RD Sharma Solutions Class 12 Maths Chapter 11 Ex 11.4

Chapter: Differentiation

Exercise: 11.4

Page Number: 11.74

1.
Solution :

$We\, have, xy = c^2$
Differentiating with respect to x,we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$$\Rightarrow x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad [Using\ product\ rule]$$

$$\Rightarrow x\frac{dy}{dx} + y = 0$$

$$\Rightarrow x\frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

2.
Solution :

$We\, have, y^3 - 3xy^2 = x^3 + 3x^2 y$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2 y)$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] \qquad [Using\ product\ rule]$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow 3\frac{dy}{dx}(y^2 - 2xy - x^2) = 3(x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

3.

Solution :

We have, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiating with respect to x, we get

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$$\Rightarrow \frac{2}{3}(x)^{\frac{2}{3}-1} + \frac{2}{3}(y)^{\frac{2}{3}-1}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}(x)^{\frac{-1}{3}} + \frac{2}{3}(y)^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}(y)^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}(x)^{\frac{-1}{3}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{3}(x)^{\frac{-1}{3}} \times \frac{3}{2y^{\frac{-1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

4.

Solution :

We have, $4x + 3y = \log(4x - 3y)$

Differentiating with respect to x,we get

$$\frac{d}{dx}(4x)+\frac{d}{dx}(3y)=\frac{d}{dx}\{\log(4x-3y)\}$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\frac{d}{dx}(4x-3y)$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\left(4-3\frac{dy}{dx}\right)$$

$$\Rightarrow 3\frac{dy}{dx}+\frac{3}{(4x-3y)}\frac{dy}{dx}=\frac{4}{(4x-3y)}-4$$

$$\Rightarrow 3\frac{dy}{dx}\left\{1+\frac{1}{(4x-3y)}\right\}=4\left\{\frac{1}{(4x-3y)}-1\right\}$$

$$\Rightarrow 3\frac{dy}{dx}\left\{\frac{4x-3y+1}{(4x-3y)}\right\}=4\left\{\frac{1-4x+3y}{(4x-3y)}\right\}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left\{\frac{1-4x+3y}{(4x-3y)}\right\}\left(\frac{4x-3y}{4x-3y+1}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left(\frac{1-4x+3y}{4x-3y+1}\right)$$

5.

Solution :

$$We\,have,\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiating with respect to x,we get

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\Rightarrow \frac{d}{dx}\left(\frac{x^2}{a^2}\right)+\frac{d}{dx}\left(\frac{y^2}{b^2}\right)=0$$

$$\Rightarrow \frac{1}{a^2}(2x)+\frac{1}{b^2}(2y)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\Rightarrow \frac{dy}{dx}=-\left(\frac{2x}{a^2}\right)\left(\frac{b^2}{2y}\right)$$

$$\Rightarrow \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

6.

Solution :

$$We\,have,x^5+y^5=5xy$$

Differentiating with respect to x,we get

$$\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(y^5\right)=\frac{d}{dx}(5xy)$$

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5\left[x\frac{dy}{dx}+y(1)\right]$$

$$\Rightarrow 5x^4+5y^4\frac{dy}{dx}=5x\frac{dy}{dx}+5y$$

$$\Rightarrow 5y^4\frac{dy}{dx}-5x\frac{dy}{dx}=5y-5x^4$$

$$\Rightarrow 5\frac{dy}{dx}\left(y^4-x\right)=5\left(y-x^4\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{5\left(y-x^4\right)}{5\left(y^4-x\right)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-x^4}{y^4-x}$$

7.

Solution :

$We\,have,(x+y)^2=2axy$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}(x+y)^2=\frac{d}{dx}(2axy)$$

$$\Rightarrow 2(x+y)\frac{d}{dx}(x+y)=2a\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 2(x+y)\left[1+\frac{dy}{dx}\right]=2a\left[x\frac{dy}{dx}+y(1)\right]$$

$$\Rightarrow 2(x+y)+2(x+y)\frac{dy}{dx}=2ax\frac{dy}{dx}+2ay$$

$$\Rightarrow \frac{dy}{dx}\left[2(x+y)-2ax\right]=2ay-2(x+y)$$

$$\Rightarrow \frac{dy}{dx}=\frac{2[ay-x-y]}{2[x+y-ax]}$$

$$\Rightarrow \frac{dy}{dx}=\left(\frac{ay-x-y}{x+y-ax}\right)$$

8.

Solution :

$We\,have,\left(x^2+y^2\right)=xy$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left[\left(x^2+y^2\right)^2\right]=\frac{d}{dx}(xy)$$

$$\Rightarrow 2\left(x^2+y^2\right)\frac{d}{dx}\left(x^2+y^2\right)=x\frac{dy}{dx}+y\frac{d}{dx}(x)$$

$$\Rightarrow 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=x\frac{dy}{dx}+y(1)$$

$$\Rightarrow 4x\left(x^2+y^2\right)+4y\left(x^2+y^2\right)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\Rightarrow 4y\left(x^2+y^2\right)\frac{dy}{dx}-x\frac{dy}{dx}=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}\left[4y\left(x^2+y^2\right)-x\right]=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-4x\left(x^2+y^2\right)}{4y\left(x^2+y^2\right)-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x\left(x^2+y^2\right)-y}{x-4y\left(x^2+y^2\right)}$$

9.

Solution :

We have, $\tan^{-1}\left(x^2+y^2\right)=a$

Differentiating with respect to x,we get

$$\frac{d}{dx}\left[\tan^{-1}\left(x^2+y^2\right)\right]=\frac{d}{dx}(a)$$

$$\Rightarrow \frac{1}{1+\left(x^2+y^2\right)^2}\times\frac{d}{dx}\left(x^2+y^2\right)=0$$

$$\Rightarrow \left[\frac{1}{1+\left(x^2+y^2\right)^2}\right]\left(2x+2y\frac{dy}{dx}\right)=0$$

$$\Rightarrow 2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow x+y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x}{y}$$

10.

Solution :

$We\,have, e^{x-y} = \log\left(\frac{x}{y}\right)$

Differentiating with respect to x,we get

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\left\{\log\left(\frac{x}{y}\right)\right\}$$

$$\Rightarrow e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right)$$

$$\Rightarrow e^{(x-y)}\left(1-\frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \frac{e^{(x-y)}}{1}\right] = \frac{1}{x} - \frac{e^{(x-y)}}{1}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right] = \frac{1-xe^{(x-y)}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\frac{1-xe^{(x-y)}}{1-ye^{(x-y)}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{-x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

11.

Solution :

$We\,have, \sin xy + \cos(x+y) = 1$

Differentiating with respect to x,we get

$$\frac{d}{dx}(\sin xy)+\frac{d}{dx}\cos(x+y)=\frac{d}{dx}(1)$$

$$\Rightarrow \cos xy\frac{d}{dx}(xy)-\sin(x+y)\frac{d}{dx}(x+y)=0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx}+y\frac{d}{dx}(x)\right]-\sin(x+y)\left[1+\frac{dy}{dx}\right]=0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx}+y(1)\right]-\sin(x+y)-\sin(x+y)\frac{dy}{dx}=0$$

$$\Rightarrow x\cos xy\frac{dy}{dx}+y\cos xy-\sin(x+y)-\sin(x+y)\frac{dy}{dx}=0$$

$$\Rightarrow \left[x\cos xy-\sin(x+y)\right]\frac{dy}{dx}=\left[\sin(x+y)-y\cos xy\right]$$

$$\Rightarrow \frac{dy}{dx}=\left[\frac{\sin(x+y)-y\cos xy}{x\cos xy-\sin(x+y)}\right]$$

12.

Solution :

We have, $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Let $x=\sin A,\ y=\sin B$

$$\Rightarrow \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B}=a(\sin A-\sin B)$$

$$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$$

$$\Rightarrow a=\frac{\cos A+\cos B}{\sin A-\sin B}$$

$$\Rightarrow a=\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}} \qquad \left[\begin{array}{l}\because \sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\\ \because \cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}\end{array}\right]$$

$$\Rightarrow a=\cot\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \cot^{-1}a=\frac{A-B}{2}$$

$$\Rightarrow 2\cot^{-1}a=A-B$$

$$\Rightarrow 2\cot^{-1}a=\sin^{-1}x-\sin^{-1}y \qquad [\because x=\sin A, y=\sin B]$$

Differentiating with respect to x,we get

$$\frac{d}{dx}\left(2cot^{-1}a\right)=\frac{d}{dx}\left(\sin^{-1}x\right)-\frac{d}{dx}\left(\sin^{-1}y\right)$$

$$\Rightarrow 0=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

13.

Solution :

$$We\ have,\ y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$$

$$Let\ x=\sin A,\ y=\sin B$$

$$\Rightarrow \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$$

$$\Rightarrow \sin B\cos A+\sin A\cos B=1 \qquad \left[\because \sin\left(x+y\right)=\sin x\cos y+\cos x\sin y\right]$$

$$\Rightarrow \sin\left(A+B\right)=1$$

$$\Rightarrow A+B=\sin^{-1}\left(1\right)$$

$$\Rightarrow \sin^{-1}x+\sin^{-1}y=\frac{\pi}{2} \qquad \left[\because\ x=\sin A,\ y=\sin B\right]$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left(\sin^{-1}x\right)+\frac{d}{dx}\left(\sin^{-1}y\right)=\frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$$

14.

Solution :

$$We\,have, xy=1$$

Differentiating with respect to x,we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad [Using\ product\ rule]$$

$$\Rightarrow x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{\frac{1}{y}} \qquad \left[\because\ x = \frac{1}{y}\right]$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} + y^2 = 0$$

15.

Solution :

$We\,have, xy^2 = 1 \qquad ...(i)$

Differentiating with respect to x,we get

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x) = 0$$

$$\Rightarrow x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

$put\ x = \frac{1}{y^2}\ from\ equation\ (i)$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2\frac{dy}{dx} = -y^3$$

$$\Rightarrow 2\frac{dy}{dx} + y^3 = 0$$

16.

Solution :

$$We\ have,\ x\sqrt{1+y}+y\sqrt{1+x}=0$$

$$\Rightarrow \quad x\sqrt{1+y}=-y\sqrt{1+x}$$

Squaring both sides, we get,

$$\Rightarrow \left(x\sqrt{1+y}\right)^2=\left(-y\sqrt{1+x}\right)^2$$

$$\Rightarrow x^2(1+y)=y^2(1+x)$$

$$\Rightarrow x^2+x^2y=y^2+y^2x$$

$$\Rightarrow x^2-y^2=y^2x-x^2y$$

$$\Rightarrow (x-y)(x+y)=xy(y-x)$$

$$\Rightarrow (x+y)=-xy$$

$$\Rightarrow y+xy=-x$$

$$\Rightarrow y(1+x)=-x$$

$$\Rightarrow y=\frac{-x}{(1+x)}$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{dy}{dx}=\left[\frac{-(1+x)\frac{d}{dx}(x)-(-x)\frac{d}{dx}(x+1)}{(1+x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}=\left[\frac{-(1+x)(1)+x(1)}{(1+x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}=\left[\frac{-1-x+x}{(1+x)^2}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{-1}{(1+x)^2}$$

$$\Rightarrow (1+x)^2\frac{dy}{dx}=-1$$

$$\Rightarrow (1+x)^2\frac{dy}{dx}+1=0$$

17.

Solution :

$$We\ have,\ \log\sqrt{x^2+y^2}=\tan^{-1}\left(\frac{x}{y}\right)$$

$$\Rightarrow \log\left(x^2+y^2\right)^{\frac{1}{2}}=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{2}\log\left(x^2+y^2\right)=\tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{1}{2}\frac{d}{dx}\log\left(x^2+y^2\right)=\frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\frac{d}{dx}\left(x^2+y^2\right)=\frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x+2y\frac{dy}{dx}\right]=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \left(\frac{1}{x^2+y^2}\right)\left(x+y\frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \left(\frac{1}{x^2+y^2}\right)\left(x+y\frac{dy}{dx}\right)=\frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right]$$

$$\Rightarrow x+y\frac{dy}{dx}=x\frac{dy}{dx}-y$$

$$\Rightarrow y\frac{dy}{dx}-x\frac{dy}{dx}=-y-x$$

$$\Rightarrow \frac{dy}{dx}(y-x)=-(y+x)$$

$$\Rightarrow \frac{dy}{dx}=\frac{-(y+x)}{y-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{x+y}{x-y}$$

18.

Solution :

$$We\ have,\ \sec\left(\frac{x+y}{x-y}\right)=a$$

$$\Rightarrow \frac{x+y}{x-y}=\sec^{-1}(a)$$

Differentiating with respect to x,we get

$$\Rightarrow \left[\frac{(x-y)\frac{d}{dx}(x+y)-(x+y)\frac{d}{dx}(x-y)}{(x-y)^2}\right]=0$$

$$\Rightarrow (x-y)\left(1+\frac{dy}{dx}\right)-(x+y)\left(1-\frac{dy}{dx}\right)=0$$

$$\Rightarrow (x-y)+(x-y)\frac{dy}{dx}-(x+y)+(x+y)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}[x-y+x+y]=x+y-x+y$$

$$\Rightarrow \frac{dy}{dx}(2x)=2y$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$$

19.

Solution :

$$We\ have,\ \tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2}=\tan a$$

$$\Rightarrow x^2-y^2=\tan a\left(x^2+y^2\right)$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left(x^2-y^2\right)=\tan a\frac{d}{dx}\left(x^2+y^2\right)$$

$$\Rightarrow \left(2x-2y\frac{dy}{dx}\right)=\tan a\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow 2x-2y\frac{dy}{dx}=2x\tan a+2y\tan a\frac{dy}{dx}$$

$$\Rightarrow 2y\tan a\frac{dy}{dx}+2y\frac{dy}{dx}=2x-2x\tan a$$

$$\Rightarrow 2y(1+\tan a)\frac{dy}{dx}=2x(1-\tan a)$$

$$\Rightarrow \frac{dy}{dx}=\frac{x}{y}\left(\frac{1-\tan a}{1+\tan a}\right)$$

20.

Solution :

$$We\,have, xy\ \log(x+y)=1$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left[xy\log(x+y)\right]=\frac{d}{dx}(1)$$

$$\Rightarrow\ xy\frac{d}{dx}\log(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)\frac{d}{dx}(x)=0 \qquad [\textit{using chain rule and product rule}]$$

$$\Rightarrow xy\left(\frac{1}{x+y}\right)\frac{d}{dx}(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)(1)=0$$

$$\Rightarrow\left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)=0$$

$$\Rightarrow\left(\frac{xy}{x+y}\right)\frac{dy}{dx}+\left(\frac{xy}{x+y}\right)+x\left(\frac{1}{xy}\right)\frac{dy}{dx}+y\left(\frac{1}{xy}\right)=0 \qquad \left[\because xy\log(x+y)=1\right]$$

$$\Rightarrow\frac{dy}{dx}\left[\frac{xy}{x+y}+\frac{1}{y}\right]=-\left[\frac{1}{x}+\frac{xy}{x+y}\right]$$

$$\Rightarrow\frac{dy}{dx}\left[\frac{xy^2+x+y}{(x+y)y}\right]=-\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\Rightarrow\frac{dy}{dx}=-\left[\frac{x+y+x^2y}{x(x+y)}\right]\left[\frac{y(x+y)}{xy^2+x+y}\right]$$

$$\Rightarrow\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

Hence, proved.

21.

Solution :

We have, $y=x\sin(a+y)$

Differentiating with respect to x,we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[x\sin(a+y)\right]$$

$$\Rightarrow\frac{dy}{dx}=x\frac{d}{dx}\{\sin(a+y)\}+\sin(a+y)\frac{d}{dx}(x) \qquad [\textit{using product rule and chain rule}]$$

$$\Rightarrow\frac{dy}{dx}=x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)$$

$$\Rightarrow\frac{dy}{dx}\{1-x\cos(a+y)\}=\sin(a+y)$$

$$\Rightarrow\frac{dy}{dx}=\frac{\sin(a+y)}{1-x\cos(a+y)}$$

$$\Rightarrow\frac{dy}{dx}=\frac{\sin(a+y)}{1-\frac{y}{\sin(a+y)}\cos(a+y)}\left[\because y=x\sin(a+y)\ \right]$$

$$\Rightarrow\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin(a+y)-y\cos(a+y)}$$

Hence,proved.

22.

Solution :

We have, $x\sin(a+y)+\sin a\cos(a+y)=0$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left[x\sin(a+y)\right]+\frac{d}{dx}\left[\sin\ a\cos(a+y)\right]=0$$

$$\Rightarrow \left[x\frac{d}{dx}\sin\ (a+y)+\sin(a+y)\frac{d}{dx}(x)\right]+\sin\ a\frac{d}{dx}\cos(a+y)=0$$

$$\Rightarrow \left[x\cos(a+y)\frac{d}{dx}(a+y)+\sin(a+y)(1)\right]+\sin\ a\left[-\sin(a+y)\frac{d}{dx}(a+y)\right]=0$$

$$\Rightarrow x\cos(a+y)\frac{dy}{dx}+\sin(a+y)-\ \sin a\sin(a+y)\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}\left[x\cos(a+y)-\sin a\sin(a+y)\right]=-\sin(a+y)$$

$$\Rightarrow \frac{dy}{dx}\left[-\sin\ a\frac{\cos^2(a+y)}{\sin(a+y)}-\sin\ a\sin(a+y)\right]=-\sin(a+y)\qquad \left[\because x=-\sin a\frac{\cos(a+y)}{\sin(a+y)}\right]$$

$$\Rightarrow -\frac{dy}{dx}\left[\frac{\sin\ a\cos^2(a+y)+\sin\ a\sin^2(a+y)}{\sin(a+y)}\right]=-\sin(a+y)$$

$$\Rightarrow \frac{dy}{dx}=\sin(a+y)\left[\frac{\sin(a+y)}{\sin a\left\{\cos^2(a+y)+\sin^2(a+y)\right\}}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

23.

Solution :

We have, $y=x\ \sin y$

Differentiating with respect to x,we get

$$\frac{dy}{dx}=\frac{d}{dx}(x\sin y)$$

$$\Rightarrow \frac{dy}{dx}=x\frac{d}{dx}(\sin y)+\sin y\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx}=x\cos y\frac{dy}{dx}+\sin y$$

$$\Rightarrow \frac{dy}{dx}(1-x\cos y)=\sin y$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin y}{1-x\cos y}$$

24.

Solution :

We have, $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left(y\sqrt{x^2+1}\right)=\frac{d}{dx}\log\left(\sqrt{x^2+1}-x\right) \qquad \left[\textit{using product rule and chain rule}\right]$$

$$\Rightarrow y\frac{d}{dx}\left(\sqrt{x^2+1}\right)+\sqrt{x^2+1}\frac{dy}{dx}=\frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\frac{d}{dx}\left(\sqrt{x^2+1}-x\right)$$

$$\Rightarrow \frac{y}{2\sqrt{x^2+1}}\times\frac{d}{dx}\left(x^2+1\right)+\sqrt{x^2+1}\frac{dy}{dx}=\frac{1}{\left(\sqrt{x^2+1}-x\right)}\times\left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}\left(x^2+1\right)-1\right]$$

$$\Rightarrow \frac{2xy}{2\sqrt{x^2+1}}+\sqrt{x^2+1}\frac{dy}{dx}=\frac{1}{\left(\sqrt{x^2+1}-x\right)}\left[\frac{2x}{2\sqrt{x^2+1}}-1\right]$$

$$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx}=\left[\frac{1}{\sqrt{x^2+1}-x}\right]\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx}=\frac{-1}{\sqrt{x^2+1}}-\frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1}\frac{dy}{dx}=\frac{-(1+xy)}{\sqrt{x^2+1}}$$

$$\Rightarrow \left(x^2+1\right)\frac{dy}{dx}=-(1+xy)$$

$$\Rightarrow \left(x^2+1\right)\frac{dy}{dx}+1+xy=0$$

25.

Solution :

$We\,have,\ \sin(xy)+\frac{y}{x}=x^2-y^2$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}(\sin xy)+\frac{d}{dx}\left(\frac{y}{x}\right)=\frac{d}{dx}(x^2)-\frac{d}{dx}(y^2)$$

$$\Rightarrow \cos(xy)\frac{d}{dx}(xy)+\left\{\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right\}=2x-2y\frac{dy}{dx}$$

$$\Rightarrow \cos(xy)\left\{x\frac{dy}{dx}+y\frac{d}{dx}(x)\right\}+\left\{\frac{x\frac{dy}{dx}-y(1)}{x^2}\right\}=2x-2y\frac{dy}{dx}$$

$$\Rightarrow \cos(xy)\left\{x\frac{dy}{dx}+y(1)\right\}+\frac{1}{x^2}\left(x\frac{dy}{dx}-y\right)=2x-2y\frac{dy}{dx}$$

$$\Rightarrow x\cos(xy)\frac{dy}{dx}+y\cos(xy)+\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2}=2x-2y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left\{x\cos(xy)+\frac{1}{x}+2y\right\}=\frac{y}{x^2}-y\cos(xy)+2x$$

$$\Rightarrow \frac{dy}{dx}\left\{\frac{x^2\cos(xy)+1+2xy}{x}\right\}=\frac{1}{x^2}\left(y-x^2y\cos(xy)+2x^3\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{2x^3+y-x^2y\cos(xy)}{x\left(x^2\cos(xy)+1+2xy\right)}$$

26

Solution :

We have, $\tan(x+y)+\tan(x-y)=1$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\tan(x+y)+\frac{d}{dx}\tan(x-y)=\frac{d}{dx}(1)$$

$$\Rightarrow \sec^2(x+y)\frac{d}{dx}(x+y)+\sec^2(x-y)\frac{d}{dx}(x-y)=0$$

$$\Rightarrow \sec^2(x+y)\left[1+\frac{dy}{dx}\right]+\sec^2(x-y)\left[1-\frac{dy}{dx}\right]=0$$

$$\Rightarrow \sec^2(x+y)\frac{dy}{dx}-\sec^2(x-y)\frac{dy}{dx}=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\sec^2(x+y)-\sec^2(x-y)\right]=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x-y)-\sec^2(x+y)}$$

27.

Solution :

$$e^x + e^y = e^{x+y}$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y}\frac{dy}{dx}$$

$$\Rightarrow e^y \frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \frac{e^x\left(e^y - 1\right)}{e^y\left(1 - e^x\right)}$$

$$= -\frac{e^x\left(e^y - 1\right)}{e^y\left(e^x - 1\right)}$$

28.

Solution :

We have, $\cos y = x\cos(a+y)$

Differentiating with respect to x, we get,

$$\frac{d}{dx}(\cos y) = \frac{d}{dx}\{x\cos(a+y)\}$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y)\frac{d}{dx}(x) + x\frac{d}{dx}\cos(a+y)$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y) + x\left[-\sin(a+y)\right]\frac{dy}{dx}$$

$$\Rightarrow \left[x\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow \left[\frac{\cos y}{\cos(a+y)}\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y) \quad \left[\because \cos y = x\cos(a+y) \Rightarrow x = \frac{\cos y}{\cos(a+y)}\right]$$

$$\Rightarrow \left[\cos y\sin(a+y) - \sin y\cos(a+y)\right]\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

29.

Solution :

$$We\ have,\ y=\left[\log_{\cos x}\ \sin x\right]\left[\log_{\sin x}\ \cos x\right]^{-1}+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow\ y=\left[\log_{\cos x}\ \sin x\right]\left[\log_{\cos x}\ \sin x\right]+\sin^{-1}\left(\frac{2x}{1+x^2}\right)\qquad\left[\because \log_a b=\left(\log_b a\right)^{-1}\right]$$

$$\Rightarrow y=\left[\frac{\log\ \sin x}{\log\ \cos x}\right]^2+\sin^{-1}\left(\frac{2x}{1+x^2}\right)\qquad\left[\because \log_a b=\frac{\log b}{\log a}\right]$$

Differentiating with respect to x,we get

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{\log\ \sin x}{\log\ \cos x}\right]^2+\frac{d}{dx}\left\{\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$$

$$\Rightarrow\frac{dy}{dx}=2\left[\frac{\log\ \sin x}{\log\ \cos x}\right]\frac{d}{dx}\left(\frac{\log\ \sin x}{\log\ \cos x}\right)+\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}\times\frac{d}{dx}\left[\frac{2x}{1+x^2}\right]$$

$$\Rightarrow\frac{dy}{dx}=2\left[\frac{\log\ \sin x}{\log\ \cos x}\right]\left[\frac{(\log\ \cos x)\frac{d}{dx}(\log\ \sin x)-\log\ \sin x\frac{d}{dx}(\log\ \cos x)}{(\log\ \cos x)^2}\right]+\left[\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}}\right]\left[\frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}\right]$$

$$\Rightarrow\frac{dy}{dx}=2\left[\frac{\log\ \sin x}{\log\ \cos x}\right]\left[\frac{\log\ \cos x\times\frac{1}{\sin x}\frac{d}{dx}(\sin x)-\log\ \sin x\times\frac{1}{\cos x}\frac{d}{dx}(\cos x)}{(\log\ \cos x)^2}\right]+\left[\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}}\right]\left[\frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}\right]$$

$$\Rightarrow\frac{dy}{dx}=2\left[\frac{\log\ \sin x}{\log\ \cos x}\right]\left[\frac{\log\ \cos x\times\left(\frac{\cos x}{\sin x}\right)+\log\ \sin x\times\left(\frac{\sin x}{\cos x}\right)}{(\log\ \cos x)^2}\right]+\left[\frac{1+x^2}{\sqrt{(1-x^2)^2}}\right]\left[\frac{2+2x^2-4x^2}{(1+x^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = 2\frac{\log\sin x}{(\log\cos x)^3}(\cot x\log\cos x + \tan x\log\sin x) + \frac{2}{1+x^2}$$

$put\ \ x = \frac{\pi}{4}$

$$\Rightarrow \frac{dy}{dx} = 2\left\{\frac{\log\sin\frac{\pi}{4}}{\left(\log\cos\frac{\pi}{4}\right)^3}\right\}\left(\cot\frac{\pi}{4}\log\cos\frac{\pi}{4} + \tan\frac{\pi}{4}\log\sin\frac{\pi}{4}\right) + 2\left\{\frac{1}{1+\left(\frac{\pi}{4}\right)^2}\right\}$$

$$\Rightarrow \frac{dy}{dx} = 2\left\{\frac{1}{\left(\log\frac{1}{\sqrt{2}}\right)^2}\right\}\left(1\times\log\frac{1}{\sqrt{2}} + 1\times\log\frac{1}{\sqrt{2}}\right) + 2\left(\frac{16}{16+\pi^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = 2\times\frac{2\log\left(\frac{1}{\sqrt{2}}\right)}{\left\{\log\left(\frac{1}{\sqrt{2}}\right)\right\}^2} + \frac{32}{16+\pi^2}$$

$$\Rightarrow \frac{dy}{dx} = 4\frac{1}{\log\left(\frac{1}{\sqrt{2}}\right)} + \frac{32}{16+\pi^2}$$

$$\Rightarrow \frac{dy}{dx} = 4\frac{1}{-\frac{1}{2}\log 2} + \frac{32}{16+\pi^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{8}{\log 2} + \frac{32}{16+\pi^2}$$

$$So, \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 8\left[\frac{4}{16+\pi^2} - \frac{1}{\log 2}\right]$$

30.

Solution :

Here,

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating with respect to x,we get

$$\Rightarrow \frac{d}{dx}\left(\sqrt{y+x}\right)+\frac{d}{dx}\sqrt{y-x}=\frac{d}{dx}(c)$$

$$\Rightarrow \frac{1}{2\sqrt{y+x}}\frac{d}{dx}(y+x)+\frac{1}{2\sqrt{y-x}}\frac{d}{dx}(y-x)=0$$

$$\Rightarrow \frac{1}{2\sqrt{y+x}}\left(\frac{dy}{dx}+1\right)+\frac{1}{2\sqrt{y-x}}\left(\frac{dy}{dx}-1\right)=0$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right)+\frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right)=\frac{1}{2\sqrt{y-x}}-\frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \frac{dy}{dx}\times\frac{1}{2}\left[\frac{1}{\sqrt{y+x}}+\frac{1}{\sqrt{y-x}}\right]=\frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y+x}+\sqrt{y-x}}\times\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y+x}-\sqrt{y-x}} \qquad [\textit{rationalizing the denominator}]$$

$$\Rightarrow \frac{dy}{dx}=\frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2y-2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2y}{2x}-\frac{2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2-x^2}{x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$$