RD Sharma Solutions Class 12 Maths Chapter 7 Ex 7.1

Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

Here, $A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 4$

$C_{12} = -2$

$C_{21} = -5$

$C_{22} = -3$

$\therefore$ $\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore$ $(\text{adj } A) = \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$

Now, $(\text{adj } A) A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also,

$A.(\text{adj } A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Therefore, $(\text{adj } A) A = |A|.I = A.(\text{adj } A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$C_{11} = d$
$C_{12} = -c$
$C_{21} = -b$
$C_{22} = a$

$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$\therefore \quad \text{adj} A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T$

$\qquad = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, $(\text{adj} A)(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

Also,

$A(\text{adj} A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

$\therefore \quad (\text{adj} A)(A) = |A|.I = A(\text{adj} A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Cofactors of A are:

$\quad C_{11} = \cos\alpha$

$\quad C_{12} = -\sin\alpha$

$\quad C_{21} = -\sin\alpha$

$\quad C_{22} = \cos\alpha$

$\therefore \quad \text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{T}$

$\quad \text{Adj } A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}^{T}$

$\quad = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $(\text{adj} A).(A) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\quad = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\quad = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

And, $A(\text{adj} A) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\quad = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -\cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\quad = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Also,

$\quad |A|.I = \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (\cos^2\alpha - \sin^2\alpha)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\quad = \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$

$\quad = \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$\therefore \ \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$\therefore \ \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(i)

Here $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = -3 \qquad C_{21} = +2 \qquad C_{31} = 2$$
$$C_{12} = +2 \qquad C_{22} = -3 \qquad C_{32} = 2$$
$$C_{13} = 2 \qquad C_{23} = 2 \qquad C_{33} = -3$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T$$

Therefore,

$$\text{adj } A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Now,

$$(\text{adj } A).A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (-3 + 4 + 4)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A.(\text{adj } A) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore \quad (\text{adj } A).A = |A|.I = A.(\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = 2 \qquad C_{21} = 3 \qquad C_{31} = -13$$
$$C_{12} = -3 \qquad C_{22} = 6 \qquad C_{32} = 9$$
$$C_{13} = 5 \qquad C_{23} = -3 \qquad C_{33} = -1$$

$$
\therefore \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T
$$

$$
= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T
$$

Therefore,

$$
\text{adj } A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}
$$

Now,

$$
(\text{adj } A).A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}
$$

$$
|A|.I = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}
$$

$$
A.(\text{adj } A) = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}
$$

$\therefore \quad (\text{adj } A).A = |A|.I = A.(\text{adj } A)$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = -22 \qquad C_{21} = 11 \qquad C_{31} = -11$$
$$C_{12} = 4 \qquad C_{22} = -2 \qquad C_{32} = 2$$
$$C_{13} = 16 \qquad C_{23} = -8 \qquad C_{33} = 8$$

$$\therefore \quad adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$\therefore \quad = \begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix}^T$$

Therefore,

$$adj\,A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$$

Now,

$$(adj\,A).A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= (44 - 4 + 48) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$A(adj\,A) = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad (adj\,A).A = |A|I = A(adj\,A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 3 \qquad\qquad C_{21} = -1 \qquad\qquad C_{31} = 1$

$C_{12} = -15 \qquad\quad C_{22} = 7 \qquad\qquad\, C_{32} = -5$

$C_{13} = 4 \qquad\qquad C_{23} = -2 \qquad\qquad C_{33} = 2$

$\therefore \qquad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$\qquad\qquad = \begin{bmatrix} 3 & -15 & 4 \\ -1 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$

Therefore,

$\text{adj}\,A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$

Now,

$(\text{adj}\,A)\,A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$|A|.I = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{vmatrix} I_3$

$\qquad\qquad = (6 - 4)\,I_3 = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A.(\text{adj}\,A) = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore \qquad (\text{adj}\,A)\,A = |A|.I = A.(\text{adj}\,A)$

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$C_{11} = 15 \qquad\qquad C_{21} = 6 \qquad\qquad C_{31} = -15$

$C_{12} = 0 \qquad\qquad\, C_{22} = -3 \qquad\qquad C_{32} = 0$

$C_{13} = -10 \qquad\quad\, C_{23} = 0 \qquad\qquad\, C_{33} = 5$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix}^{T}$$

Therefore,

$$\operatorname{adj} A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$$

Now,

$$(\operatorname{adj} A) A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$|A|.I = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix} \quad I_{3}$$

$$= (-15) I_{3} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$A.(\operatorname{adj} A) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\therefore \quad (\operatorname{adj} A) A = |A|.I = A.(\operatorname{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q3

Here

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$$

Cofactors of A are

$$
\begin{array}{lll}
C_{11} = 30 & C_{21} = 12 & C_{31} = -3 \\
C_{12} = -20 & C_{22} = -8 & C_{32} = 2 \\
C_{13} = -50 & C_{23} = -20 & C_{33} = 5
\end{array}
$$

Therefore,

$$adj\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 30 & -20 & -50 \\ 12 & -8 & -20 \\ -3 & 2 & 5 \end{bmatrix}^{T}$$

So,

$$adjA = \begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$$

Now,

$$A(adjA) = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}\begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}(0)$$

$$= 0$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q4

Here, $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = -4 \qquad C_{21} = -3 \qquad C_{31} = -3$$
$$C_{12} = 1 \qquad C_{22} = 0 \qquad C_{32} = 1$$
$$C_{13} = 4 \qquad C_{23} = 4 \qquad C_{33} = 3$$

$$\therefore \qquad adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T$$

Therefore, $\quad adj\,A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

So, $\quad adj\,A = A$

Adjoint and Inverse of Matrix Ex 7.1 Q5

Here $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = -3 \qquad C_{21} = 6 \qquad C_{31} = 6$$
$$C_{12} = -6 \qquad C_{22} = 3 \qquad C_{32} = -6$$
$$C_{13} = -6 \qquad C_{23} = -6 \qquad C_{33} = 3$$

$$\therefore \quad adj A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$$

Therefore, $adj A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$ $\qquad\qquad ---(i)$

Now, $3.A^T = 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$ $\qquad ---(ii)$

$$\therefore \quad adj A = 3.A^T$$

Adjoint and Inverse of Matrix Ex 7.1 Q6

Here, $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = 9 \qquad C_{21} = 19 \qquad C_{31} = -4$$
$$C_{12} = 4 \qquad C_{22} = 14 \qquad C_{32} = 1$$
$$C_{13} = 8 \qquad C_{23} = 3 \qquad C_{33} = 2$$

$$\therefore \quad adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}^T$$

Therefore,

$$adj\,A = \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$$

Now, $A\,adj\,A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$= 25\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 25 I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (i)

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now, $|A| = 1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = \cos\theta \qquad C_{21} = -\sin\theta$$
$$C_{12} = \sin\theta \qquad C_{22} = \cos\theta$$

$\therefore$
$$\text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$\therefore$
$$\text{adj}\,A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (ii)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $|A| = -1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = 0 \qquad C_{12} = -1$$
$$C_{21} = -1 \qquad C_{22} = 0$$

$$\therefore \quad adjA = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Also, $A^{-1} = \frac{1}{|A|}(adjA)$

$$A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iii)

$$A = \begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix}$$

Now, $\quad |A| = \dfrac{a + abc}{a} - bc = \dfrac{a + abc - abc}{a} = 1 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$$C_{11} = \dfrac{1+bc}{a} \qquad C_{12} = -c$$

$$C_{21} = -b \qquad C_{22} = a$$

$\therefore \quad adj\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} \dfrac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$$

$\therefore \quad adj\,A = \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Also, $\quad A^{-1} = \dfrac{1}{|A|} . adj\,A$

$$A^{-1} = \dfrac{1}{1} \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

or $\quad A^{-1} = \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iv)

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Now, $|A| = 2 + 15 = 17 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$$C_{11} = 1 \qquad C_{12} = 3$$
$$C_{21} = -5 \qquad C_{22} = 2$$

$\therefore \quad adj A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$

$$= \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$\therefore \quad adj A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \dfrac{1}{|A|} \cdot (adj A)$

$A^{-1} = \dfrac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Expanding using 1st row, we get

$$|A| = 1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 1(6-1) - 2(4-3) + 3(2-9)$$
$$= 5 - 2 \times 1 + 3 \times (-7)$$
$$= 5 - 2 - 21 = -18 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 5 & C_{21} = -1 & C_{31} = -7 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 5 \\ C_{13} = -7 & C_{23} = 5 & C_{33} = -1 \end{array}$$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\,A$$

Hence, $A^{-1} = \dfrac{1}{(-18)}\begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \begin{bmatrix} \dfrac{-5}{18} & \dfrac{1}{18} & \dfrac{7}{18} \\ \dfrac{1}{18} & \dfrac{7}{18} & \dfrac{-5}{18} \\ \dfrac{7}{18} & \dfrac{-5}{18} & \dfrac{1}{18} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + 5\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= 1 \times (1+3) - 2(-1+2) + 5(3+2)$$

$$= 4 - 2(1) + 5(5) = 27 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$\quad C_{11} = 4 \qquad\qquad C_{21} = +17 \qquad\qquad C_{31} = 3$

$\quad C_{12} = -1 \qquad\qquad C_{22} = -11 \qquad\qquad C_{32} = +6$

$\quad C_{13} = 5 \qquad\qquad C_{23} = +1 \qquad\qquad C_{33} = -3$

$$\therefore \quad adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.adj\,A$

$$\therefore \quad A^{-1} = \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} = \begin{bmatrix} \dfrac{4}{27} & \dfrac{17}{27} & \dfrac{3}{27} \\ \dfrac{-1}{27} & \dfrac{-11}{27} & \dfrac{6}{27} \\ \dfrac{5}{27} & \dfrac{1}{27} & \dfrac{-3}{27} \end{bmatrix} = \begin{bmatrix} \dfrac{4}{27} & \dfrac{17}{27} & \dfrac{1}{9} \\ \dfrac{-1}{27} & \dfrac{-11}{27} & \dfrac{2}{9} \\ \dfrac{5}{27} & \dfrac{1}{27} & \dfrac{-1}{9} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= 2(4-1) + 1(-2+1) + 1(1-2)$$

$$= 2(3) + 1(-1) + 1(-1) = 6 - 2 = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 3$ $C_{21} = +1$ $C_{31} = -1$

$C_{12} = +1$ $C_{22} = 3$ $C_{32} = +1$

$C_{13} = -1$ $C_{23} = +1$ $C_{33} = 3$

$\therefore$ $\text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} \text{adj} A$

$\therefore$ $A^{-1} = \dfrac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \dfrac{3}{4} & \dfrac{1}{4} & \dfrac{-1}{4} \\ \dfrac{1}{4} & \dfrac{3}{4} & \dfrac{1}{4} \\ \dfrac{-1}{4} & \dfrac{1}{4} & \dfrac{3}{4} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2 \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0 \begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 2(3 - 0) - 0 - 1(5)$$

$$= 2(3) - 1(5) = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 6 & C_{32} = -5 \\ C_{13} = 5 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} .\text{adj} A$$

$$= \frac{1}{1} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(v)

Here, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 0 \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix}$$

$$= 0 - 1(6 - 12) - 1(-12 + 9)$$

$$= -1(4) - 1(-3) = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$
\begin{array}{lll}
C_{11} = 0 & C_{21} = -1 & C_{31} = 1 \\
C_{12} = -4 & C_{22} = 3 & C_{32} = -4 \\
C_{13} = -3 & C_{23} = +3 & C_{33} = -4
\end{array}
$$

$\therefore$ $\quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$$= \begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} \cdot \operatorname{adj} A$

$$A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vi)

Here, $A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 0 \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1 \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$$

$$= 0 - 0 - 1(-12 + 8)$$

$$= -1(-4) = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -8$ $C_{21} = +4$ $C_{31} = 4$

$C_{12} = +11$ $C_{22} = -2$ $C_{32} = -3$

$C_{13} = -4$ $C_{23} = +0$ $C_{33} = 0$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{adj}\,A$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vii)

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Expanding using first column, we get

$$|A| = 1 \begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - 0 + 0$$

$$= -\cos^2\alpha + \sin^2\alpha$$

$$= -\left(\cos^2\alpha + \sin^2\alpha\right)$$

$$|A| = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 0 & C_{31} = 0 \\ C_{12} = 0 & C_{22} = -\cos\alpha & C_{32} = -\sin\alpha \\ C_{13} = 0 & C_{23} = -\sin\alpha & C_{33} = \cos\alpha \end{array}$$

$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} \text{adj} A$

$\therefore \quad A^{-1} = \dfrac{1}{(-1)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Hence, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(i)

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Expanding using 1st row, we get

$$|A| = 1\begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix}$$

$$= 1(16 - 9) - 3(4 - 3) + 3(3 - 4)$$

$$= 7 - 3(1) + 3(-1)$$

$$= 7 - 3 - 3 = +1 = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 7 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = -1 & C_{22} = 1 & C_{32} = -0 \\ C_{13} = -1 & C_{23} = -0 & C_{33} = 1 \end{array}$$

$$\therefore \quad adj\,A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.adj\,A$

$$\therefore \quad A^{-1} = \frac{1}{1}\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Also, $A^{-1}.A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q9(ii)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Expanding 1st row, we get

$$|A| = 2\begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} - 3\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix}$$

$$= 2(8-7) - 3(6-3) + 1(21-12)$$

$$= 2 - 3(3) + 1(9) = 2 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 1 & C_{21} = +1 & C_{31} = -1 \\ C_{12} = -3 & C_{22} = 1 & C_{32} = +1 \\ C_{13} = 9 & C_{23} = -5 & C_{33} = -1 \end{array}$$

$$\therefore \quad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.\text{adj}\, A \Rightarrow A^{-1} = \dfrac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

Also, $A^{-1}.A = \dfrac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

$$= \dfrac{1}{2}\begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix}$$

$$= \dfrac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore A^{-1}.A = I_3$

Adjoint and Inverse of Matrix Ex 7.1 Q10(i)

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \qquad \therefore \ |A| = 1 \neq 0$$

$$\text{adj}\,A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \quad \Rightarrow \ A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} \qquad \therefore \ |B| = -10 \neq 0$$

$$\text{adj}\,B = \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \quad \Rightarrow \ B^{-1} = \frac{\text{adj}\,B}{|B|} = \frac{1}{-10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also,} \quad AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$$

$$|AB| = 936 - 946 = -10 \neq 0$$

$$\text{adj}\,(AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|} = \frac{1}{|AB|}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} +52 & -22 \\ -43 & +18 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$B^{-1}.A^{-1} = \frac{-1}{10}\begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \frac{-1}{10}\begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

Hence, $(AB)^{-1} = B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q10(ii)

$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $\therefore |A| = 1 \neq 0$ and adj $A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ $\therefore |A| = -1 \neq 0$ and adj$B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

$\therefore B^{-1} = \dfrac{\text{adj}B}{|B|} = \dfrac{1}{1}\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

Also, $A.B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 27 \end{bmatrix}$

$\qquad |AB| = 407 - 406 = 1 \neq 0$

and, $\text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

$\therefore \qquad (AB)^{-1} = \dfrac{1}{|AB|}.\text{adj}(AB)$

$\qquad\qquad = \dfrac{1}{1}\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Again, $B^{-1}.A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$\qquad\qquad = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$

Hence, $(AB)^{-1} = B^{-1}.A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \qquad \therefore |A| = 1 \neq 0 \text{ and adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 7 \\ 7 & 9 \end{bmatrix} \qquad \therefore |B| = -2 \neq 0 \text{ and adj } B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{\text{adj} B}{|B|} = \frac{1}{(-2)}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}.A^{-1}$

$$(AB)^{-1} = \frac{1}{(-2)}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(AB)^{-1} = -\frac{1}{2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} -47 & \dfrac{39}{2} \\ 41 & -17 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} \qquad \therefore |A| = 2 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

To show: $\qquad 2A^{-1} = 9I - A$

LHS: $\quad 2A^{-1} = 2.\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

RHS: $\quad 9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Therefore, $\qquad 2A^{-1} = 9I - A$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ $\therefore |A| = -6$ and adj $A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$ $\therefore A^{-1} = \dfrac{1}{-6}\begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

To show: $A - 3I = 2\left(I + 3A^{-1}\right)$

$\therefore$ LHS $= A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

RHS: $2\left(I + 3A^{-1}\right) = 2I + 2.3.A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 2.3.\dfrac{1}{6}\begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$

$\therefore$ $A - 3I = 2\left(I + 3A^{-1}\right)$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$A = \begin{bmatrix} a & b \\ c & \dfrac{1 + bc}{a} \end{bmatrix}$

$\Rightarrow |A| = (1 + bc) - bc = 1 \neq 0$

$A^{-1} = \dfrac{1}{|A|}.adj A = \dfrac{1}{1}\begin{bmatrix} \dfrac{1 + bc}{a} & -b \\ -c & a \end{bmatrix}$

Now $aA^{-1} = \left(a^2 + bc + 1\right)I - aA$

LHS : $aA^{-1} = a\begin{bmatrix} \dfrac{1 + bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

RHS : $\left(a^2 + bc + 1\right)I - aA = \begin{bmatrix} a^2 + bc + 1 & 0 \\ 0 & a^2 + bc + 1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1 + bc \end{bmatrix} = \begin{bmatrix} 1 + bc & -ab \\ -ac & a^2 \end{bmatrix}$

Since, $L.H.S = RHS$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q15

Here

$$(AB)^{-1} = B^{-1}A^{-1}$$

Now we need to find A^{-1}.

We have

$$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

So,

$$|A| = -5 + 4 = -1$$

Co-factors of A are

$$C_{11} = -1 \qquad C_{21} = 8 \qquad C_{31} = -12$$
$$C_{12} = 0 \qquad C_{22} = 1 \qquad C_{32} = -2$$
$$C_{13} = 1 \qquad C_{23} = -10 \qquad C_{33} = 15$$

Therefore,

$$adjA = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

Hence,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(i)

$$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |F(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$$

$$\begin{array}{lll} C_{11} = \cos\alpha & C_{21} = +\sin\alpha & C_{31} = 0 \\ C_{12} = -\sin\alpha & C_{22} = \cos\alpha & C_{32} = 0 \\ C_{13} = 0 & C_{23} = 0 & C_{33} = 1 \end{array}$$

$$[F(\alpha)]^{-1} = \frac{adj(F(\alpha))}{|F(\alpha)|} = \frac{1}{1}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad --- (1)$$

Now

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad --- (2)$$

From (1) & (2) $F(-\alpha) = [F(\alpha)]^{-1}$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q16(ii)

$$G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \Rightarrow |G(\beta)| = \cos^2\beta + \sin^2$$

$$\begin{array}{lll} C_{11} = \cos\beta & C_{21} = 0 & C_{31} = \sin\beta \\ C_{12} = +0 & C_{22} = 1 & C_{32} = 0 \\ C_{13} = \sin\beta & C_{23} = 0 & C_{33} = \cos\beta \end{array}$$

$$[G(\beta)]^{-1} = \frac{adj(G(\beta))}{|G(\beta)|} = \frac{1}{1}\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad --- (1)$$

Now

$$G(-\beta) = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad --- (2)$$

From (1) & (2)

$$[G(\beta)]^{-1} = G(-\beta)$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(iii)

We have to show that

$$\left[F\left(\alpha\right)G\left(\beta\right)\right]^{-1} = G\left(-\beta\right)F\left(-\alpha\right)$$

We have already shown that

$$G\left(-\beta\right) = \left[G\left(\beta\right)\right]^{-1}$$

and $F\left(-\beta\right) = \left[F\left(\beta\right)\right]^{-1}$

∴ $\text{LHS} = \left[F\left(\alpha\right)G\left(\beta\right)\right]^{-1}$

$= \left[G\left(\beta\right)\right]^{-1}\left[F\left(\alpha\right)\right]^{-1}$ $\left[\because \left(AB\right)^{-1} = B^{-1}A^{-1}\right]$

$= G\left(-\beta\right) \times F\left(-\alpha\right)$

$= \text{RHS}$

Adjoint and Inverse of Matrix Ex 7.1 Q17

We have $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

Hence $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Now, $A^2 - 4A + I = O$

$\Rightarrow A.A - 4A = -I$

Post multiplying both sides by A^{-1}, since $|A| \neq 0$

$AA\left(A^{-1}\right) - 4AA^{-1} = -IA^{-1}$

$\Rightarrow A\left(AA^{-1}\right) - 4I = -A^{-1}$

$\Rightarrow AI - 4I = -A^{-1}$

or $A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q18

$$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

Now $A^2 + 4A - 42I = 0$

For this $A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

Hence,

$$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence proved.

Now, $A^2 + 4A - 42I = 0$

$\Rightarrow \qquad A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$

$\Rightarrow \qquad IA + 4I - 42A^{-1} = 0$

$\Rightarrow \qquad 42A^{-1} = A + 4I$

$\Rightarrow \qquad A^{-1} = \dfrac{1}{42}[A + 4I] = \dfrac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\} = \dfrac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now,

$$A^2 - 5A + 7I = 0$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$A^2 - 5A + 7I = 0$$

Px-multipling with A^{-1}

$$A^{-1}A^2 - 5A^{-1}A + 7IA^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$A^{-1} = \frac{1}{7}[5I - A]$$

$$= \frac{1}{7}\left\{\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\therefore \ A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

Now $A^2 - xA + yI = 0$

$$\Rightarrow \qquad \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & 5x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \qquad 22 - 4x + y = 0$ or $4x - y = 22$

$\Rightarrow \qquad 18 - 2x = 0$ or $x = 9$

$\therefore \qquad y = 14$

Again,

$A^2 - 9A + 14I = 0$

$\Rightarrow \qquad 9A = A^2 + 14I = 0$

$\Rightarrow \qquad 9A^{-1}A = A^{-1}.A.A + 14A^{-1}$

$\Rightarrow \qquad 9I = IA + 14A^{-1}$

$\Rightarrow \qquad A^{-1} = \dfrac{1}{14}\{9I - A\} = \dfrac{1}{14}\left\{\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\right\} = \dfrac{1}{14}\left\{\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\right\}$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

If $A^2 = \lambda A - 2I$

$$\lambda A = A^2 + 2I$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$3\lambda = 3$$
$$\lambda = 1 \qquad\qquad \text{Ans } \lambda = 1$$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$
$$A = I - 2A^{-1}$$
$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q22

We have $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$

To prove: $A^2 - 3A - 7 = 0$

Now, $A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$

So, $A^2 - 3A - 7 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $A^2 - 3A - 7 = 0$

$\Rightarrow \quad AA^{-1}.A - 3A^{-1}.A - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad A - 3I - 7A^{-1} = 0$

$\Rightarrow \quad 7A^{-1} = A - 3I$

$\Rightarrow \quad A^{-1} = \frac{1}{7}\left(\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ \frac{-1}{7} & \frac{-5}{7} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q23

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + I = 0$. Thus, find A^{-1}.

$A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix}$

Now $A^2 - 12A + I = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix} - 12\begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 12A + I = 0$

$\Rightarrow \quad A - 12I + A^{-1} = 0$

$\Rightarrow \quad A^{-1} = 12I - A = \left\{\begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}\right\} = \left\{\begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix}\right\}$

Adjoint and Inverse of Matrix Ex 7.1 Q24

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Thus, $A^3 - 6A^2 + 5A + 11I = O.$

Now,

$A^3 - 6A^2 + 5A + 11I = O$

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 5AA^{-1} + 11IA^{-1} = 0$ $\qquad$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0\right]$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = -11(IA^{-1})$

$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$

$\Rightarrow A^{-1} = -\dfrac{1}{11}\left(A^2 - 6A + 5I\right)$ $\qquad$ ...(1)

Now,

$A^2 - 6A + 5I$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = -\frac{1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q25

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$$

Now $A^3 - A^2 - 3A - I_3 = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ $A^3 - A^2 - 3A - I_3 = 0$

⇒ $A^2 - A - 3I - A^{-1} = 0$

⇒ $A^{-1} = A^2 - A - 3I = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now,

$A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4I = O$

Now,

$A^3 - 6A^2 + 9A - 4I = O$

$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 9AA^{-1} - 4IA^{-1} = O$ $\left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \ne 0\right]$

$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 9(AA^{-1}) = 4(IA^{-1})$

$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$$\Rightarrow A^{-1} = \frac{1}{4}\left(A^2 - 6A + 9I\right) \qquad \ldots(1)$$

$A^2 - 6A + 9I$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q27

$$A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} \text{ and } A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$|A| = \frac{1}{9}\left[-8(16+56) - 1(9) + 4(-36)\right] = -81$$

$C_{11} = 72$	$C_{21} = -36$	$C_{31} = -9$
$C_{12} = -9$	$C_{22} = -36$	$C_{32} = +72$
$C_{13} = -36$	$C_{23} = -63$	$C_{33} = -36$

$$A^{-1} = \frac{1}{-81}\begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = A^T$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q28

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } |A| = 3 + 6 - 8 = 1$$

$C_{11} = 1$ $C_{21} = -1$ $C_{31} = 0$

$C_{12} = -2$ $C_{22} = 3$ $C_{32} = -4$

$C_{13} = -2$ $C_{23} = +3$ $C_{33} = -3$

$$A^{-1} = \frac{1}{|A|} \, adjA = \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad \text{---(1)}$$

Now

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad \text{---(2)}$$

From (1) and (2)

$A^{-1} = A^3$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q29

$$A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Expanding using 1st row, we get

$$|A| = -1 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 2 \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 0$$

$$= -1(0-1) - 2(0) + 0$$

$$= 1 - 0 + 0$$

$$|A| = 1$$

$$A^2 = AA = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Cofactors of A are:

$$C_{11} = -1 \qquad C_{21} = 0 \qquad C_{31} = 2$$
$$C_{12} = 0 \qquad C_{22} = 0 \qquad C_{32} = 1$$
$$C_{13} = -1 \qquad C_{23} = +1 \qquad C_{33} = 1$$

$$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Now, $\quad A^{-1} = \dfrac{1}{|A|} \text{adj} A$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = A^2$$

Hence, $A^2 = A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q30

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

So, $AX = B$

or $X = A^{-1}B$ $---(i)$

$\quad |A| = 1 \neq 0$

Cofactors of A are:

$\qquad C_{11} = 1 \qquad\qquad C_{12} = -1$

$\qquad C_{21} = -4 \qquad\quad C_{22} = 5$

$\therefore \qquad adj\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{T}$

$\qquad\qquad = \begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^{T}$

$\qquad\qquad = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

Now, $\quad A^{-1} = \dfrac{1}{|A|}.adj\,A$

$\qquad A^{-1} = \dfrac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$

So from (i)

$\qquad\qquad X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

$\qquad\qquad X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$

Ans.

Adjoint and Inverse of Matrix Ex 7.1 Q31

$$X \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$$

Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

So, $XB = C$

$XBB^{-1} = CB^{-1}$

$XI = C.B^{-1}$

$X = C.B^{-1}$ $---(i)$

Now, $|B| = -7 \neq 0$

Cofactors of B are:

$C_{11} = -2$ $C_{12} = 1$

$C_{21} = -3$ $C_{22} = 5$

$\therefore$ $\text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^{T}$

$= \begin{bmatrix} -2 & 1 \\ -3 & 5 \end{bmatrix}^{T}$

$= \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \dfrac{1}{|B|}.\text{adj}(B)$

$= \dfrac{1}{(-7)} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} \dfrac{-1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

Now from (i)

$X = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \cdot \dfrac{1}{7} \cdot \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \dfrac{7}{7} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$

$= \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q32

Let $\quad A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$

$\quad\quad B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$

$\quad\quad C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

Then the given equation becomes

$\quad\quad A \times B = C$

$\Rightarrow \quad X = A^{-1}CB^{-1}$

Now $\quad |A| = 35 - 14 = 21$

$\quad\quad |B| = -1 + 2 = 1$

$\quad\quad A^{-1} = \dfrac{adj\,(A)}{|A|} = \dfrac{1}{21}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$\quad\quad B^{-1} = \dfrac{adj\,(B)}{|B|} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$\therefore \quad X = A^{-1}CB^{-1} = \dfrac{1}{21}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$\quad\quad = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q33

Let $\quad A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$\quad\quad B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

Then the given equation can be written as

$\quad\quad A \times B = I$

$\Rightarrow \quad X = A^{-1}B^{-1}$

Now $\quad |A| = 6 - 5 = 1$

$\quad\quad |B| = 10 - 9 = 1$

$\quad\quad A^{-1} = \dfrac{adj\,(A)}{|A|} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$\quad\quad B^{-1} = \dfrac{adj\,(B)}{|B|} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

$\therefore \quad X = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q34

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, $A^2 + 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Also, $A^2 - 4A + 5I = O$

$$A^{-1}.AA - 4A^{-1}.A - 5A^{-1}.I = 0$$

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}\left[A - 4I\right]$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\left(\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\right) = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q35

$$|A \text{ adj } A| = |A|^n$$

LHS $= |A \text{ Adj } A|$

$= |A|.|\text{Adj } A|$

$= |A|.|A|^{n-1}$

$= |A|^{n-1+1}$

$= |A|^n$

$= $ RHS

Adjoint and Inverse of Matrix Ex 7.1 Q36

Here

$$B^{-1} = \frac{1}{|B|} adj\,|B|$$

Co-factors of B are

$$
\begin{array}{lll}
C_{11} = 3 & C_{21} = 2 & C_{31} = 6 \\
C_{12} = 1 & C_{22} = 1 & C_{32} = 2 \\
C_{13} = 2 & C_{33} = 2 & C_{33} = 5
\end{array}
$$

Therefore,

$$
adj\,B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T
$$

$$
= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T
$$

$$
= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}
$$

Therefore,

$$
B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}
$$

Now,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$
= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}
$$

$$
= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 61 & -24 & 22 \end{bmatrix}
$$

Adjoint and Inverse of Matrix Ex 7.1 Q37

Let $B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$

So, B is invertible matrix.

$B_{11} = (-1)^{1+1}(-9) = -9; \quad B_{12} = (-1)^{1+2}(-8) = 8; \quad B_{13} = (-1)^{1+3}(-5) = -5$

$B_{21} = (-1)^{2+1}(8) = -8; \quad B_{22} = (-1)^{2+2}(7) = 7; \quad B_{23} = (-1)^{2+3}(4) = -4$

$B_{31} = (-1)^{3+1}(-2) = -2; \quad B_{32} = (-1)^{3+2}(-2) = 2; \quad B_{33} = (-1)^{3+3}(-1) = -1$

$\text{adj } B = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|} \text{adj} B$

$\Rightarrow B^{-1} = \frac{1}{1} \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

$\Rightarrow \left(A^T\right)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q38

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -1(1-4) + 2(2+4) - 2(-4-2) = 3 + 12 + 12 = 27$$

$A_{11} = (-1)^{1+1}(-3) = -3$; $A_{12} = (-1)^{1+2}(6) = -6$; $A_{13} = (-1)^{1+3}(-6) = -6$

$A_{21} = (-1)^{2+1}(-6) = 6$; $A_{22} = (-1)^{2+2}(3) = 3$; $A_{23} = (-1)^{2+3}(6) = -6$

$A_{31} = (-1)^{3+1}(6) = 6$; $A_{32} = (-1)^{3+2}(6) = -6$; $A_{33} = (-1)^{3+3}(3) = 3$

$$\text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A(\text{adj}A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = 27\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A(\text{adj}A) = |A|I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q39

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 - 1(0-1) + 1(1-0) = 0 + 1 + 1 = 2 \neq 0$$

So, A is invertible matrix.

$$A_{11} = (-1)^{1+1}(-1) = -1; \quad A_{12} = (-1)^{1+2}(-1) = 1; \quad A_{13} = (-1)^{1+3}(1) = 1$$
$$A_{21} = (-1)^{2+1}(-1) = 1; \quad A_{22} = (-1)^{2+2}(-1) = -1; \quad A_{23} = (-1)^{2+3}(-1) = 1$$
$$A_{31} = (-1)^{3+1}(1) = 1; \quad A_{32} = (-1)^{3+2}(-1) = 1; \quad A_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \dots\dots\dots(i)$$

$$A^2 - 3I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \dots\dots\dots\dots(ii)$$

From (i) and (ii) we can see that,

$$A^{-1} = \frac{1}{2}(A^2 - 3I)$$