Rightarrow \quad 5y + 14 = -6$$

$$\Rightarrow \quad 5y = -20$$

$$\Rightarrow \quad y = -4$$

Algebra of Matrices Ex 5.3 Q48

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2\times 2}$ $\quad A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$\Rightarrow$ A is a matrix of order 2×3

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+d & b+e & c+c \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$d = 1,\ e = 0,\ f = 1$

And $a + d = 3$

$a + 1 = 3$

$a = 3 - 1$

$a = 2$

$b + e = 3$

$b + 0 = 3$

$b = 3$

And $c + f = 5$

$c + 1 = 5$

$c = 4$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q48(ii)

It is given that:

$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7, \quad 2a+5c=-8, \quad 3a+6c=-9$

$b+4d=2, \quad 2b+5d=4, \quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1,\ b=2,\ c=-2,\ d=0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

Therefore,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1\times3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3},$$

$$\Rightarrow \begin{bmatrix} 4\times(x_1) & 4\times(x_2) & 4\times(x_3) \\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3) \\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3) \end{bmatrix}_{3\times3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

$$\Rightarrow \begin{bmatrix} 4x_1 & 4x_2 & 4x_3 \\ x_1 & x_2 & x_3 \\ 3x_1 & 3x_2 & 3x_3 \end{bmatrix}_{3\times3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times3}$$

$\Rightarrow \quad 4x_1 = -4 \;,\; 4x_2 = 8 \,,\; 4x_3 = 4$

Solving $\quad x_1 = -1 \,,\; x_2 = 2 \,,\; x_3 = 1$

So, matrix $A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1.A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$$= \begin{bmatrix} (2\times-1)+(1\times-1)+(3\times0) & (2\times0)+(1\times1)+(3\times1) & (2\times-1)+(1\times0)+(3\times1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$$

and $(A_1.A_2)A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$= \begin{bmatrix} (-3\times1)+(4\times0)+(1\times-1) \end{bmatrix}$$

$(A_1.A_2)A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating (A_2A_3) first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Algebra of Matrices Ex 5.3 Q49

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$a + b = 6$ ---(i)

$-2a + 4b = 0$ ---(ii)

$c + d = 0$ ---(iii)

$-2c + 4d = 6$ ---(iv)

Solving equation (i) and (ii)

$$4a + 4b = 24$$
$$-2a + 4b = 0$$
$$(+) \quad (-)$$
$$6a = 24$$

$$\Rightarrow a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$$a + b = 6$$
$$4 + b = 6$$
$$b = 6 - 4$$
$$b = 2$$

Solving equation (iii) and (iv)

$$2c + 2d = 0$$
$$-2c + 4d = 6$$
$$6d = 6$$
$$d = \frac{6}{6}$$
$$d = 1$$

Put $d = 1$ in equation (iii)

$$c + d = 0$$
$$c = -1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q50

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O$$

$$A^4 = A^2 \times A^2$$

$$= O \times O$$

$$= O$$

$$A^{16} = A^4 \times A^4$$

$$= O \times O$$

$$= O$$

So,

A^{16} is a null matrix

Algebra of Matrices Ex 5.3 Q51

Solving the LHS of the given equation we have ,

$$\Rightarrow \quad A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} (0 \times 0) + ((-x + 1) \times (x + 1)) & (0 \times (-x + 1)) + ((-x + 1) \times 0) \\ ((x + 1) \times 0) + (0 \times (x + 1)) & ((x + 1) \times (-x + 1)) + (0 \times 0) \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}.$$

Solving the RHS we get,

$$\Rightarrow \quad A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0 \times 0) + ((-x) \times (x)) & (0 \times (-x)) + ((-x) \times 0) \\ ((x) \times 0) + (0 \times (x)) & ((x) \times (-x)) + (0 \times 0) \end{bmatrix} + \begin{bmatrix} (0 \times 0) + (1 \times 1) & (0 \times 1) + (1 \times 0) \\ (1 \times 0) + (0 \times 1) & (1 \times 1) + (0 \times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}$$

Subsituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad (A + B)^2 = A^2 + B^2.$$

Algebra of Matrices Ex 5.3 Q52

Solving the LHS i.e.

$$A^2 + A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Solving the RHS i.e.

$$A(A+I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

So, LHS = RHS verified.

Algebra of Matrices Ex 5.3 Q53

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now, $$A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3\times 3)+(-5\times -4) & (3\times -5)+(-5\times 2) \\ (-4\times 3)+(2\times -4) & (-4\times -5)+(2\times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \text{ and } -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\therefore \quad A^2-5A-14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 + 0 \\ -20 + 20 + 0 & 24 - 10 + - 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A^2-5A-14I = 0$$

$$\Rightarrow \quad A^2 = 5A + 14I$$

$$\Rightarrow \quad A^3 = A^2.A = (5A + 14I)A$$

$$\Rightarrow \quad A^3 = A^2.A = 5A^2 + 14A \quad \left[\text{By using dist. of matrices over matrix addition}\right]$$

$$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q54

We have,

$$P(x).\ P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$\Rightarrow P(x).\ P(y) = \begin{bmatrix} \cos x\cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$\Rightarrow P(x).\ P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

Now,

$$P(y).\ P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow P(y).\ P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$$

$$\Rightarrow P(y).\ P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

$$\therefore\ P(x).\ P(y) = P(x+y) = P(y).\ P(x)$$

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix},\ Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

So, $$PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix}$$

$$= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

and $$QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix}$$

$$= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$$\therefore\ PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times-1 & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+0\times1+1\times-1 & 2\times1+1\times3+3\times0 \\ 1\times2+-1\times2+0\times1 & 1\times0+-1\times1+0\times-1 & 1\times1+-1\times3+0\times0 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence, $A^2\text{-}5A+4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$A^2\text{-}5A+4I = \begin{bmatrix} 5\text{-}10+4 & -1+0+0 & 5\text{-}5+0 \\ 9\text{-}10+0 & -2\text{-}5+4 & 5\text{-}15+0 \\ 0\text{-}5\text{-}0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is $A^2\text{-}5A+4I+X=O$

$$\Rightarrow X = -(A^2\text{-}5A+4I)$$

$$X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \quad \text{---(i)}$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that A^n is true for $n = k+1$ whenever it is true for $n = k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q52

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \frac{b(a^1 - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \frac{b(ak - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \quad \text{---(i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} a^k & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} a^{k+1} + 0 & a^k b + \frac{b(a^k - 1)}{a - 1} \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & \frac{a^{k+1}b - a^k b + a^k b - b}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q57 pending

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1} - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q58

Given,

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

To show that,

$$A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in N.$$

Put $n = 1$

$$A^1 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

So,

A^n is true for $n = 1$

Let, A^n is true for $n = k$, so

$$A^k = \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix} \qquad ---(i)$$

Now, we have to show that,

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now, $A^{k+1} = A^k \times A$

$$= \begin{bmatrix} \cos k\theta & i\sin k\theta \\ i\sin k\theta & \cos k\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta + i^2\sin k\theta\sin\theta & i^2\cos k\theta\sin\theta + i\sin k\theta\cos\theta \\ i\sin k\theta\cos\theta + i\cos k\theta\sin\theta & i^2\sin k\theta\sin\theta + \cos\theta\cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta\cos\theta - \sin k\theta\sin\theta & i(\cos k\theta\sin\theta + \sin k\theta\cos\theta) \\ i(\sin k\theta\cos k\theta\sin\theta) & \cos k\theta\cos\theta - \sin k\theta\sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & i\sin(k+1)\theta \\ i\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

So, A^n is true for $n = k+1$ whenever it is true for $n = k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q59

Given,

$$A = \begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

To prove P(n): $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2}\sin n\alpha \\ -\sqrt{2}\sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show P(1) is true.

A^n is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2}\sin k\alpha \\ -\sqrt{2}\sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha + \sin\alpha & \sqrt{2}\sin\alpha \\ -\sqrt{2}\sin\alpha & \cos\alpha - \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos\alpha + \sin\alpha) - 2\sin\alpha\sin k\alpha & (\cos k\alpha + \sin k\alpha)\sqrt{2}\sin\alpha + \sqrt{2}\sin k\alpha(\cos\alpha - \sin\alpha) \\ (\cos\alpha + \sin\alpha)\left(-\sqrt{2}\sin k\alpha\right) - \sqrt{2}\sin\alpha(\cos k\alpha - \sin k\alpha) & -2\sin k\alpha\sin\alpha + (\cos k\alpha - \sin k\alpha)(\cos\alpha - \sin\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\alpha\cos\alpha + \sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha + \sin\alpha\sin k\alpha - 2\sin\alpha\sin k\alpha & \sqrt{2}\cos k\alpha\sin\alpha + \sqrt{2}\sin\alpha\sin k\alpha + \sqrt{2}\sin k\alpha\cos\alpha - \sqrt{2}\sin k\alpha\sin\alpha \\ -\sqrt{2}\cos\alpha\sin\alpha - \sqrt{2}\sin\alpha\sin k\alpha - \sqrt{2}\sin\alpha\cos k\alpha + \sqrt{2}\sin\alpha\sin k\alpha & -2\sin k\alpha\sin\alpha + \cos k\alpha\cos\alpha - \cos\alpha\sin k\alpha - \sin\alpha\cos k\alpha\sin\alpha\sin k\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos k\alpha + \sin\alpha\sin k\alpha \ \sin\alpha\cos k\alpha + \sin k\alpha\cos\alpha & \sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) \\ -\sqrt{2}(\sin k\alpha\cos\alpha + \cos k\alpha\sin\alpha) & \cos k\alpha\cos\alpha - \sin k\alpha\sin\alpha - (\sin k\alpha\cos\alpha + \sin\alpha\cos k\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha \\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Algebra of Matrices Ex 5.3 Q60

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k + 1$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n.

Algebra of Matrices Ex 5.3 Q61

We will prove P(n): $A^{n+1} = B^n\left[B + (n+1)C\right]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C,\ BC = CB,\ C^2 = 0$$
$$A = B + C$$

Squaring both the sides, so

$$A^2 = (B + C)^2$$
$$\Rightarrow \quad A^2 = (B + C)(B + C)$$
$$\Rightarrow \quad A^2 = B \times B + BC + CB + C \times C \qquad \{\text{using distributive property}\}$$
$$\Rightarrow \quad A^2 = B^2 + BC + BC + C^2 \qquad \{\text{using } BC = CB \text{ given}\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC + 0 \qquad \{\text{since, given } C^2 = 0\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC \qquad ---(1)$$
$$A^2 = B(B + 2C)$$

Now, consider

$$P(n):\ A^{n+1} = B^n\left[B + (n+1)C\right]$$

Step 1: Tp prove P(1) is true, put $n = 1$

$$A^{1+1} = B^1\left[B + (1+1)C\right]$$
$$A^2 = B\left[B + 2C\right]$$
$$A^2 = B^2 + 2BC$$

From equation (i), P(1) is true.

Step 2: Suppose P(k) is true.

$$\therefore\ A^{k+1} = B^k\left[B + (k+1)C\right] \qquad ---(2)$$

Step 3: Now, we have to show that P(k+1) is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1}\left[B + (k+2)C\right]$$

Now,

$$A^{k+2} = A^k \times A^2$$

$$= B^{(k-1)}\left[B + kC\right] \times \left[B\left(B + 2C\right)\right]$$

$$= B^k\left[B + kC\right] \times \left[B + 2C\right]$$

$$= B^k\left[B \times B + B \times 2C + kC \times B + 2kC^2\right]$$

$$= B^k\left[B^2 + 2BC + kBC + 2k \times 0\right] \qquad \left\{\text{since } BC = CB,\ C^2 = 0\right\}$$

$$= B^k\left[B^2 + BC\left(2 + k\right)\right]$$

$$= B^k \times B\left[B + (k+2)C\right]$$

$$= B^{k+1}\left[B + (k+2)C\right]$$

So, $P(n)$ is true for $n = k+1$ whenever $P(n)$ is true for $n = k$

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Algebra of Matrices Ex 5.3 Q62

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}\left(a^n, b^n, c^n\right)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}\left(a^1, b^1, c^1\right)$$

$$A = \text{diag}(a, b, c)$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}\left(a^k, b^k, c^k\right) \quad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

Now,

$$A^{k+1} = A^k \times k3$$

$$= \text{diag}\left(a^k, b^k, c^k\right) \times \text{diag}(a, b, c) \quad \{\text{using equation (i) and given}\}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q64

Given,

order of matrix $X = (a+b)\times(a+2)$

order of matrix $Y = (b+1)\times(a+3)$

Given, $X_{(a+b)\times(a+2)} . Y_{(b+1)\times(a+3)}$ exist.

$\Rightarrow \quad a+2 = b+1$

$\Rightarrow \quad a-b = -1 \qquad$ ---(i)

And

$Y_{(b+1)\times(a+3)} . X_{(a+b)\times(a+2)}$ exists.

$\Rightarrow \quad a+3 = a+b$

$\Rightarrow \quad b = 3$

Put $b = 3$ in equation (i),

$a-b = -1$

$a-3 = -1$

$a = 3-1$

$a = 2$

So, $\quad a = 2, b = 3$

So,

Order of $X = (a+b)\times(a+2)$

$= (2+3)\times(2+2)$

$= 5\times4$

Order of $Y = (b+1)\times(a+3)$

$= (3+1)\times(2+3)$

$= 4\times5$

Order of $X_{5\times4} . Y_{4\times5} = 5\times5$

Order of $X_{4\times5} . Y_{5\times4} = 4\times4$

So, order of XY and YX are not same and they are not equal but both are square matrices.

Algebra of Matrices Ex 5.3 Q65(i)

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation (i) and (ii)

$$AB \neq BA$$

when $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q65(ii)

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$

$$AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence,

$$AB = 0$$

When,

$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$

Algebra of Matrices Ex 5.3 Q65(iii)

Let, $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = 0$$

$$BA = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix}$$

$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q65(iv)

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$$A \neq 0, B \neq C$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LHS = RHS

So,

for $A \neq 0$, $BC \neq 0$ but $AB = AC$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matices of same order

$(A+B)^2 = (A+B)(A+B)$

$= A(A+B)+B(A+B)$ {using distributive property}

$= A \times A + AB + BA + B^2$

$= A^2 + AB + BA + B^2$

But,

$(A+B)^2 = A^2 + 2AB + B^2$ is possible only when $AB = BA$

Here, we can not say that $AB = BA$

So,

$(A+B)^2 = A^2 + 2AB + B^2$ does not hold.

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

(i) $(A+B)^2 = (A+B)(A+B)$

$= A(A+B)+B(A+B)$ {using distributive property}

$= A \times A + AB + BA + B \times B$

$= A^2 + AB + BA + B^2$

$\neq A^2 + 2AB + B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$

So, $(A+B)^2 \neq A^2 + 2AB + B^2$

(ii) $(A-B)^2 = (A-B)(A-B)$

$= A(A-B) - B(A-B)$ {using distributive property}.

$= A \times A - AB - BA + B \times B$

$= A^2 - AB - BA + B^2$

$\neq A^2 - 2AB + B^2$

Since, in general matrix multiplication is not commutative $(AB \neq BA)$, so

So, $(A-B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A+B)(A-B) = A(A-B) + B(A-B)$ {using distributive property}

$= A \times A - AB + BA - B \times B$

$= A^2 - AB + BA - B^2$

$= A^2 - B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$,

So, $(A+B)(A-B) \neq A^2 - B^2$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that AB = BA else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Here $AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times 0+0\times 2+0\times 0 & 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 0+1\times 2+0\times 0 & 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $BA = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$AB \neq BA$

Now, $(AB)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0\times 0+1\times 1+0\times 0 & 0\times 1+1\times 2+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 1\times 0+2\times 1+0\times 0 & 1\times 1+1\times 2+0\times 0 & 1\times 0+2\times 0+0\times 1 \\ 0\times 0+0\times 1+1\times 0 & 0\times 1+0\times 2+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times 0+1\times 2+0\times 0 & 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 0+1\times 2+0\times 0 & 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 1+0\times 1+0\times 0 & 1\times 1+0\times 2+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 1+1\times 2+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 1+0\times 2+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

Algebra of Matrices Ex 5.3 Q69

Given,

A and B two square matrices of same order such that $AB = BA$.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$(A+B)^2 = (A+B)(A+B)$$

$$= A(A+B) + B(A+B) \quad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + AB + BA + B^2 \quad \text{[by dist. of matrix multiplication over addition]}$$

$$= A^2 + 2AB + B^2 \quad \text{[As, } AB = BA\text{]}$$

$$= \text{RHS}$$

Hence proved.

Algebra of Matrices Ex 5.3 Q70

Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \qquad ---(i)$$

$$AC = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii)

$$AB = AC$$

Algebra of Matrices Ex 5.3 Q71

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{matrix} & \text{Notebook} & \text{Pens} & \text{Pencils} \\ A & 144 & 60 & 72 \\ B & 120 & 72 & 84 \\ C & 132 & 156 & 96 \end{matrix}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{matrix} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{matrix}$$

Individual bill can be calculated by

$$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix}$$

$$XY = \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix}$$

$$XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

So,

Bill of A = Rs 157.80

Bill of B = Rs 167.40

Bill of C = Rs 281.40

Algebra of Matrices Ex 5.3 Q72

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{bmatrix} \overset{\text{Physics}}{120} & \overset{\text{Chemistry}}{96} & \overset{\text{Mathematics}}{60} \end{bmatrix}$$

Matrix representation of sellin price (Rs.) of each book is given by

$$Y = \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \begin{matrix} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{matrix}$$

So, totaol amount recieved by the store from sellin all the items is given by,

$$XY = \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix}$$

$$= \begin{bmatrix} (120)(8.30) + (96)(3.45) + (60)(4.50) \end{bmatrix}$$

$$= \begin{bmatrix} 996 + 331.20 + 270 \end{bmatrix}$$

$$= \begin{bmatrix} 1597.20 \end{bmatrix}$$

Required amount = Rs 1597.20

Algebra of Matrices Ex 5.3 Q73

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and y is given by.

$$B = \begin{bmatrix} \overset{\text{Telephone}}{1000} & \overset{\text{Housecall}}{500} & \overset{\text{Letter}}{5000} \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Total amount spent by the group in the two cities X and y can be given by

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix}$$

$$= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Algebra of Matrices Ex 5.3 Q74

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$
$$\Rightarrow 5x + 210000 - 7x = 180000$$
$$\Rightarrow 210000 - 2x = 180000$$
$$\Rightarrow 2x = 210000 - 180000$$
$$\Rightarrow 2x = 30000$$
$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

Algebra of Matrices Ex 5.3 Q75

The cost for each mode per attempt is represented by 3 x 1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3 x 3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the prganization for the three villages seperately is given by matrix multiplication

$$BA = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

$$BA = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 30{,}000 \\ 23{,}000 \\ 39{,}000 \end{bmatrix}$$

Note: The answer given in the book is incorrect.

Algebra of Matrices Ex 5.3 Q76

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Men} \\ \text{Women} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Portein} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R thus ,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Protein} \end{array} \\ \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix} \end{array}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Algebra of Matrices Ex 5.3 Q77

The cost per contact (in paisa) is given in matrix A as

$$A=\begin{bmatrix}140\\200\\150\end{bmatrix}\begin{matrix}\text{Telephone}\\\text{House calls}\\\text{Letters}\end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{matrix}\\\text{City X}\\\text{City Y}\end{matrix}\begin{matrix}\begin{matrix}\text{Telephone} & \text{House calls} & \text{Letters}\end{matrix}\\\begin{bmatrix}1000 & 500 & 5000\\3000 & 1000 & 10000\end{bmatrix}\end{matrix}$$

The total amout of money spent by party in each of the city for the election is given by the matrix multiplication :

$$BA = \begin{bmatrix}1000 & 500 & 5000\\3000 & 1000 & 10000\end{bmatrix}\begin{bmatrix}140\\200\\150\end{bmatrix}$$

$$=\begin{bmatrix}1000\times 140+500\times 200+5000\times 150\\3000\times 140+1000\times 200+10000\times 150\end{bmatrix}$$

$$=\begin{matrix}\text{City X}\\\text{City Y}\end{matrix}\begin{bmatrix}990000\\2120000\end{bmatrix}$$

The total amout of money spent by party in each of the city for the election in rupees is given by

$$=\left(\frac{1}{100}\right)\begin{matrix}\text{City X}\\\text{City Y}\end{matrix}\begin{bmatrix}990000\\2120000\end{bmatrix}$$

$$=\begin{matrix}\text{City X}\\\text{City Y}\end{matrix}\begin{bmatrix}9900\\21200\end{bmatrix}$$

One sould consider social activities before casting his/her vote to the party.