

**RD Sharma**  
**Solutions**  
**Class 12 Maths**  
**Chapter 11**  
**Ex 11.6**

## Chapter: Differentiation

### Exercise: 11.6

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**Q1.**

**Answer :**

We have,  $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \text{ to } \infty}}$

$$\Rightarrow y = \sqrt{x + y}$$

Squaring both sides, we get,

$$y^2 = x + y$$

$$\Rightarrow 2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y - 1) = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y - 1}$$

**Q2.**

**Answer :**

We have,  $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \dots \text{ to } \infty}}$

$$\Rightarrow y = \sqrt{\cos x + y}$$

Squaring both sides, we get,

$$y^2 = \cos x + y$$

$$\Rightarrow 2y \frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y - 1) = -\sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin x}{2y - 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

**Q3.**

**Answer :**

We have,  $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \dots \text{ to } \infty}}}$

$$\Rightarrow y = \sqrt{\log x + y}$$

Squaring both sides, we get,

$$y^2 = \log x + y$$

$$\Rightarrow 2y \frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} (2y - 1) = \frac{1}{x}$$

**Q4.**

**Answer :**

We have,  $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \dots \text{ to } \infty}}}$

$$\Rightarrow y = \sqrt{\tan x + y}$$

Squaring both sides, we get,

$$y^2 = \tan x + y$$

$$\Rightarrow 2y \frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} (2y - 1) = \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

**Q5.**

**Answer :**

We have,  $y = (\sin x)^{(\sin x)^{(\sin x)^{\dots x}}}$

$$\Rightarrow y = (\sin x)^y$$

Taking log on both sides,

$$\log y = \log (\sin x)^y$$

$$\Rightarrow \log y = y \log (\sin x)$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y \frac{d}{dx} \{ \log(\sin x) \} + \log \sin x \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y \left( \frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{1}{y} - \log \sin x \right) = \frac{y}{\sin x} (\cos x)$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{1 - y \log \sin x}{y} \right) = y \cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y \log \sin x)}$$

**Q6.**

**Answer :**

*We have,*  $y = (\tan x)^{(\tan x)^{\tan^{-1} x}}$

$$\Rightarrow y = (\tan x)^y$$

Taking log on both sides,

$$\log y = \log (\tan x)^y$$

$$\Rightarrow \log y = y \log \tan x$$

Differentiating with respect to  $x$  using chain rule ,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= y \frac{d}{dx} \{ \log \tan x \} + \log \tan x \frac{dy}{dx} \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \frac{y}{\tan x} \frac{d}{dx} (\tan x) + \log \tan x \frac{dy}{dx} \\ \Rightarrow \frac{dy}{dx} \left( \frac{1}{y} - \log \tan x \right) &= \frac{y}{\tan x} \sec^2 x \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{\tan x} \sec^2 x \times \left( \frac{y}{1 - y \log \tan x} \right)\end{aligned}$$

$$\text{Now, } \left( \frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = \frac{y \sec^2 \left( \frac{\pi}{4} \right)}{\tan \left( \frac{\pi}{4} \right)} \times \frac{y}{1 - y \log \tan \left( \frac{\pi}{4} \right)}$$

$$\Rightarrow \left( \frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = \frac{y^2 (\sqrt{2})^2}{1(1 - y \log \tan 1)}$$

$$\Rightarrow \left( \frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = \frac{2(1)^2}{(1-0)} \left[ \because (y)_{x=\frac{\pi}{4}} = \left( \tan \frac{\pi}{4} \right)^{\left( \tan \frac{\pi}{4} \right)^{\left( \tan \frac{\pi}{4} \right)^{\dots}}} = 1 \right]$$

$$\Rightarrow \left( \frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = 2$$

**Q7.**

**Answer :**

$$\text{We have, } y = e^{x+e^x} + x^{e^{e^x}} + e^{x+e}$$

$$\Rightarrow y = u + v + w$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \quad \dots(i)$$

$$\text{where } u = e^{x+e^x}, v = x^{e^{e^x}} \text{ and } w = e^{x+e}$$

$$\text{Now, } u = e^{x+e^x} \quad \dots(ii)$$

Taking log on both sides,

$$\log u = \log e^{x+e^x}$$

$$\Rightarrow \log u = x^{e^x} \log e$$

$$\Rightarrow \log u = x^{e^x} \quad \dots(iii)$$

$$\log \log u = \log x^{e^x} \Rightarrow \log \log u = e^x \log x$$

Differentiating with respect to  $x$ ,

$$\Rightarrow \frac{1}{\log u} \frac{d}{dx}(\log u) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$$

$$\Rightarrow \frac{1}{\log u} \frac{1}{u} \frac{du}{dx} = \frac{e^x}{x} + e^x \log x$$

$$\Rightarrow \frac{du}{dx} = u \log u \left[ \frac{e^x}{x} + e^x \log x \right]$$

$$\Rightarrow \frac{du}{dx} = e^{x+e^x} \times x^{e^x} \left[ \frac{e^x}{x} + e^x \log x \right] \quad \dots(A)$$

$$\text{Now, } v = x^{e^{e^x}} \quad \dots(iv)$$

Taking log on both sides,

$$\log v = \log x^{e^{e^x}}$$

$$\Rightarrow \log v = e^{e^x} \log x$$

Taking log on both sides,

$$\Rightarrow \frac{1}{\log w} \frac{d}{dx}(\log w) = x^e \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^e)$$

$$\Rightarrow \frac{1}{\log w} \left( \frac{1}{w} \right) \frac{dw}{dx} = x^e \left( \frac{1}{x} \right) + \log x e x^{e-1}$$

$$\Rightarrow \frac{dw}{dx} = w \log w \left[ x^{e-1} + e \log x x^{e-1} \right]$$

$$\Rightarrow \frac{dw}{dx} = e^{x+x^e} x^{x^e} x^{e-1} (1 + e \log x) \quad \text{-----}(C)$$

Using equation (A), (B) and (C) in equation (i), we get

$$\frac{dy}{dx} = e^{x+e^x} x^{e^x} \left[ \frac{e^x}{x} + e^x \log x \right] + x^{e^{e^x}} \times e^{e^x} \left[ \frac{1}{x} + e^x \log x \right] + e^{x+x^e} x^{x^e} x^{e-1} (1 + e \log x)$$

**Q8.**

**Answer :**

$$\text{We have, } y = (\cos x)^{(\cos x)^{(\cos x)^{\dots \infty}}}$$

$$\Rightarrow y = (\cos x)^y$$

Taking log on both sides,

$$\log y = \log (\cos x)^y$$

$$\Rightarrow \log y = y \log (\cos x)$$

Differentiating with respect to  $x$  using chain rule,

$$\frac{1}{y} \frac{dy}{dx} = y \frac{d}{dx} \{ \log \cos x \} + \log \cos x \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y \left( \frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{1}{y} - \log \cos x \right) = \frac{y}{\cos x} (-\sin x)$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{1 - y \log \cos x}{y} \right) = -y \tan x$$

$$\Rightarrow \frac{dy}{dx} = - \frac{y^2 \tan x}{(1 - y \log \cos x)}$$