

RD Sharma
Solutions
Class 12 Maths
Chapter 9
Ex 9.1

We have to check the continuity of function at $x = 0$.

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{-h}{|-h|} = \lim_{h \rightarrow 0} \frac{-h}{h} = -1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h}{|h|} = 1$$

Thus, LHL \neq R.H.L

So, the given function is discontinuous and the discontinuity is of first kind.

Continuity Ex 9.1 Q2

We have, to check the continuity at $x = 3$.

$$\text{L.H.L} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3-h) = \lim_{h \rightarrow 0} \frac{(3-h)^2 - (3-h) - 6}{(3-h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 - 5h}{-h} = \lim_{h \rightarrow 0} -h + 5 = 5$$

$$\text{R.H.L} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} \frac{(3+h)^2 - (3+h) - 6}{(3+h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 + 5h}{h} = \lim_{h \rightarrow 0} h + 5 = 5$$

$$f(3) = 5$$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 5$

So, The function is continuous at $x = 3$

Continuity Ex 9.1 Q3

We have, to check the continuity of the function at $x = 3$.

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3-h) = \lim_{h \rightarrow 0} \frac{(3-h)^2 - 9}{(3-h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 - 6h}{-h} = \lim_{h \rightarrow 0} -h + 6 = 6$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{(3+h) - 3} = \lim_{h \rightarrow 0} \frac{h^2 + 6h}{h} = \lim_{h \rightarrow 0} h + 6 = 6$$

$$f(3) = 6$$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 6$

So, the given function is continuous at $x = 3$.

Continuity Ex 9.1 Q4

We want, to check the continuity of the function at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 - 2h}{-h} = \lim_{h \rightarrow 0} -h + 2 = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{(1+h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 + 2h}{h} = \lim_{h \rightarrow 0} h + 2 = 2$$

$$f(1) = 2$$

we find that $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

Continuity Ex 9.1 Q5

We have, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 3(-h)}{-h} = \lim_{h \rightarrow 0} \frac{\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\sin 3h}{h} = 3$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

\Rightarrow Function is discontinuous at $x = 0$. It is removable discontinuity.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} e^{1/h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} e^{1/h} = e^{\infty} = \infty$$

So, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of Ist kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos(-h)}{(-h)^2} \\ &= \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2} \quad [\because \cos(-\theta) = \cos \theta] \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h^2} \quad \left[\because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \right] \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{\sin \frac{h}{2}}{h} \right)^2 = 2 \times \frac{1}{4} = \frac{1}{2} \end{aligned}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2} = \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h^2} = \lim_{h \rightarrow 0} 2 \left(\frac{\sin \frac{h}{2}}{h} \right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuity.

Continuity Ex 9.1 Q8

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{-h - |-h|}{2} = \lim_{h \rightarrow 0} \frac{-h - h}{2} = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h - (|h|)}{2} = 0$$

$$f(0) = 2$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuity.

Continuity Ex 9.1 Q9

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a - h) = \lim_{h \rightarrow 0} \frac{|a - h - a|}{a - h - a} = \lim_{h \rightarrow 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a + h) = \lim_{h \rightarrow 0} \frac{|a + h - a|}{a + h - a} = \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, function is discontinuous at $x = a$. And the discontinuity is of first kind.

Continuity Ex 9.1 Q10(i)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} |-h| \cos\left(\frac{1}{-h}\right) = \lim_{h \rightarrow 0} h \cos\left(\frac{1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} |h| \cos\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \rightarrow a^-} f(x) = \lim_{h \rightarrow 0} f(a - h) = \lim_{h \rightarrow 0} (a - h - a) \sin\left(\frac{1}{a - h - a}\right) = \lim_{h \rightarrow 0} -h \sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \rightarrow a^+} f(x) = \lim_{h \rightarrow 0} f(a + h) = \lim_{h \rightarrow 0} (a + h - a) \sin\left(\frac{1}{a + h - a}\right) = \lim_{h \rightarrow 0} h \sin\left(\frac{1}{h}\right) = 0$$

$$f(a) = 0$$

Thus, $\text{LHL} = \text{RHL} = f(a) = 0$

Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{\log(1 + 2(-h))} = \lim_{h \rightarrow 0} \frac{e^{-h} - 1}{\log(1 - 2h)} = DNE$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{e^h - 1}{\log(1 + 2h)} = DNE$$

Thus, Both LHL and RHL do not exist

\therefore Function is discontinuous and the discontinuity is of IInd kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{1 - (1-h)^n}{1 - (1-h)} = \lim_{h \rightarrow 0} \frac{1 - \left[1 - nh + \frac{n(n-1)}{2!}h^2 + \dots\right]}{h} \\ &= \lim_{h \rightarrow 0} n - \frac{n(n-1)}{2!}h + \dots \\ &= n \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{1 - (1+h)^n}{1 - (1+h)} = \lim_{h \rightarrow 0} \frac{1 - \left[1 + nh + \frac{n(n-1)}{2!}h^2 + \dots\right]}{-h} \\ &= \lim_{h \rightarrow 0} n + \frac{n(n-1)}{2!}h + \dots \\ &= n \end{aligned}$$

$$f(1) = n - 1$$

Thus, $\text{LHL} = \text{RHL} \neq f(1)$

Hence, function is discontinuous at $x = 1$

This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{|(1-h)^2 - 1|}{(1-h) - 1} = \lim_{h \rightarrow 0} \frac{|h^2 - 2h|}{-h} = \lim_{h \rightarrow 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{|(1+h)^2 - 1|}{1+h-1} = \lim_{h \rightarrow 0} \frac{h^2 + 2h}{h} = 2$$

$$f(1) = 2$$

$$\therefore \text{LHL} = \text{RHL} = f(1) = 2$$

Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{2(-h) + (-h)^2}{-h} = \lim_{h \rightarrow 0} \frac{2h + h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{2 \times |h| + h^2}{h} = 2$$

Thus, $\text{LHL} \neq \text{RHL}$

Function is not continuous at $x = 0$

This is discontinuity of Ist kind.

Continuity Ex 9.1 Q11

We want to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} 1 + (1-h)^2 = \lim_{h \rightarrow 0} 1 + 1 - 2h + h^2 = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} 2 - (1+h) = 1$$

LHL \neq RHL

Hence, the function is discontinuous at $x = 1$

This is discontinuity of Ist kind.

Continuity Ex 9.1 Q12

We want to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin(3 \times (-h))}{\tan(2 \times (-h))} = \lim_{h \rightarrow 0} \frac{-\sin 3h}{-\tan 2h} = \lim_{h \rightarrow 0} \frac{\frac{\sin 3h}{3h} \times 3h}{\frac{\tan 2h}{2h} \times 2h} = \frac{3}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\log(1+3h)}{e^{2h}-1} = \lim_{h \rightarrow 0} \frac{\frac{\log(1+3h)}{3h} \times 3h}{\frac{e^{2h}-1}{2h} \times 2h} = \frac{3}{2}$$

$$f(0) = \frac{3}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = \frac{3}{2}$$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q13

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 2(-h) - |-h| = \lim_{h \rightarrow 0} -2h - h = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 2h - |h| = 0$$

$$f(0) = 0$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = 0$$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q14

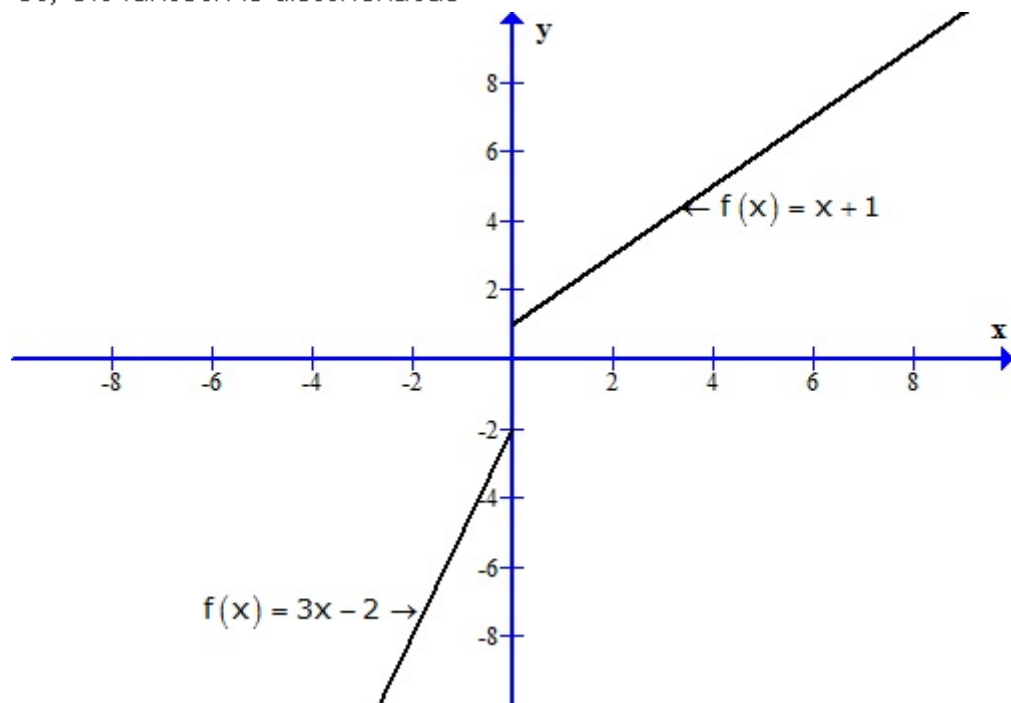
We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 3(-h) - 2 = \lim_{h \rightarrow 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} h + 1 = 1 \neq 0$$

LHL \neq RHL

So, the function is discontinuous



Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} (-h) = 0$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} h = 0$$

$$f(0) = 1$$

Thus, LHL = RHL \neq $f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

$$\text{LHL} = \lim_{x \rightarrow \frac{1}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{1}{2} - h\right) = \lim_{h \rightarrow 0} \frac{1}{2} - h = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \left(\frac{1}{2}\right)^+} f(x) = \lim_{h \rightarrow 0} f\left(\frac{1}{2} + h\right) = \lim_{h \rightarrow 0} 1 - \left(\frac{1}{2} + h\right) = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f\left(\frac{1}{2}\right) = \frac{1}{2}$$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} 2(-h) - 1 = -1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} 2h + 1 = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of 1st kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$

$$\text{LHL} = \text{RHL} = f(1) \dots (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} \frac{(1 - h)^2 - 1}{(1 - h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 - 2h}{-h} = 2$$

$$f(1) = k$$

$$\text{From (1), LHL} = f(1)$$

$$\therefore 2 = k$$

Continuity Ex 9.1 Q19

We have that the function is continuous at $x = 1$

$$\therefore \text{LHL} = \text{RHL} = f(1) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} \frac{(1 - h)^2 - 3(1 - h) + 2}{(1 - h) - 1} = \lim_{h \rightarrow 0} \frac{h^2 + h}{-h} = \lim_{h \rightarrow 0} -h - 1 = -1$$

$$f(1) = k$$

From (1), we get,

$$k = -1$$

Continuity Ex 9.1 Q20

We know that a function is continuous at 0 if

$$\text{LHL} = \text{RHL} = f(0) \quad \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 5(-h)}{3(-h)} = \lim_{h \rightarrow 0} \frac{-\sin 5h}{-3h} = \lim_{h \rightarrow 0} \frac{\sin 5h}{5h} \times \frac{5h}{3h} = \frac{5}{3}$$

$$f(0) = k$$

Thus, from (1),

$$k = \frac{5}{3}$$

Continuity Ex 9.1 Q21

$$\text{The given function is } f(x) = \begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} (kx^2) = \lim_{x \rightarrow 2^+} (3) = 4k$$

$$\Rightarrow k \times 2^2 = 3 = 4k$$

$$\Rightarrow 4k = 3 = 4k$$

$$\Rightarrow 4k = 3$$

$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of k is $\frac{3}{4}$.

Continuity Ex 9.1 Q22

We have given that the function is continuous at $x = 0$

$$\text{So, LHL} = \text{RHL} = f(0) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 2(-h)}{5(-h)} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-5h} = \lim_{h \rightarrow 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$$

$$f(0) = k$$

$$\text{Using (1), } k = \frac{2}{5}$$

Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

$$\text{LHL} = \text{RHL} = f(2) \dots (1)$$

Now,

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} a(2-h) + 5 = 2a + 5$$

$$f(2) = 2a + 5$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} 2+h-1 = 1$$

\therefore Using (1),

$$2a + 5 = 1 \Rightarrow a = -2$$

Continuity Ex 9.1 Q24

We have, at $x = 0$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{-h}{|-h| + 2(-h)^2} = \lim_{h \rightarrow 0} \frac{-h}{h + 2h^2} = \lim_{h \rightarrow 0} \frac{-1}{1 + 2h} = -1$$

$$f(0) = k$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{h}{|h| + 2h^2} = \lim_{h \rightarrow 0} \frac{1}{1 + 2h} = 1$$

Since, $\text{LHL} \neq \text{RHL}$, function will remain discontinuous at $x = 0$, regardless the choice of k .

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, $\text{L.H.Limit} = \text{R.H.Limit}$.

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = c$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin(a+h) - \sin h}{-h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(a+h)h}{h} + \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= a+1 + 1 = a+2 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h+bh^2} + \sqrt{h}}{\sqrt{h+bh^2} + \sqrt{h}} \\ &= \lim_{h \rightarrow 0} \frac{h+bh^2-h}{bh^{\frac{3}{2}}(\sqrt{h+bh^2} + \sqrt{h})} = \lim_{h \rightarrow 0} \frac{bh^2}{bh^{\frac{3}{2}}(\sqrt{1+bh} + 1)} = \frac{1}{2} \end{aligned}$$

\therefore from (1),

$$a+2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$$

$$c = \frac{1}{2} \text{ and}$$

$$b \in \mathbb{R} - \{0\}$$

$$\text{Hence, } a = \frac{-3}{2}, b \in \mathbb{R} - \{0\}, c = \frac{1}{2}$$

Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos k(-h)}{-h \sin(-h)} = \lim_{h \rightarrow 0} \frac{1 - \cos kh}{+h \sin h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{kh}{2}}{h \cdot 2 \sin \frac{h}{2} \cdot \cos \frac{h}{2}} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \times \frac{\frac{k^2 h^2}{4}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \times \frac{h}{2}} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}} \\ &= \frac{k^2}{2} \end{aligned}$$

\therefore Using (1) we get,

$$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$$\therefore \text{LHL} = \text{RHL} = f(4) \dots (1)$$

$$f(4) = a + b \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow 4^-} f(x) = \lim_{h \rightarrow 0} f(4-h) = \lim_{h \rightarrow 0} \frac{(4-h) - 4}{|(4-h) - 4|} + a = \lim_{h \rightarrow 0} \frac{-h}{h} + a = a - 1 \quad \dots (B)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4+h) = \lim_{h \rightarrow 0} \frac{(4+h) - 4}{|(4+h) - 4|} + b = \lim_{h \rightarrow 0} \frac{h}{h} + b = b + 1 \quad \dots (C)$$

\therefore from (1)

$$a - 1 = b + 1 \Rightarrow a - b = 2 \quad \dots (D)$$

from (A) and (B)

$$a + b = a - 1 \Rightarrow b = -1$$

from (A) and (C)

$$a + b = b + 1 \Rightarrow a = 1$$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k$$

$$\text{LHL} = \lim_{x \rightarrow 0} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sin 2(0-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-h} = 2$$

\therefore using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\log\left(1 - \frac{h}{a}\right) - \log\left(1 + \frac{h}{b}\right)}{(-h)} = \lim_{h \rightarrow 0} \frac{\log\left(1 + \left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right) \times a} + \frac{\log\left(1 + \frac{h}{b}\right)}{h} \\ &= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \end{aligned}$$

from (1),

$$f(0) = \frac{a+b}{ab}$$

Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$$\therefore \text{LHL} = \text{RHL} = f(2) \quad \dots (1)$$

Now,

$$f(2) = k \quad \dots (A)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} \frac{2^{(2-h)+2} - 16}{4^{(2-h)} - 16} = \lim_{h \rightarrow 0} \frac{2^{4-h} - 16}{4^{2-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{2^4 \cdot 2^{-h} - 16}{4^2 \cdot 4^{-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{16 \cdot 2^{-h} - 16}{16 \cdot 4^{-h} - 16} \\ &= \lim_{h \rightarrow 0} \frac{16(2^{-h} - 1)}{16(4^{-h} - 1)} \\ &= \lim_{h \rightarrow 0} \frac{2^{-h} - 1}{(2^{-h})^2 - 1^2} \quad \left[\because 2^{-2h} = (2^{-h})^2 = 4^{-h} \right] \\ &= \lim_{h \rightarrow 0} \frac{2^{-h} - 1}{(2^{-h} - 1)(2^{-h} + 1)} = \frac{1}{2} \quad \dots (B) \end{aligned}$$

\therefore Using (1) from (A) & (B)

$$k = \frac{1}{2}$$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

$$\text{LHL} = \text{RHL} = \text{value of the function at } x = \pi \dots (1)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow \pi^-} f(x) = \lim_{h \rightarrow 0} f(\pi - h) = \lim_{h \rightarrow 0} \frac{1 - \cos 7(\pi - h - \pi)}{5((\pi - h) - \pi)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 7h}{5h^2} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{7}{2} h}{5h^2} \\ &= \lim_{h \rightarrow 0} \frac{2}{5} \left(\frac{\sin \frac{7}{2} h}{\frac{7}{2} h} \right)^2 \times \left(\frac{7}{2} \right)^2 \\ &= \frac{2}{5} \times \frac{49}{4} = \frac{49}{10} \dots (B) \end{aligned}$$

Thus, using (1) we get,

$$f(\pi) = \frac{49}{10}$$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{2(-h) + 3 \sin(-h)}{3(-h) + 2 \sin(-h)} = \lim_{h \rightarrow 0} \frac{-2h - 3 \sin h}{-3h - 2 \sin h} \\ &= \lim_{h \rightarrow 0} \frac{2h + 3 \sin h}{3h + 2 \sin h} \\ &= \lim_{h \rightarrow 0} \frac{2 + 3 \frac{\sin h}{h}}{3 + 2 \frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \quad \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \end{aligned}$$

Using (1) we get,

$$f(0) = 1$$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos 4(-h)}{8(-h)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 4h}{8h^2} = \lim_{h \rightarrow 0} \frac{2 \sin^2 2h}{8h^2} = \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h} \right)^2 = 1$$

Thus, using (1) we get,

$$k = 1$$

Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = 8 \dots (A)$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos 2k(-h)}{(-h)^2} = \lim_{h \rightarrow 0} \frac{1 - \cos 2kh}{h^2} = \lim_{h \rightarrow 0} \frac{2 \sin^2 kh}{h^2} \\ &= \lim_{h \rightarrow 0} 2 \left(\frac{\sin kh}{kh} \right)^2 \cdot k^2 \\ &= 2k^2 \end{aligned}$$

Thus, using (1) we get,

$$2k^2 = 8 \Rightarrow k^2 = 4 \Rightarrow k = \pm 2$$

Hence, $k = \pm 2$

Let $x - 1 = y$

$$\Rightarrow x = y + 1$$

Thus,

$$\begin{aligned} \lim_{x \rightarrow 1} (x-1) \tan \frac{\pi x}{2} &= \lim_{y \rightarrow 0} y \tan \frac{\pi(y+1)}{2} \\ &= \lim_{y \rightarrow 0} y \tan \left(\frac{\pi y}{2} + \frac{\pi}{2} \right) \\ &= - \lim_{y \rightarrow 0} y \cot \frac{\pi y}{2} \\ &= - \lim_{y \rightarrow 0} y \frac{\cos \frac{\pi y}{2}}{\sin \frac{\pi y}{2}} \\ &= - \lim_{y \rightarrow 0} y \frac{\cos \frac{\pi y}{2}}{\left(\sin \frac{\pi y}{2} \right) \frac{\pi}{2}} \\ &= - \lim_{y \rightarrow 0} \frac{\cos \frac{\pi y}{2}}{\frac{\left(\sin \frac{\pi y}{2} \right) \frac{\pi}{2}}{y}} \\ &= - \lim_{y \rightarrow 0} \frac{2}{\pi} \frac{\cos \frac{\pi y}{2}}{\left(\sin \frac{\pi y}{2} \right)} \\ &= - \frac{2}{\pi} \lim_{y \rightarrow 0} \cos \frac{\pi y}{2} \\ &= - \frac{2}{\pi} \end{aligned}$$

Since the function is continuous, L.H.Limit = R.H.Limit

$$\text{Thus, } k = -\frac{2}{\pi}$$

Since the function is continuous at every point, therefore

$$LHL = RHL = f(0)$$

Now

$$\begin{aligned} f(0) &= \cos 0 \\ &= 1 \end{aligned}$$

Again

$$\begin{aligned} LHL &= \lim_{x \rightarrow 0} k(x^2 - 2x) \\ &= \lim_{h \rightarrow 0^+} k(h^2 - 2h) \\ &= 0 \end{aligned}$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL = RHL = f(\pi)$$

Now

$$f(\pi) = k\pi + 1$$

Again

$$\begin{aligned} RHL &= \lim_{x \rightarrow \pi^+} \cos x \\ &= \lim_{h \rightarrow 0^+} \cos(\pi - h) \\ &= -\lim_{h \rightarrow 0^+} \cos h \\ &= -1 \end{aligned}$$

Therefore we can write

$$k\pi + 1 = -1$$

$$k = -\frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.

$$\therefore LHL = RHL = f(5) \dots (1)$$

$$f(5) = 5k + 1$$

$$LHL = \lim_{x \rightarrow 5^+} f(x) = \lim_{h \rightarrow 0} f(5+h) = \lim_{h \rightarrow 0} 3(5+h) - 5 = 10$$

Thus, using (1), we get,

$$5k + 1 = 10$$

$$5k = 9$$

$$k = \frac{9}{5}$$

We know that the function will be continuous at $x = 5$. if

$$\text{LHL} = \text{RHL} = f(5) \dots (1)$$

$$f(5) = k$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} \frac{(5-h)^2 - 25}{(5-h) - 5} = \lim_{h \rightarrow 0} \frac{h^2 - 10h}{-h} = \lim_{h \rightarrow 0} -h + 10 = 10$$

Thus, using (1), we get,

$$k = 10$$

We know that a function will be continuous at $x = 1$. if

$$\text{LHL} = \text{RHL} = f(1) \dots (1)$$

$$f(1) = k \cdot 1^2 = k$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} 4 = 4$$

Thus, using (1), we get,

$$k = 4$$

We know that a function will be continuous at $x = 0$. if

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k(0+2) = 2k$$

$$\text{LHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 3(h) + 1 = 1$$

Thus, using (1), we get,

$$2k = 1$$

$$k = \frac{1}{2}$$

Continuity Ex 9.1 Q37

It is given that the function is continuous at $x = 3$ and at $x = 5$

$$\therefore \text{LHL} = \text{RHL} = f(3) \dots (1) \text{ and}$$

$$\text{LHL} = \text{RHL} = f(5) \dots (2)$$

Now,

$$f(3) = 1$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} a(3+h) + b = 3a + b$$

Thus, using (1), we get,

$$3a + b = 1 \dots (3)$$

$$f(5) = 7$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} a(5-h) + b = 5a + b$$

Thus, using (2), we get

$$5a + b = 7 \dots (4)$$

Now, solving (3) and (4) we get,

$$a = 3 \text{ and } b = -8$$

Continuity Ex 9.1 Q38

We want to discuss the continuity of the function at $x = 1$

We need to prove that

$$\text{LHL} = \text{RHL} = f(1)$$

$$f(1) = \frac{1^2}{2} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2}{2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f(1) = \frac{1}{2}$$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q39

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,

$$f(0) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} |-h| + |-h-1| = 1.$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} |h| + |h-1| = 1$$

\therefore LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$$f(1) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} |1-h| + |1-h-1| = 1$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} |1+h| + |1+h-1| = 1$$

\therefore LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$$f(-1) = |-1-1| + |-1+1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow -1^-} f(x) = \lim_{h \rightarrow 0} f(-1-h) = \lim_{h \rightarrow 0} |-1-h-1| + |-1-h+1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow -1^+} f(x) = \lim_{h \rightarrow 0} f(-1+h) = \lim_{h \rightarrow 0} |-1+h-1| + |-1+h+1| = 2$$

Thus, LHL = RHL = $f(-1) = 2$

Hence, function is continuous at $x = -1$

For $x = 1$

$$f(1) = |1-1| + |1+1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} |1-h-1| + |1-h+1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} |1+h-1| + |1+h+1| = 2$$

Thus, LHL = RHL = $f(1) = 2$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since $f(x)$ is continuous at $x = 0$, $L.H.Limit = R.H.Limit$.

Thus, we have

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} a \sin \frac{\pi}{2}(x+1) = \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left(\frac{1}{\cos x} - 1 \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left(\frac{1 - \cos x}{\cos x} \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{\cos x} \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1+1}$$

$$\Rightarrow a = \frac{1}{2}$$

Continuity Ex 9.1 Q41

It is given that function is continuous at $x = 0$. then,

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

Now,

$$f(0) = 2 \cdot 0 + k = k$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} 2(-h)^2 + k = k$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} 2(h^2) + k = k$$

Thus, the function will be continuous for any $k \in \mathbb{R}$.

Continuity Ex 9.1 Q42

The given function f is $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$

If f is continuous at $x = 0$, then

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} \lambda(x^2 - 2x) = \lim_{x \rightarrow 0^+} (4x + 1) = \lambda(0^2 - 2 \times 0)$$

$$\Rightarrow \lambda(0^2 - 2 \times 0) = 4 \times 0 + 1 = 0$$

$$\Rightarrow 0 = 1 = 0, \text{ which is not possible}$$

Therefore, there is no value of λ for which f is continuous at $x = 0$

At $x = 1$,

$$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$$

$$\lim_{x \rightarrow 1} (4x + 1) = 4 \times 1 + 1 = 5$$

$$\therefore \lim_{x \rightarrow 1} f(x) = f(1)$$

Therefore, for any values of λ , f is continuous at $x = 1$

Continuity Ex 9.1 Q43

The function will be continuous at $x = 2$

if $\text{LHL} = \text{RHL} = f(2) \dots (1)$

Now,

$$f(2) = k$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} 2(2-h) + 1 = 5.$$

Thus, using (1) we get,

$$k = 5$$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$$\therefore \text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \dots (1)$$

Now,

$$f\left(\frac{\pi}{2}\right) = a$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \rightarrow 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \rightarrow 0} \frac{1 - \cos^3 h}{3\sin^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{(1 - \cosh)(1 + \cos^2 h + \cosh)}{3\sin^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{2\sin^2 \frac{h}{2} (1 + \cos^2 h + \cosh)}{3\sin^2 h}$$

$$= \lim_{h \rightarrow 0} \frac{2\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4} \cdot (1 + \cos^2 h + \cosh)}{3\left(\frac{\sin h}{h}\right)^2 \cdot h^2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \cdot \frac{1}{4} (1 + \cos^2 h + \cosh)}{3} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \rightarrow 0} \frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \rightarrow 0} \frac{b(1 - \cosh)}{(\pi - \pi - 2h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{b \cdot 2\sin^2 \frac{h}{2}}{(2h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{b}{2} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \lim_{h \rightarrow 0} \frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$$a = \frac{1}{2}$$

And

$$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$$

Thus, $a = \frac{1}{2}$ and $b = 4$

Continuity Ex 9.1 Q45

It is given that the function is continuous at $x = 0$, then

$$LHL = RHL = f(0) \dots (1)$$

Now,

$$f(0) = k$$

$$RHL = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} \frac{h}{|h|} = 1 \dots (B)$$

Thus, using (1) we get,

$$k = 1$$

Continuity Ex 9.1 Q46

Since the function is continuous at $x = 3$, therefore

$$LHL = RHL = f(3)$$

Now

$$\begin{aligned} RHL &= \lim_{x \rightarrow 3^+} f(x) \\ &= \lim_{h \rightarrow 0} f(3 + h) \\ &= \lim_{h \rightarrow 0} b(3 + h) + 3 \\ &= \lim_{h \rightarrow 0} 3b + 3h + 3 \\ &= 3b + 3 \end{aligned}$$

Again

$$\begin{aligned} f(3) &= a(3) + 1 \\ &= 3a + 1 \end{aligned}$$

Thus we can write

$$f(3) = RHL$$

$$3a + 1 = 3b + 3$$

$$3a - 3b = 2$$