

RD Sharma
Solutions
Class 12 Maths
Chapter 11
Ex 11.7

Chapter: Differentiation

Exercise: 11.7

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Q1.

Solution :

We have, $x = at^2$ and $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2at \text{ and } \frac{dy}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Q2.

Solution :

We have, $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$

$$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos \theta) \text{ and } \frac{dy}{d\theta} = a \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 + \cos \theta)} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = \tan \frac{\theta}{2}$$

Q3.

Solution :

We have, $x = a \cos \theta$ and $y = b \sin \theta$

$$\Rightarrow \frac{dx}{d\theta} = -a \sin \theta \text{ and } \frac{dy}{d\theta} = b \cos \theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \cos \theta}{-a \sin \theta} = -\frac{b}{a} \cot \theta$$

Q4.

Solution :

We have, $x = ae^\theta (\sin \theta - \cos \theta)$ and $y = ae^\theta (\sin \theta + \cos \theta)$

$$\begin{aligned} \Rightarrow \frac{dx}{d\theta} &= a \left[e^\theta \frac{d}{d\theta} (\sin \theta - \cos \theta) + (\sin \theta - \cos \theta) \frac{d}{d\theta} (e^\theta) \right] \text{ and } \frac{dy}{d\theta} \\ &= a \left[e^\theta \frac{d}{d\theta} (\sin \theta + \cos \theta) + (\sin \theta + \cos \theta) \frac{d}{d\theta} (e^\theta) \right] \\ \Rightarrow \frac{dx}{d\theta} &= a \left[e^\theta (\cos \theta + \sin \theta) + (\sin \theta - \cos \theta) e^\theta \right] \text{ and } \frac{dy}{d\theta} = a \left[e^\theta (\cos \theta - \sin \theta) + (\sin \theta + \cos \theta) e^\theta \right] \\ \Rightarrow \frac{dx}{d\theta} &= a \left[2e^\theta \sin \theta \right] \text{ and } \frac{dy}{d\theta} = a \left[2e^\theta \cos \theta \right] \\ \therefore \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} &= \frac{a(2e^\theta \cos \theta)}{a(2e^\theta \sin \theta)} = \cot \theta \end{aligned}$$

Q5.

Solution :

We have, $x = b \sin^2 \theta$ and $y = a \cos^2 \theta$

$$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta} (b \sin^2 \theta) = 2b \sin \theta \cos \theta$$

and,

$$\frac{dy}{d\theta} = \frac{d}{d\theta} (a \cos^2 \theta) = -2a \cos \theta \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a \cos \theta \sin \theta}{2b \sin \theta \cos \theta} = -\frac{a}{b}$$

Q6.

Solution :

We have, $x = a(1 - \cos \theta)$ and $y = a(\theta + \sin \theta)$

$$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta} [a(1 - \cos \theta)] = a(\sin \theta)$$

and

$$\frac{dy}{d\theta} = \frac{d}{d\theta} [a(\theta + \sin \theta)] = a(1 + \cos \theta)$$

$$\therefore \left[\frac{dy}{dx} \right]_{\theta=\frac{\pi}{2}} = \left[\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \right]_{\theta=\frac{\pi}{2}} = \left[\frac{a(1 + \cos \theta)}{a(\sin \theta)} \right]_{\theta=\frac{\pi}{2}} = \frac{a(1+0)}{a} = 1$$

Q7.

Solution :

We have, $x = \frac{e^t + e^{-t}}{2}$ and $y = \frac{e^t - e^{-t}}{2}$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2} \left[\frac{d}{dt}(e^t) + \frac{d}{dt}(e^{-t}) \right] \text{ and } \frac{dy}{dt} = \frac{1}{2} \left[\frac{d}{dt}(e^t) - \frac{d}{dt}(e^{-t}) \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2} \left[e^t + e^{-t} \frac{d}{dt}(-t) \right] \text{ and } \frac{dy}{dt} = \frac{1}{2} \left[e^t - e^{-t} \frac{d}{dt}(e^{-t}) \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{2}(e^t - e^{-t}) = y \text{ and } \frac{dy}{dt} = \frac{1}{2}(e^t + e^{-t}) = x$$

$$\therefore \frac{dy}{dt} = \frac{dx}{dt} = \frac{x}{y}$$

Q8.

Solution :

$$\text{We have, } x = \frac{3at}{1+t^2}$$

Differentiating with respect to t ,

$$\frac{dx}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(3at) - 3at \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right]$$

[using quotient rule]

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2)(3a) - 3at(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{3a + 3at^2 - 6at^2}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{3a - 3at^2}{(1-t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{3a(1-t^2)}{(1+t^2)^2} \quad \dots(i)$$

$$\text{and, } y = \frac{3at^2}{1+t^2}$$

Q9.

Solution :

$$\frac{dx}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(3at^2) - 3at^2 \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \quad [\text{using quotient rule}]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2)(6at) - 3at^2(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{6at + 6at^3 - 6at^3}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{6at}{(1+t^2)^2} \quad \dots(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{(1+t^2)^2} \times \frac{(1+t^2)^2}{3a(1-t^2)} = \frac{2t}{1-t^2}$$

We have, $x = a(\cos \theta + \theta \sin \theta)$ and $y = a(\sin \theta - \theta \cos \theta)$

$$\Rightarrow \frac{dx}{d\theta} = a \left[\frac{d}{d\theta} \cos \theta + \frac{d}{d\theta} (\theta \sin \theta) \right] \text{ and } \frac{dy}{d\theta} = a \left[\frac{d}{d\theta} (\sin \theta) - \frac{d}{d\theta} (\theta \cos \theta) \right]$$

$$\Rightarrow \frac{dx}{d\theta} = a \left[-\sin \theta + \theta \frac{d}{d\theta} (\sin \theta) + \sin \theta \frac{d}{d\theta} (\theta) \right] \text{ and } \frac{dy}{d\theta} = a \left[\cos \theta - \left\{ \theta \frac{d}{d\theta} (\cos \theta) + \cos \theta \frac{d}{d\theta} (\theta) \right\} \right]$$

$$\Rightarrow \frac{dx}{d\theta} = a[-\sin \theta + \theta \cos \theta] \text{ and } \frac{dy}{d\theta} = a[\cos \theta + \theta \sin \theta - \cos \theta]$$

$$\Rightarrow \frac{dx}{d\theta} = a\theta \cos \theta \text{ and } \frac{dy}{d\theta} = a\theta \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta \sin \theta}{a\theta \cos \theta} = \tan \theta$$

Q10.

Solution :

$$\text{We have, } x = e^\theta \left(\theta + \frac{1}{\theta} \right)$$

$$\begin{aligned} \frac{dx}{d\theta} &= e^\theta \frac{d}{d\theta} \left(\theta + \frac{1}{\theta} \right) + \left(\theta + \frac{1}{\theta} \right) \frac{d}{d\theta} (e^\theta) && \text{[using product rule]} \\ \Rightarrow \frac{dx}{d\theta} &= e^\theta \left(1 - \frac{1}{\theta^2} \right) + \left(\frac{\theta^2 + 1}{\theta} \right) e^\theta \\ \Rightarrow \frac{dx}{d\theta} &= e^\theta \left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta} \right) \\ \Rightarrow \frac{dx}{d\theta} &= e^\theta \left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2} \right) \\ \Rightarrow \frac{dx}{d\theta} &= \frac{e^\theta (\theta^3 + \theta^2 + \theta - 1)}{\theta^2} && \dots(i) \end{aligned}$$

and,

$$y = e^\theta \left(\theta - \frac{1}{\theta} \right)$$

$$\begin{aligned} \frac{dy}{d\theta} &= e^{-\theta} \frac{d}{d\theta} \left(\theta - \frac{1}{\theta} \right) + \left(\theta - \frac{1}{\theta} \right) \frac{d}{d\theta} (e^{-\theta}) && \text{[using product rule]} \\ \Rightarrow \frac{dy}{d\theta} &= e^{-\theta} \left(1 + \frac{1}{\theta^2} \right) + \left(\theta - \frac{1}{\theta} \right) e^{-\theta} \frac{d}{d\theta} (-\theta) \\ \Rightarrow \frac{dy}{d\theta} &= e^{-\theta} \left(1 + \frac{1}{\theta^2} \right) + \left(\theta - \frac{1}{\theta} \right) e^{-\theta} (-1) \\ \Rightarrow \frac{dy}{d\theta} &= e^{-\theta} \left(1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta} \right) \\ \Rightarrow \frac{dy}{d\theta} &= e^{-\theta} \left(\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2} \right) \\ \Rightarrow \frac{dy}{d\theta} &= \frac{e^{-\theta} (-\theta^3 + \theta^2 + \theta + 1)}{\theta^2} && \dots(ii) \end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned} \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} &= e^{-\theta} \left(\frac{\theta^2 - \theta^3 + \theta + 1}{\theta^2} \right) \times \frac{\theta^2}{e^\theta (\theta^3 + \theta^2 + \theta - 1)} \\ &= e^{-2\theta} \left(\frac{\theta^2 - \theta^3 + \theta + 1}{\theta^3 + \theta^2 + \theta - 1} \right) \end{aligned}$$

Q11.

Solution :

$$\text{We have, } x = \frac{2t}{1+t^2}$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(2t) - 2t \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \quad [\text{using quotient rule}]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{2+2t^2-4t^2}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{2-2t^2}{(1+t^2)^2} \right] \quad \dots(i)$$

and,

$$y = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{-4t}{(1+t^2)^2} \right] \quad \dots(ii)$$

Dividing equation (ii) by (i), we get,

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t}{(1+t^2)^2} \times \frac{(1+t^2)^2}{2(1-t^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2t}{1-t^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y} \quad \left[\because \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \right]$$

Q12.

Solution :

$$\text{We have, } x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}} \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{\sqrt{1-\frac{1}{(1+t^2)}}} \left\{ \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt}(1+t^2)$$

$$\Rightarrow \frac{dx}{dt} = \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2}-1} \times \frac{1}{2(1+t^2)^{\frac{3}{2}}} (2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{t}{\sqrt{t^2} \times (1+t^2)}$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{1+t^2} \quad \dots(i)$$

$$\text{Now, } y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1 - \left(\frac{1}{\sqrt{1+t^2}}\right)^2}} \frac{d\left(\frac{1}{\sqrt{1+t^2}}\right)}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1 - \frac{1}{(1+t^2)}}} \left\{ \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt}(1+t^2)$$

$$\Rightarrow \frac{dx}{dt} = \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2}-1} \times \frac{-1}{2(1+t^2)^{\frac{3}{2}}} (2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{2\sqrt{t^2} \times (1+t^2)} (2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{1+t^2} \quad \dots(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{(1+t^2)} \times \frac{(1+t^2)}{-1}$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Q13.

Solution :

$$\text{We have, } y = \frac{2t}{1+t^2}$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(2t) - 2t \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \quad [\text{using quotient rule}]$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{2 + 2t^2 - 4t^2}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = \left[\frac{2 - 2t^2}{(1+t^2)^2} \right] \quad \dots(i)$$

and,

$$x = \frac{1-t^2}{1+t^2}$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{-4t}{(1+t^2)^2} \right] \quad \dots(ii)$$

Dividing equation (i) by (ii), we get,

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(1-t^2)}{(1+t^2)^2} \times \frac{(1+t^2)^2}{-4t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(1-t^2)}{-4t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{t^2 - 1}{2t}$$

Q14.

Solution :

We have, $x = 2 \cos \theta - \cos 2\theta$

$$\Rightarrow \frac{dx}{d\theta} = 2(-\sin \theta) - (-\sin 2\theta) \frac{d}{d\theta}(2\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = -2 \sin \theta + 2 \sin 2\theta$$

$$\Rightarrow \frac{dx}{d\theta} = 2(\sin 2\theta - \sin \theta) \quad \dots(i)$$

and,

$$y = 2 \sin \theta - \sin 2\theta$$

$$\Rightarrow \frac{dy}{d\theta} = 2 \cos \theta - \cos 2\theta \frac{d}{d\theta}(2\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = 2 \cos \theta - \cos 2\theta(2)$$

$$\Rightarrow \frac{dy}{d\theta} = 2 \cos \theta - 2 \cos 2\theta$$

$$\Rightarrow \frac{dy}{d\theta} = 2(\cos \theta - \cos 2\theta) \quad \dots(ii)$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos \theta - \cos 2\theta)}{2(\sin 2\theta - \sin \theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos \theta - \cos 2\theta}{\sin 2\theta - \sin \theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2 \sin\left(\frac{\theta + 2\theta}{2}\right) \sin\left(\frac{\theta - 2\theta}{2}\right)}{2 \cos\left(\frac{2\theta + \theta}{2}\right) \sin\left(\frac{2\theta - \theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\left(\frac{3\theta}{2}\right) \sin\left(\frac{-\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\left(\frac{3\theta}{2}\right) \left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Q15.

Solution :

We have, $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$

$$\begin{aligned}
\Rightarrow \frac{dx}{dt} &= \frac{d}{dt}(e^{\cos 2t}) \text{ and } \frac{dy}{dt} = \frac{d}{dt}(e^{\sin 2t}) \\
\Rightarrow \frac{dx}{dt} &= e^{\cos 2t} \frac{d}{dt}(\cos 2t) \text{ and } \frac{dy}{dt} = e^{\sin 2t} \frac{d}{dt}(\sin 2t) \\
\Rightarrow \frac{dx}{dt} &= e^{\cos 2t} (-\sin 2t) \frac{d}{dt}(2t) \text{ and } \frac{dy}{dt} = e^{\sin 2t} (\cos 2t) \frac{d}{dt}(2t) \\
\Rightarrow \frac{dx}{dt} &= -2 \sin 2t e^{\cos 2t} \text{ and } \frac{dy}{dt} = 2 \cos 2t e^{\sin 2t} \\
\frac{dy}{dx} & \\
\therefore \frac{dy}{dx} &= \frac{2 \cos 2t e^{\sin 2t}}{-2 \sin 2t e^{\cos 2t}} \\
\Rightarrow \frac{dy}{dx} &= -\frac{y \log x}{x \log y} \quad \left[\begin{array}{l} \because x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ y = e^{\sin 2t} \Rightarrow \log y = \sin 2t \end{array} \right]
\end{aligned}$$

Q16.

Solution :

We have, $x = \cos t$ and $y = \sin t$

$$\begin{aligned}
\Rightarrow \frac{dx}{dt} &= \frac{d}{dt}(\cos t) \text{ and } \frac{dy}{dt} = \frac{d}{dt}(\sin t) \\
\Rightarrow \frac{dx}{dt} &= -\sin t \text{ and } \frac{dy}{dt} = \cos t \\
\frac{dy}{dx} & \\
\therefore \frac{dy}{dx} &= \frac{\cos t}{-\sin t} = -\cot t
\end{aligned}$$

$$\text{Now, } \left(\frac{dy}{dx} \right)_{t=\frac{2\pi}{3}} = -\cot \left(\frac{2\pi}{3} \right) = \frac{1}{\sqrt{3}}$$

Q17.

Solution :

We have, $x = a \left(t + \frac{1}{t} \right)$ and $y = a \left(t - \frac{1}{t} \right)$

$$\Rightarrow \frac{dx}{dt} = a \frac{d}{dt} \left(t + \frac{1}{t} \right) \text{ and } \frac{dy}{dt} = a \frac{d}{dt} \left(t - \frac{1}{t} \right)$$

$$\Rightarrow \frac{dx}{dt} = a \left(1 - \frac{1}{t^2} \right) \text{ and } \frac{dy}{dt} = a \left(1 + \frac{1}{t^2} \right)$$

$$\Rightarrow \frac{dx}{dt} = a \left(\frac{t^2 - 1}{t^2} \right) \text{ and } \frac{dy}{dt} = a \left(\frac{t^2 + 1}{t^2} \right)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a(t^2 + 1)}{t^2} \times \frac{t^2}{a(t^2 - 1)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a(t^2 + 1)}{t} \times \frac{t}{a(t^2 - 1)}$$

$$\Rightarrow \frac{dy}{dx} = a \left(t + \frac{1}{t} \right) \times \frac{1}{a \left(t - \frac{1}{t} \right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

Q18.

Solution :

$$\text{We have, } x = \sin^{-1} \left(\frac{2t}{1+t^2} \right)$$

$$\text{Put } t = \tan \theta$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$$

$$\therefore x = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow x = \sin^{-1} (\sin 2\theta)$$

$$\Rightarrow x = 2\theta \quad \left[\because -\frac{\pi}{2} < 2\theta < \frac{\pi}{2} \right]$$

$$\Rightarrow x = 2 \left(\tan^{-1} t \right) \quad \left[\because t = \tan \theta \right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{2}{1+t^2} \quad \dots(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\Rightarrow \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dx}{dt} = \frac{2}{1+t^2} \quad \dots(i)$$

$$\text{Now, } y = \tan^{-1} \left(\frac{2t}{1-t^2} \right)$$

$$\text{put } t = \tan \theta$$

$$\Rightarrow y = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

$$\Rightarrow y = \tan^{-1} (\tan 2\theta)$$

$$\Rightarrow y = 2\theta \quad \left[\because -\frac{\pi}{2} < 2\theta < \frac{\pi}{2} \right]$$

$$\Rightarrow y = 2 \tan^{-1} t \quad \left[\because t = \tan \theta \right]$$

Q19.

Solution :

$$\text{We have, } x = \frac{\sin^3 t}{\sqrt{\cos 2t}} \text{ and } y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} \left[\frac{\sin^3 t}{\sqrt{\cos 2t}} \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{\sqrt{\cos 2t} \frac{d}{dt} (\sin^3 t) - \sin^3 t \frac{d}{dt} \sqrt{\cos 2t}}{\cos 2t} \quad [\text{Using quotient rule}]$$

$$\Rightarrow \frac{dx}{dt} = \frac{\sqrt{\cos 2t} (3 \sin^2 t) \frac{d}{dt} (\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}} \frac{d}{dt} (\cos 2t)}{\cos 2t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3\sqrt{\cos 2t} (\sin^2 t \cos t) - \frac{\sin^3 t}{2\sqrt{\cos 2t}} (-2 \sin 2t)}{\cos 2t}$$

$$\Rightarrow \frac{dx}{dt} = \frac{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}}$$

$$\text{Now, } \frac{dy}{dt} = \frac{d}{dt} \left[\frac{\cos^3 t}{\sqrt{\cos 2t}} \right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{\sqrt{\cos 2t} \frac{d}{dt} (\cos^3 t) - \cos^3 t \frac{d}{dt} \sqrt{\cos 2t}}{\cos 2t} \quad [\text{Using quotient rule}]$$

$$\Rightarrow \frac{dy}{dt} = \frac{\sqrt{\cos 2t} (3 \cos^2 t) \frac{d}{dt} (\cos t) - \cos^3 t \times \frac{1}{2\sqrt{\cos 2t}} \frac{d}{dt} (\cos 2t)}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dt} = \frac{3\sqrt{\cos 2t} \cos^2 t (-\sin t) - \frac{\cos^3 t}{2\sqrt{\cos 2t}} (-2 \sin 2t)}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-3 \cos 2t \cos^2 t \sin t + \cos^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3 \cos 2t \cos^2 t \sin t + \cos^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}} \times \frac{\cos 2t \sqrt{\cos 2t}}{3 \cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin t \cos t [-3 \cos 2t \cos t + 2 \cos^3 t]}{\sin t \cos t [3 \cos 2t \sin t + 2 \sin^3 t]}$$

$$\Rightarrow \frac{dy}{dx} = \frac{[-3(2 \cos^2 t - 1) \cos t + 2 \cos^3 t]}{[3(1 - 2 \sin^2 t) \sin t + 2 \sin^3 t]} \quad \left[\begin{array}{l} \cos 2t = 2 \cos^2 t - 1 \\ \cos 2t = 1 - 2 \sin^2 t \end{array} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4 \cos^3 t + 3 \cos t}{3 \sin t - 4 \sin^3 t}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\cos 3t}{\sin 3t} \quad \left[\begin{array}{l} \cos 3t = 4 \cos^3 t - 3 \cos t \\ \sin 3t = 3 \sin t - 4 \sin^3 t \end{array} \right]$$

$$\therefore \frac{dy}{dx} = -\cot 3t$$

Q20.

Solution :

$$\text{We have, } x = \left(t + \frac{1}{t}\right)^a$$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} \left[\left(t + \frac{1}{t}\right)^a \right]$$

$$\Rightarrow \frac{dx}{dt} = a \left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt} \left(t + \frac{1}{t}\right)$$

$$\Rightarrow \frac{dx}{dt} = a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right) \quad \dots(i)$$

and,

$$y = a^{\left(t + \frac{1}{t}\right)}$$

$$\Rightarrow \frac{dy}{dt} = \frac{d}{dt} \left[a^{\left(t + \frac{1}{t}\right)} \right]$$

$$\Rightarrow \frac{dy}{dt} = a^{\left(t + \frac{1}{t}\right)} \times \log a \frac{d}{dt} \left(t + \frac{1}{t}\right)$$

$$\Rightarrow \frac{dy}{dt} = a^{\left(t + \frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right) \quad \dots(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a \left(1 - \frac{1}{t^2}\right)}{a \left(t + \frac{1}{t}\right)^{a-1} \left(1 - \frac{1}{t^2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a^{\left(t + \frac{1}{t}\right)} \times \log a}{a \left(t + \frac{1}{t}\right)^{a-1}}$$

Q21.

Solution :

$$\text{We have, } x = a \left(\frac{1+t^2}{1-t^2} \right)$$

$$\Rightarrow \frac{dx}{dt} = a \left[\frac{(1-t^2) \frac{d}{dt}(1+t^2) - (1+t^2) \frac{d}{dt}(1-t^2)}{(1-t^2)^2} \right] \quad [\text{Using quotient rule}]$$

$$\Rightarrow \frac{dx}{dt} = a \left[\frac{(1-t^2)(2t) - (1+t^2)(-2t)}{(1-t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = a \left[\frac{2t - 2t^3 + 2t + 2t^3}{(1-t^2)^2} \right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{4at}{(1-t^2)^2} \quad \dots(i)$$

and,

$$y = \frac{2t}{1-t^2}$$

$$\Rightarrow \frac{dy}{dt} = 2 \left[\frac{(1-t^2) \frac{d}{dt}(t) - t \frac{d}{dt}(1-t^2)}{(1-t^2)^2} \right] \quad [\text{Using quotient rule}]$$

$$\Rightarrow \frac{dy}{dt} = 2 \left[\frac{(1-t^2)(1) - t(-2t)}{(1-t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = 2 \left[\frac{1-t^2 + 2t^2}{(1-t^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{2(1+t^2)}{(1-t^2)^2} \quad \dots(ii)$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(1+t^2)}{(1-t^2)^2} \times \frac{(1-t^2)^2}{4at}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+t^2)}{2at}$$

Q22.

Solution :

We have, $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}[10(t - \sin t)] \text{ and } \frac{dy}{dt} = \frac{d}{dt}[12(1 - \cos t)]$$

$$\Rightarrow \frac{dx}{dt} = 10 \frac{d}{dt}(t - \sin t) \text{ and } \frac{dy}{dt} = 12 \frac{d}{dt}(1 - \cos t)$$

$$\Rightarrow \frac{dx}{dt} = 10(1 - \cos t) \text{ and } \frac{dy}{dt} = 12[0 - (-\sin t)] = 12 \sin t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{12 \sin t}{10(1 - \cos t)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{12 \times 2 \sin \frac{t}{2} \cos \frac{t}{2}}{10 \times 2 \sin^2 \frac{t}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6}{5} \cot \frac{t}{2}$$

Q23.

Solution :

We have, $x = a(\theta - \sin \theta)$ and $y = a(1 + \cos \theta)$

$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}[a(\theta - \sin \theta)] \text{ and } \frac{dy}{d\theta} = \frac{d}{d\theta}[a(1 + \cos \theta)]$$

$$\Rightarrow \frac{dx}{d\theta} = a(1 - \cos \theta) \text{ and } \frac{dy}{d\theta} = a(-\sin \theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a \sin \theta}{a(1 - \cos \theta)}$$

$$\text{Now, } \left[\frac{dy}{dx} \right]_{\theta = \frac{\pi}{3}} = -\frac{\sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} = -\frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}} = -\sqrt{3}$$