RD Sharma Solutions Class 12 Maths Chapter 6 Ex 6.2

Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-ii

Consider the determinant

$$\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Applying $C_1 \to C_1 - 4C_3$, *we get,*

$$\Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 - R_2 \text{ and } R_1 \to R_1 + R_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_2 \to 3R_1 + R_2]$$

$$\Rightarrow \Delta = 1(109 \times 12 - 119 \times 11)$$

$$\Rightarrow \Delta = -1$$

Chapter 6 Determinants Ex 6.2 Q1-iii

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$$

$$= abc - af^2 - h^2c + hfg + ghf - bg^2$$

Chapter 6 Determinants Ex 6.2 Q1-iv

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix} = 2(24 - 4) = 40$$

Chapter 6 Determinants Ex 6.2 Q1-v

Let Δ be the determinant.

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get,

$$\Delta = \begin{vmatrix} 1 & 4 & 9-4 \\ 4 & 9 & 16-9 \\ 9 & 16 & 25-16 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix} \quad [\text{Applying } C_2 \to C_1 + C_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix} \quad [\text{Applying } C_2 \to 5C_1 - C_2 \text{ and } C_3 \to 5C_1 - C_3]$$

$$\Rightarrow \Delta = 1(7 \times 36 - 13 \times 20) = 252 - 260 = -8$$

Chapter 6 Determinants Ex 6.2 Q1-vi

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Apply: $R_1 \to R_1 + (-3)R_2$ and $R_3 \to R_3 + 5R_2$

$$= \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-vii

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+3+3^2+3^3 & 3 & 3^2 & 3^3 \\ 1+3+3^2+3^3 & 3^2 & 3^3 & 1 \\ 1+3+3^2+3^3 & 3^3 & 1 & 3 \\ 1+3+3^2+3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2-3 & 3^3-3^2 & 1-3^3 \\ 0 & 3^3-3 & 1-3^2 & 3-3^3 \\ 0 & 1-3 & 3-3^2 & 3^2-3^3 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \times 40(36+4) = 512000$$

Chapter 6 Determinants Ex 6.2 Q1-viii

$$Let\ \Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Applying $R_3 \to 17R_2 - R_3$, *we get,*

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 0 & 48 & 62 \end{vmatrix}$$

Applying $R_2 \to 102R_2 - R_1$, *we get,*

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 0 & 288 & 372 \\ 0 & 48 & 62 \end{vmatrix}$$

Thus,

$$\Delta = 102(288 \times 62 - 372 \times 48)$$

$$\Rightarrow \Delta = 0$$

Chapter 6 Determinants Ex 6.2 Q2(i)

$$\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

Apply: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 4 & 1 & -2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} 8 & 2 & 7 \\ 4 & 1 & -2 \\ 4 & 1 & -2 \end{vmatrix}$$

Since, $R_3 = R_2$, the value of the determinant is zero.

Chapter 6 Determinants Ex 6.2 Q2(ii)

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Taking (-2) common from C_1, we get

$$= (-2)\begin{vmatrix} -3 & -3 & 2 \\ -1 & -1 & 2 \\ 5 & 5 & 2 \end{vmatrix}$$

$$= 0$$

$\because C_1$ and C_2 are identical.

Chapter 6 Determinants Ex 6.2 Q2(iii)

$$\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$$

Use: $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$$= 0$$

$\because R_3 = R_1$

Chapter 6 Determinants Ex 6.2 Q2(iv)

$$\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$$

Multiply: R_1, R_2 and R_3 by a, b and c respectively, we get

$$= \frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & bca \\ 1 & c^3 & cab \end{vmatrix}$$

Take abc common from C_3, we get,

$$= \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$$

$= 0$

$\because \quad C_1 = C_3$

Chapter 6 Determinants Ex 6.2 Q2(v)

$$\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$$

Apply: $C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a+b & 2a+b & a \\ 2a+b & 3a+b & a \\ 4a+b & 5a+b & a \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$

$$= \begin{vmatrix} a+b & a & a \\ 2a+b & a & a \\ 4a+b & a & a \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(vi)

$$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ac \\ 1 & c & c^2-ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 0 & b-a & (a-b)c \\ 0 & c-a & (a-c)b \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix} - (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$$= (b-a)(c-a)(c+a-b-a) - (b-a)(c-a)(-b+c)$$

$$= (b-a)(c-a)(c-b) - (b-a)(c-a)(-b+c)$$

$$= 0$$

Chapter 6 Determinants Ex 6.2 Q2(vii)

$$\begin{vmatrix} 49 & 1 & 6 \\ 39 & 7 & 4 \\ 26 & 2 & 3 \end{vmatrix}$$

Apply: $C_1 \to C_1 + (-8)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(viii)

$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix}$$

Multiply C_1, C_2 and C_3 by z, y, and x respectively

$$= \frac{1}{xyz}\begin{vmatrix} 0 & xy & yx \\ -xz & 0 & zx \\ -yz & -zy & 0 \end{vmatrix}$$

Take y, x, and z common from R_1, R_2 and R_3 respectively

$$= \begin{vmatrix} 0 & x & x \\ -z & 0 & z \\ -y & -y & 0 \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & x \\ -z & -z & z \\ -y & -y & 0 \end{vmatrix}$$

$$= 0$$

$\because C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(ix)

$$\begin{vmatrix} 1 & 43 & 6 \\ 7 & 35 & 4 \\ 3 & 17 & 2 \end{vmatrix}$$

Apply: $C_2 \rightarrow C_2 + (-7)C_3$

$$= \begin{vmatrix} 1 & 1 & 6 \\ 7 & 7 & 4 \\ 3 & 3 & 2 \end{vmatrix}$$

$= 0$

$\because \quad C_1 = C_2$

Chapter 6 Determinants Ex 6.2 Q2(x)

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$$

$Apply: C3 \rightarrow C3 - C2, C4 \rightarrow C4 - C1$

$$= \begin{vmatrix} 1^2 & 2^2 & 3^2 - 2^2 & 4^2 - 1^2 \\ 2^2 & 3^2 & 4^2 - 3^2 & 5^2 - 2^2 \\ 3^2 & 4^2 & 5^2 - 4^2 & 6^2 - 3^2 \\ 4^2 & 5^2 & 6^2 - 5^2 & 7^2 - 4^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1^2 & 2^2 & 5 & 15 \\ 2^2 & 3^2 & 7 & 21 \\ 3^2 & 4^2 & 9 & 27 \\ 4^2 & 5^2 & 11 & 33 \end{vmatrix}$$

Take 3 common from C_4

$$= 3\begin{vmatrix} 1^2 & 2^2 & 5 & 5 \\ 2^2 & 3^2 & 7 & 7 \\ 3^2 & 4^2 & 9 & 9 \\ 4^2 & 5^2 & 11 & 11 \end{vmatrix}$$

$= 0$

$\because \quad C_3 = C_4$

Chapter 6 Determinants Ex 6.2 Q2(xi)

$$\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ 2a+2x & 2b+2y & 2c+2z \\ x+a & y+b & z+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ a+x & b+y & c+z \\ x+a & y+b & z+c \end{vmatrix}$$

$= 0$

Chapter 6 Determinants Ex 6.2 Q3

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Apply: $C_2 \to C_2 + C_1$.

$$= \begin{vmatrix} a & b+c+a & a^2 \\ b & c+a+b & b^2 \\ c & a+b+c & c^2 \end{vmatrix}$$

Take $(a+b+c)$ common from C_2

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= (b+c+a)\begin{vmatrix} a & 1 & a^2 \\ b-a & 0 & b^2-a^2 \\ c-a & 0 & c^2-a^2 \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)\begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b+a \\ 1 & 0 & c+a \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q4

$$Let\ \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$ *we get,*

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-ba \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

Taking $(a-b)$ *and* $(a-c)$ *common, we have*

$$\Delta = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q5

$$Let\ \Delta = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}$$

$Taking\ (3x+\lambda)\ common,\ we\ have$

$$\Delta = (3x+\lambda) \begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (3x+\lambda) \begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \lambda^2(3x+\lambda)$$

Chapter 6 Determinants Ex 6.2 Q6

$$Let\ \Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$Taking\ (a+b+c)\ common,\ we\ have$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b+c)[(a-b)(a-c) - (b-c)(c-b)]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 - ac - ab + bc + b^2 + c^2 - 2bc\right]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 + b^2 + c^2 - ac - ab - bc\right]$$

Chapter 6 Determinants Ex 6.2 Q7

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix} = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

$$= (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

$$= (2+x)(x-1)^2$$

Chapter 6 Determinants Ex 6.2 Q8

$$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$$

$$= 0\left(0 - y^3z^3\right) - xy^2\left(0 - x^2yz^3\right) + xz^2\left(x^2y^3z - 0\right)$$

$$= 0 + x^3y^3z^3 + x^3y^3z^3$$

$$= 2x^3y^3z^3$$

Chapter 6 Determinants Ex 6.2 Q9

Let $\Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$$

$$\Delta = a\left[a(a+x+y) + az\right] + 0 + 0$$

$$\Delta = a^2(a+x+y+z)$$

Chapter 6 Determinants Ex 6.2 Q10

$$\Delta + \Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix} \ldots\ldots\ldots \left[\because |A| = \left|A^T\right|\right]$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$\ldots\ldots$ [If any two rows (columns) of the determinant are interchanged then value of the determinant changes in sign.]

$$= \begin{vmatrix} 0 & 0 & x^2 - yz \\ 0 & 0 & y^2 - zx \\ 0 & 0 & z^2 - xy \end{vmatrix}$$

$= 0 \ldots\ldots\ldots [\because C_1$ and C_2 are identical$]$

Chapter 6 Determinants Ex 6.2 Q11

Chapter 6 Determinants Ex 6.2 Q12

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$$\text{LHS} = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} b+c+a & -b & a \\ c+a+b & -c & b \\ a+b+c & -a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & a & c \end{vmatrix}$$

$$= -(b+c+a)\begin{vmatrix} 1 & b & a \\ 0 & c-b & b-a \\ 0 & a-b & c-a \end{vmatrix}$$

$$= -(b+c+a)\left[(c-b)(c-a)-(b-a)(a-b)\right]$$

$$= 3abc - a^3 - b^3 - c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q13

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\text{LHS} = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & b+c & c+a \\ a+b+c & c+a & a+b \\ a+b+c & a+b & b+c \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$$= 2\begin{vmatrix} a+b+c & -a & -b \\ a+b+c & -b & -c \\ a+b+c & -c & -a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a & b \\ a+b+c & b & c \\ a+b+c & c & a \end{vmatrix}$$

$$= 2\left(\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix} + \begin{vmatrix} a & a & b \\ b & b & c \\ c & c & a \end{vmatrix} + \begin{vmatrix} b & a & b \\ c & b & c \\ a & c & a \end{vmatrix}\right)$$

$$= 2\begin{vmatrix} c & a & b \\ a & b & c \\ b & c & a \end{vmatrix}$$

$$= 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q14

We need to prove the following identity:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, *we have,*

$$L.H.S = \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix}$$

Taking the term $2a+2b+2$ *as common, we have*

$$L.H.S = (2a+2b+2c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$\Rightarrow L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ *and* $R_3 \rightarrow R_3 - R_1$ *we have*

$$L.H.S = 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$$

Thus, we have,

$$\begin{aligned} L.H.S &= 2(a+b+c)\left[1 \times (a+b+c)^2\right] \\ &= 2(a+b+c)(a+b+c)^2 \\ &= 2(a+b+c)^3 \end{aligned}$$

Chapter 6 Determinants Ex 6.2 Q15

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$\text{LHS} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$.

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Take $(a+b+c)$ common from R_1

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Apply: $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & b+c+a & 0 \\ 2c & 0 & b+c+a \end{vmatrix}$$

$$= (a+b+c)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q16

$$LHS = \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & a-b & a^2-b^2 \\ 0 & a-c & a^2-c^2 \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 1 & a+c \end{vmatrix}$$

$$= (a-b)(a-c)\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 0 & 1 & a+b \\ 0 & 0 & c-b \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q17

$$LHS = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= \begin{vmatrix} 3a+3b & 3a+3b & 3a+3b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= (3a+3b)\begin{vmatrix} 1 & 1 & 1 \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}$$

$$= 3(a+b)\begin{vmatrix} 0 & 1 & 0 \\ 2b & a & b \\ -b & a+2b & -2b \end{vmatrix}$$

$$= 3(a+b)b^2\begin{vmatrix} 0 & 1 & 0 \\ 2 & a & 1 \\ -1 & a+2b & -2 \end{vmatrix}$$

$$= 9(a+b)b^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q18

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Apply $R_1 \to R_1 a,\ R_2 \to R_2 b,\ R_3 \to R_3 c$

$$= \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & cab \\ c & c^2 & abc \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$= -\begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Chapter 6 Determinants Ex 6.2 Q19

$$\begin{vmatrix} z & x & y \\ z^2 & x^2 & y^2 \\ z^4 & x^4 & y^4 \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix} = \begin{vmatrix} x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \\ x & y & z \end{vmatrix} = xyz\,(x-y)(y-z)(z-x)(x+y+z)$$

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^4 & y^4 & z^4 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz \begin{vmatrix} 0 & 1 & 0 \\ x-y & y & z-y \\ x^3-y^3 & y^3 & z^3-y^3 \end{vmatrix}$$

$$= xyz\,(x-y)(z-y) \begin{vmatrix} 0 & 1 & 0 \\ 1 & y & 1 \\ x^2+y^2+xy & y^3 & z^2+y^2+zy \end{vmatrix}$$

$$= -xyz\,(x-y)(z-y)\left[z^2+y^2+zy-x^2-y^2-xy\right]$$

$$= -xyz\,(x-y)(z-y)\left[(z-x)(z+x)+y(z-x)\right]$$

$$= -xyz\,(x-y)(z-y)(z-x)\left[z+x+y\right]$$

$$= xyz\,(x-y)(y-z)(z-x)(x+y+z)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q20

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 - 2C_3$

$$= \begin{vmatrix} (b+c)^2 + a^2 - 2bc & a^2 & bc \\ (c+a)^2 + b^2 - 2ca & b^2 & ca \\ (a+b)^2 + c^2 - 2ab & c^2 & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2 & a^2 & bc \\ a^2+b^2+c^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

Take $\left(a^2+b^2+c^2\right)$ common from C_1

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ca-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b+a & -c \\ 0 & c+a & -b \end{vmatrix}$$

$$= \left(a^2+b^2+c^2\right)(b-a)(c-a)\left[(b+a)(-b)-(-c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q21

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

$$\text{LHS} = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

$$= \left[(2a+4)(1) - (1)(2a+6)\right]$$

$$= -2$$

$$= \text{RHS}$$

Chapter 6 Determinants Ex 6.2 Q22

$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

Apply: $C_2 \rightarrow C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2-2b^2-2ca & ca \\ c^2 & c^2-(a-b)^2-2c^2-2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -(b^2+c^2+a^2) & bc \\ b^2 & -(b^2+c^2+a^2) & ca \\ c^2 & -(b^2+c^2+a^2) & ab \end{vmatrix}$$

Take $-(a^2+b^2+c^2)$ common from C_2

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\left[(-(b+a))(-b)-(c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q23

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & b^2+ca-a^2-bc & b^3-a^3 \\ 0 & c^2+abb^2+ca-a^2-bc & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b^2-a^2)-c(b-a) & b^3-a^3 \\ 0 & (c^2-a^2)-b(c-a) & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b-a)(b+a-c) & b^3-a^3 \\ 0 & (c-a)(c+a-b) & c^3-a^3 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b+a-c) & b^2+a^2+ab \\ 0 & (c+a-b) & c^2+a^2+ac \end{vmatrix}$$

$$= (b-a)(c-a)\left[((b+a-c))(c^2+a^2+ac)-(b^2+a^2+ab)(c^2+a^2+ac)\right]$$

$$= -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q24

We need to prove the following identity:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Taking the term a, b, c *common from* C_1, C_2 *and* C_3, *respectively, we have,*

$$L.H.S = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, *we have,*

$$L.H.S = abc\begin{vmatrix} 2a+2c & c & a+c \\ 2a+2b & b & a \\ 2b+2c & b+c & c \end{vmatrix}$$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ *and* $C_3 \rightarrow C_3 - C_1$, *we have,*

$$L.H.S = 2abc\begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, *we have,*

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

Taking c, a, *and* b *from* C_1, C_2 *and* C_3 *respectively, we have,*

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$, *we have*

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

Chapter 6 Determinants Ex 6.2 Q25

We need to prove the following identity:

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we get,*

$$\Delta = \begin{vmatrix} 3x+4 & x & x \\ 3x+4 & x+4 & x \\ 3x+4 & x & x+4 \end{vmatrix}$$

Taking the common term $3x+4$, *we get,*

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we get,*

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \Delta = 16(3x+4)$$

Chapter 6 Determinants Ex 6.2 Q26

We need to prove the following identity:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Let us consider the L.H.S of the above equation.

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

Applying $C_3 \to C_3 - pC_2$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

$$\Rightarrow \Delta = 1[7-6] = 1$$

Chapter 6 Determinants Ex 6.2 Q27

$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} -a+c+b & -b-c+a & -c-b+a \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a) \begin{vmatrix} 1 & -1 & -1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a) \begin{vmatrix} 1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

$$= (a+b-c)(b+c-a)(c+a-b)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q28

$$LHS = \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+2ab & 2ab & b^2 \\ a^2+b^2+2ab & a^2 & 2ab \\ a^2+b^2+2ab & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-2ab & 2ab-b^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= \left(a^2+b^2+2ab\right)\begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-b^2 & 2ab-a^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix}$$

$$= (a+b)^2\left[\left(a^2-b^2\right)\left(a^2-b^2\right)-\left(2ab-a^2\right)\left(b^2-2ab\right)\right]$$

$$= (a+b)^2\left(a^2+b^2-ab\right)^2$$

$$= \left(a^3+b^3\right)^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q29

We need to prove the following identity:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ *and* $R_3 \to R_3(c)$, *we get*

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{vmatrix}$$

Taking a,b, and c common from C_1, C_2 *and* C_3, *respectively, we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} (a^2+1) & a^2 & a^2 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we get,*

$$\Delta = \frac{abc}{abc}\begin{vmatrix} (a^2+b^2+c^2+1) & (a^2+b^2+c^2+1) & (a^2+b^2+c^2+1) \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Taking the term, $(a^2+b^2+c^2+1)$ *common from the above equation, we have,*

$$\Delta = (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, *we get,*

$$\Delta = (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2+b^2+c^2+1)$$

Chapter 6 Determinants Ex 6.2 Q30

Let us consider the L.H.S of the given equation.

$$\text{Let } \Delta = \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta = \begin{vmatrix} 1+a+a^2 & a & a^2 \\ 1+a+a^2 & 1 & a \\ 1+a+a^2 & a^2 & 1 \end{vmatrix}$$

Taking the term $(1+a+a^2)$ *common, we have,*

$$\Delta = (1+a+a^2)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have*

$$\Delta = (1+a+a^2)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & -a(1-a) & (1-a)(1+a) \end{vmatrix}$$

Taking the term $(1-a)$ *common from* R_2 *and* R_3, *we have*

$$\Rightarrow \Delta = (1+a+a^2)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & (1+a) \end{vmatrix}$$

$$\Rightarrow \Delta = (1+a+a^2)(1-a)^2(1+a+a^2)$$

$$\Rightarrow \Delta = (1+a+a^2)^2(1-a)^2$$

$$\Rightarrow \Delta = [(1+a+a^2)(1-a)]^2$$

$$\Rightarrow \Delta = [(a^3-1)]^2$$

Chapter 6 Determinants Ex 6.2 Q31

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

$$\text{LHS} = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c & -(c+b) & -b \\ -(c+a) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$= (c+a)(c+b) \begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$$= (c+a)(c+b) \begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -a-b \\ 0 & 2 & a+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q34

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

$$\text{LHS} = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$$= a^3b^3c^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= a^3b^3c^3 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2a^3b^3c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q36

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 0 & 1 & 0 \\ ab+bc+ac & -ac & bc+ab+ac \\ 0 & bc+ac & -ab-bc-ac \end{vmatrix}$$

$$= (ab+bc+ca)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & -ac & 1 \\ 0 & bc+ac & -1 \end{vmatrix}$$

$$= (ab+bc+ca)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q37

L.H.S.,

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [C_1 \to C_1 - C_3, C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} \lambda - x & 0 & 2x \\ 0 & \lambda - x & 2x \\ x-\lambda & x-\lambda & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)(\lambda - x)\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2[1(x+\lambda) + 2x + 2x(0+1)]$$

$$= (\lambda - x)^2[x + \lambda + 2x + 2x]$$

$$= (\lambda - x)^2[5x + \lambda]$$

= R.H.S

Hence Proved

Chapter 6 Determinants Ex 6.2 Q39

$$\text{Let } \Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$

$$\Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Delta = 2x\left[z(x+y) - xy\right] - 2x\left[zx - y(z+x)\right]$$

$$\Delta = 2x\left[zx + zy - xy - zx + yz + yx\right]$$

$$\Delta = 4xyz$$

Chapter 6 Determinants Ex 6.2 Q40

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix} = abc\left(a^2+b^2+c^2\right)^3$$

$$\text{LHS} = \begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$=abc\begin{vmatrix} -\left(b^2+c^2-a^2\right) & 2b^2 & 2c^2 \\ 2a^2 & -\left(c^2+a^2-b^2\right) & 2c^2 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Apply: $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$=abc\begin{vmatrix} -\left(b^2+c^2-a^2\right)-2a^2 & 0 & 2c^2+\left(a^2+b^2-c^2\right) \\ 0 & -\left(c^2+a^2-b^2\right)-2b^2 & 2c^2+\left(a^2+b^2-c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=abc\begin{vmatrix} -\left(b^2+c^2+a^2\right) & 0 & \left(a^2+b^2+c^2\right) \\ 0 & -\left(c^2+a^2+b^2\right) & \left(a^2+b^2+c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$=abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right)+2a^2 \end{vmatrix}$$

$$=abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -b^2+c^2+a^2 \end{vmatrix}$$

$$=-abc\left(b^2+c^2+a^2\right)^2\left[(-1)\left(-b^2+c^2+a^2\right)-(1)\left(2b^2\right)\right]$$

$$abc\left(a^2+b^2+c^2\right)^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q48

Since, α, β, γ are in A.P, $2\beta = \alpha + \gamma$

$$LHS = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$R_2 \to R_2 - \frac{R_1}{2} - \frac{R_3}{2}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ (x-2) - \frac{x-3}{2} - \frac{x-1}{2} & (x-3) - \frac{x-4}{2} - \frac{x-2}{2} & (x-\beta) - \frac{x-\alpha}{2} - \frac{x-\gamma}{2} \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ 0 & 0 & 0 \\ x-1 & x-2 & x-\gamma \end{vmatrix} \qquad [\because 2\beta = \alpha + \gamma]$$

$$= 0$$

Chapter 6 Determinants Ex 6.2 Q52-v

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)\begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+9)(x-1)^2 = 0$$

$$\Rightarrow \quad (x+9) = 0 \quad or \quad (x-1)^2 = 0$$

$$\Rightarrow \quad x = -9 \quad or \quad x = 1$$

Chapter 6 Determinants Ex 6.2 Q52-vii

$$\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 15-2x & -x-4 & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 8-x & -x-4 & 7-x \\ 0 & 0 & 1 \\ -3 & 6 & 13 \end{vmatrix} = 0$$

$$\Rightarrow -\left[(8-x)(6)-(-x-4)(-3)\right] = 0$$

$$\Rightarrow -\left[36-9x\right] = 0$$

$$\Rightarrow x = 4$$

Chapter 6 Determinants Ex 6.2 Q52-viii

$$\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p\begin{vmatrix} 1 & 1 & x \\ 0 & 0 & 1-x \\ 2 & x & 2 \end{vmatrix} = 0$$

$$\Rightarrow p(x-1)(x-2) = 0$$

$$\Rightarrow (x-1) = 0 \qquad (x-2) = 0$$

$$\Rightarrow x = 1 \qquad x = 2$$