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Solutions
Class 12 Maths
Chapter 6
Ex 6.1

Determinants Ex 6.1 Q1(i)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$$M_{11} = -1$$

[In a 2×2 matrix, the minor is obtained for a particular element, by deleting that row and column where the element is present.]

$$M_{21} = 20$$

$$C_{11} = (-1)^{1+1} \times M_{11}$$

$$[\because C_{ij} = (-1)^{i+j} \times M_{ij}]$$

$$= (+1)(-1)$$

$$= -1$$

$$C_{21} = (-1)^{2+1} M_{21}$$

$$= (-1)^3 \times 20$$

$$= -20$$

Also,

$$|A| = 5(-1) - (0) \times (20)$$

$$\left[\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ then } |A| = a_{11}a_{22} - a_{21}a_{12} \right]$$

$$= -5$$

Determinants Ex 6.1 Q1(ii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is present at the i^{th} row and j^{th} column.

Now,

$$M_{11} = 3$$

[In a 2×2 matrix, the minor of an element is obtained by deleting that row and that column in which it is present.]

$$M_{21} = 4$$

$$C_{11} = (-1)^{1+1} \times M_{11}$$

$$[C_{ij} = (-1)^{i+j} \times M_{ij}]$$

$$C_{21} = (-1)^{2+1} \times M_{21}$$

$$= (-1)^3 \times 4$$

$$= -4$$

Also,

$$|A| = (-1) \times (3) - (2) \times (4)$$

$$= -3 - 8$$

$$= -11$$

Determinants Ex 6.1 Q1(iii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$$M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix} \quad \left[\text{In a } 3 \times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2 \times 2 \right. \\ \left. \text{sub-matrix obtained by leaving the } i^{\text{th}} \text{ row and } j^{\text{th}} \text{ column of } A. \right]$$
$$= (-1) \times (2) - (5) \times (2)$$
$$= -2 - 10$$
$$= -12$$

$$M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix} = (-3) \times (2) - (5) \times (2) = -6 - 10 = -16$$

$$M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} = (-3)(2) - (-1)(2) = -6 + 2 = -4$$

$$C_{11} = (-1)^{1+1} M_{11} \quad \left(C_{ij} = (-1)^{i+j} \times M_{ij} \right)$$
$$= (+)(-12) = -12$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-16) = 16$$

$$C_{31} = (-1)^{3+1} M_{31} = (-1)^4 (-4) = -4$$

Also, expanding the determinant along the first column.

$$|A| = a_{11} \times \left((-1)^{1+1} \times M_{11} \right) + a_{21} \times \left((-1)^{2+1} \times M_{21} \right) + a_{31} \times \left((-1)^{3+1} \times M_{31} \right)$$
$$= a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$$
$$= 1 \times (-12) + 4 \times 16 + 3 \times (-4)$$
$$= -12 + 48 - 12 = 24$$

Determinants Ex 6.1 Q1(iv)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij} .

Now,

$$\begin{aligned}M_{11} &= \begin{vmatrix} b & ca \\ c & ab \end{vmatrix} \\ &= ab^2 - ac^2\end{aligned}$$

$$\begin{aligned}M_{21} &= \begin{vmatrix} a & bc \\ c & ab \end{vmatrix} \\ &= a^2b - c^2b\end{aligned}$$

$$\begin{aligned}M_{31} &= \begin{vmatrix} a & bc \\ b & ca \end{vmatrix} \\ &= a^2c - b^2c\end{aligned}$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +(ab^2 - ac^2)$$

$$C_{21} = (-1)^{2+1} \times M_{21} = -(a^2b - c^2b)$$

$$C_{31} = (-1)^{3+1} \times M_{31} = +(a^2c - b^2c)$$

Also, expanding the determinant, along the first column.

$$\begin{aligned}|A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 1(ab^2 - ac^2) + 1(c^2b - a^2b) + 1 \times (a^2c - b^2c) \\ &= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c\end{aligned}$$

Determinants Ex 6.1 Q1(v)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij} .

Now,

$$M_{11} = \begin{vmatrix} 5 & 0 \\ 7 & 1 \end{vmatrix} = 5 - 0 = 5$$

$$M_{21} = \begin{vmatrix} 2 & 6 \\ 7 & 1 \end{vmatrix} = 2 - 42 = -40$$

$$M_{31} = \begin{vmatrix} 2 & 6 \\ 5 & 0 \end{vmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$

$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$

$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 0 \times 5 + 1 \times (40) + 3 \times (-30) \\ &= 40 - 90 \\ &= -50 \end{aligned}$$

Determinants Ex 6.1 Q1(vi)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij} .

Now,

$$M_{11} = \begin{vmatrix} b & f \\ f & c \end{vmatrix} = bc - f^2$$

$$M_{21} = \begin{vmatrix} h & g \\ f & c \end{vmatrix} = hc - gf$$

$$M_{31} = \begin{vmatrix} h & g \\ b & f \end{vmatrix} = hf - bg$$

$$\text{Also } C_{11} = (-1)^{1+1} M_{11} = bc - f^2$$

$$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$$

$$C_{31} = (-1)^{3+1} M_{31} = hf - bg$$

Also, expanding along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= a(bc - f^2) + h(-)(hc - gf) + g(hf - bg) \\ &= abc - af^2 + hgf - h^2c + ghf - bg^2 \end{aligned}$$

Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

$$\text{Here, } M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0+10) - 1(1-2) = -9$$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix} = 0$$

$$\therefore C_{11} = (-1)^{1+1} M_{11} = -9$$

$$C_{21} = (-1)^3 M_{21} = -9$$

$$C_{31} = (-1)^4 M_{31} = -9$$

$$C_{41} = (-1)^5 M_{41} = 0$$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3)C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2 - 3 + 1] = 0$$

Determinants Ex 6.1 Q2(i)

$$\text{Let } A = \begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$$

$$\begin{aligned} |A| &= x(5x+1) + 7 \times x \\ &= 5x^2 + x + 7x \\ &= 5x^2 + 8x \end{aligned}$$

$$\text{Hence } |A| = 5x^2 + 8x$$

Determinants Ex 6.1 Q2(ii)

$$\text{Let } A = \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$$

$$\begin{aligned} |A| &= \cos \theta \times \cos \theta + \sin \theta \times \sin \theta \\ &= \cos^2 \theta + \sin^2 \theta \\ &= 1 \end{aligned}$$

$$\text{Hence } |A| = 1$$

Determinants Ex 6.1 Q2(iii)

$$\text{Let } A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$$

$$\begin{aligned} |A| &= \cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ \\ &= \cos (75 + 15) && (\because \cos A \cos B - \sin A \sin B = \cos (A + B)) \\ &= \cos 90^\circ \\ &= 0 \end{aligned}$$

$$\text{Hence } |A| = 0$$

Determinants Ex 6.1 Q2(iv)

$$\text{Let } A = \begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$$

$$\begin{aligned} |A| &= (a + ib)(a - ib) - (c + id)(-c + id) \\ &= (a^2 + b^2) + (c + id)(c - id) && (\text{Taking } (-) \text{ sign common from } -c + id) \\ & && (\text{Also } (a + ib)(a - ib) = a^2 + b^2) \\ &= a^2 + b^2 + c^2 + d^2 \end{aligned}$$

$$\text{Hence } |A| = a^2 + b^2 + c^2 + d^2$$

Determinants Ex 6.1 Q3

$$\text{Since } |AB| = |A| \times |B|$$

$$\text{Hence } |A|^2 = |A| \times |A| \quad \text{--- (1)}$$

$$\text{Now let } A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Expanding along the first column, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3 \begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7 \begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix} \\ &= 2(204 - 100) - 3(156 - 75) + 7(260 - 255) \\ &= 2(104) - 3(81) + 7(5) \\ &= 208 - 243 + 35 \\ &= 243 - 243 \\ &= 0 \end{aligned}$$

Hence from eq. (1)

$$|A|^2 = |A| \times |A| = 0 \times 0 = 0$$

Determinants Ex 6.1 Q4

Evaluating the given determinant

$$\sin 10^\circ \times \cos 80^\circ + \cos 10^\circ \sin 80^\circ$$

$$= \sin (10^\circ + 80^\circ) \quad [\because \sin A \cos B + \cos A \sin B = \sin (A + B)]$$

$$= \sin 90^\circ$$

$$= 1$$

Hence proved

Determinants Ex 6.1 Q5

We will evaluate the given determinant

(i) Along the first row

(ii) Along the first column

(i) Along the first row

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3 \begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5 \begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix} \\ &= 2(1+8) - 3(7-6) - 5(28+3) \\ &= 2(9) - 3(1) - 5(31) \\ &= 18 - 3 - 155 = -140 \end{aligned}$$

(ii) Along the first column

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 7 \begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} - 3 \begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} \\ &= 2(1+8) - 7(3+20) - 3(-6+5) \\ &= 18 - 7(23) - 3(-1) \\ &= 18 - 161 + 3 \\ &= 21 - 161 \\ &= -140 \end{aligned}$$

We can see, the answer is same with both the methods.

Determinants Ex 6.1 Q6

$$\begin{aligned} \Delta &= \begin{vmatrix} 0 & \sin \alpha & -\cos \alpha \\ -\sin \alpha & 0 & \sin \beta \\ \cos \alpha & -\sin \beta & 0 \end{vmatrix} \\ &= -\sin \alpha (-\sin \beta \cos \alpha) - \cos \alpha (\sin \alpha \sin \beta) \\ &= 0 \end{aligned}$$

Determinants Ex 6.1 Q7

$$\Delta = \begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Expanding along C_3 , we have:

$$\begin{aligned} \Delta &= -\sin \alpha (-\sin \alpha \sin^2 \beta - \cos^2 \beta \sin \alpha) + \cos \alpha (\cos \alpha \cos^2 \beta + \cos \alpha \sin^2 \beta) \\ &= \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta) + \cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) \\ &= \sin^2 \alpha (1) + \cos^2 \alpha (1) \\ &= 1 \end{aligned}$$

Determinants Ex 6.1 Q8

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow |A| = 2 - 10 = -8$$

$$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow |B| = 20 + 6 = 26$$

$$\begin{aligned} \text{Now } AB &= \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 4 + 5 \times 2 & 2 \times (-3) + 5 \times 5 \\ 2 \times 4 + 1 \times 2 & 2 \times (-3) + 1 \times 5 \end{bmatrix} \\ &= \begin{bmatrix} 8 + 10 & -6 + 25 \\ 8 + 2 & -6 + 5 \end{bmatrix} \\ &= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \Rightarrow |AB| &= 18 \times (-1) - (10)(19) \\ &= -18 - 190 = -208 \end{aligned}$$

$$\begin{aligned} \text{Now } |AB| &= |A| \times |B| \\ -208 &= (-8) \times (26) \\ -208 &= -208 \end{aligned}$$

Hence verified.

Determinants Ex 6.1 Q9

$$\text{Let } A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$$

Evaluating the determinant along the first column

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \\ &= 1 \times (4 - 0) - 0 + 0 \\ &= 4 \end{aligned}$$

$$\text{Again } 3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix} \quad (\text{every element of } A \text{ will be multiplied by } 3)$$

Now, evaluating this determinant

$$\begin{aligned} |3A| &= 3 \begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix} \\ &= 3(36 - 0) - 0 + 0 \\ &= 108 \end{aligned}$$

Now, according to the question

$$\begin{aligned} |3A| &= 27|A| \\ 108 &= 27(4) && (\text{Substituting values}) \\ 108 &= 108 \end{aligned}$$

Hence proved

Determinants Ex 6.1 Q10

$$(i) \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$

$$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow 2x^2 = 6$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

$$(ii) \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

$$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

$$\Rightarrow x = 2$$

(iii)

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$3 - x^2 = 3 - 8$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

(iv)

$$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$$

$$12x - 14 = 10$$

$$12x = 24$$

$$x = 2$$

Determinants Ex 6.1 Q11

$$\text{Let } A = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

Expanding the given determinant along the first column

$$|A| = x^2 \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - 0 \begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + 3 \begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}$$

$$28 = x^2(8 - 1) - 0(4x - 1) + 3(x - 2)$$

$$28 = 7x^2 + 3x - 6$$

or

$$7x^2 + 3x - 6 = 28$$

$$7x^2 + 3x - 34 = 0$$

Solving using quadratic formula, we get $x = 2$.

Determinants Ex 6.1 Q12(i)

A matrix A is called singular if $|A| = 0$

Now expanding along the first row $|A|$

$$\begin{aligned}
&= (x-1) \begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ x-1 & 1 \end{vmatrix} \\
&= (x-1) [(x-1)^2 - 1] - 1[x-1-1] + 1[1-x+1] \\
&= (x-1)(x^2 + 1 - 2x - 1) - 1(x-2) + 1(2-x) \\
&= (x-1)(x^2 - 2x) - x + 2 + 2 - x \\
&= (x-1) \times x \times (x-2) + (4 - 2x) \\
&= (x-1) \times x \times (x-2) + 2(2-x) \\
&= (x-1) \times x \times (x-2) - 2(x-2) \\
&= (x-2)[x(x-1) - 2] \qquad \qquad \qquad \text{(Taking } (x-2) \text{ common)}
\end{aligned}$$

Since A is a singular matrix, so $|A| = 0$

$$\text{i.e. } (x-2)(x^2 - x - 2) = 0$$

$$\begin{aligned}
\text{either } (x-2) &= 0 & \text{or } x^2 - x - 2 &= 0 \\
x &= 2 & \text{or } x^2 - 2x + x - 2 &= 0 \\
& & x(x-2) + 1(x-2) &= 0 \\
& & (x-2)(x+1) &= 0 \\
& & x &= 2, -1
\end{aligned}$$

$$x = 2 \text{ or } -1$$

Determinants Ex 6.1 Q12(ii)

A matrix A is said to be singular if $|A| = 0$

Now

$$\begin{aligned}
&\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0 \\
&8 + 8x - 21 + 7x = 0 \\
&15x = 13 \\
&x = \frac{13}{15}
\end{aligned}$$

Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2 \begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$