

RD Sharma
Solutions
Class 12 Maths
Chapter 11
Ex 11.8

Chapter: Differentiation

Exercise: 11.8

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Q1.

Answer :

$$\text{Let } u = x^2 \text{ and } v = x^3$$

$$\Rightarrow \frac{du}{dx} = 2x \text{ and } \frac{dv}{dx} = 3x^2$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2} = \frac{2}{3x}$$

Q2.

Answer :

$$\text{Let } u = \log(1+x^2) \text{ and } v = \tan^{-1} x$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{(1+x^2)} \frac{d}{dx}(1+x^2) = \frac{2x}{(1+x^2)} \text{ and } \frac{dv}{dx} = \frac{1}{1+x^2}$$

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{1+x^2} \times \frac{1+x^2}{1} = 2x$$

Q3.

Answer :

$$\text{Let } u = (\log x)^x$$

Taking log on both sides,

$$\log u = \log(\log x)^x$$

$$\Rightarrow \log u = x \log(\log x)$$

$$\begin{aligned}
\Rightarrow \frac{1}{u} \frac{du}{dx} &= x \frac{d}{dx} \{ \log(\log x) \} + \log(\log x) \frac{d}{dx}(x) \\
\Rightarrow \frac{1}{u} \frac{du}{dx} &= x \left(\frac{1}{\log x} \right) \frac{d}{dx}(\log x) + \log \log x (1) \\
\Rightarrow \frac{du}{dx} &= u \left[\frac{x}{\log x} \left(\frac{1}{x} \right) + \log \log x \right] \\
\Rightarrow \frac{du}{dx} &= (\log x)^x \left[\frac{1}{\log x} + \log \log x \right] \quad \dots(i)
\end{aligned}$$

Again, let $v = \log x$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{x} \quad \dots(ii)$$

Dividing equation (i) by (ii), we get

$$\begin{aligned}
\frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{(\log x)^x \left[\frac{1}{\log x} + \log \log x \right]}{\frac{1}{x}} \\
\Rightarrow \frac{du}{dv} &= \frac{(\log x)^x \left[\frac{1 + \log x (\log \log x)}{\log x} \right]}{\frac{1}{x}} \\
\Rightarrow \frac{du}{dv} &= x (\log x)^{x-1} (1 + \log x \times \log \log x)
\end{aligned}$$

Q4.

Answer :

$$(i) \text{ Let, } u = \sin^{-1} \sqrt{1-x^2}$$

$$\text{Put } x = \cos \theta$$

$$\Rightarrow u = \sin^{-1} \sqrt{1-\cos^2 \theta}$$

$$\Rightarrow u = \sin^{-1} (\sin \theta) \quad \dots(i)$$

$$\text{And, } v = \cos^{-1} x \quad \dots(ii)$$

$$\text{Now, } x \in (0,1)$$

$$\Rightarrow \cos \theta \in (0,1)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow u = \cos^{-1} x \quad [\text{Since, } \cos \theta = x]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iii)$$

from equation (ii),

$$v = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1} \therefore \frac{du}{dv} = 1$$

$$(ii) \text{ Let, } u = \sin^{-1} \sqrt{1-x^2}$$

$$\text{Put } x = \cos \theta$$

$$\Rightarrow u = \sin^{-1} \sqrt{1-\cos^2 \theta}$$

$$\Rightarrow u = \sin^{-1} (\sin \theta) \quad \dots(i)$$

$$\text{And, } v = \cos^{-1} x \quad \dots(ii)$$

$$\text{Now, } x \in (-1, 0)$$

$$\Rightarrow \cos \theta \in (-1, 0)$$

$$\Rightarrow \theta \in \left(\frac{\pi}{2}, \pi \right)$$

So, from equation (i),

$$u = \pi - \theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \pi - \theta \text{ if } \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2} \right) \right]$$

$$\Rightarrow u = \pi - \cos^{-1} x \quad [\text{Since, } x = \cos \theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(iii)$$

from equation (ii),

$$v = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\therefore \frac{du}{dx} = -1$$

Q5.

Answer :

$$(i) \text{ Let, } u = \sin^{-1}(4x\sqrt{1-4x^2})$$

$$\text{put } 2x = \cos \theta$$

$$\Rightarrow u = \sin^{-1}(2 \times \cos \theta \sqrt{1 - \cos^2 \theta})$$

$$\Rightarrow u = \sin^{-1}(2 \cos \theta \sin \theta)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let, } v = \sqrt{1-4x^2} \quad \dots(ii)$$

Here,

$$x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow 2x \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

So, from equation (i),

$$u = \pi - 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \pi - \theta, \text{ if } \theta \in \left(\frac{\pi}{2}, \pi\right) \right]$$

$$\Rightarrow u = \pi - 2 \cos^{-1}(2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}} \quad (2)$$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \dots(iii)$$

from equation (ii)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

$$\text{but, } x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \dots(iv)$$

Diferentiating equation (ii) with respect to x,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} \frac{d}{dx}(1-4x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} (-8x)$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \quad \dots(v)$$

Dividing equation (iii) by (v)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\therefore \frac{du}{dv} = -\frac{1}{x}$$

$$(ii) \text{ Let, } u = \sin^{-1}(4x\sqrt{1-4x^2})$$

$$\text{put } 2x = \cos \theta$$

$$u = \sin^{-1}(2 \times \cos \theta \sqrt{1 - \cos^2 \theta})$$

$$\Rightarrow u = \sin^{-1}(2 \cos \theta \sin \theta)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let, } v = \sqrt{1-4x^2} \quad \dots(ii)$$

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$$

$$\Rightarrow 2x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \cos \theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow u = 2 \cos^{-1}(2x) \quad \left[\text{Since, } 2x = \cos \theta \right]$$

Differentiate it with respect to x,

$$\frac{du}{dx} = 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx}(2x)$$

$$\frac{du}{dx} = \frac{-2}{\sqrt{1-4x^2}} (2)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \quad \dots(iii)$$

Diferentiating equation (ii) with respect to x,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} \frac{d}{dx}(1-4x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} (-8x)$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\therefore \frac{du}{dv} = \frac{1}{x}$$

$$(iii) \text{ Let, } u = \sin^{-1}(4x\sqrt{1-4x^2})$$

$$\text{put, } 2x = \cos \theta$$

$$\Rightarrow u = \sin^{-1}(2 \times \cos \theta \sqrt{1 - \cos^2 \theta})$$

$$\Rightarrow u = \sin^{-1}(2 \cos \theta \sin \theta)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let, } v = \sqrt{1-4x^2} \quad \dots(ii)$$

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \pi - \theta, \text{ if } \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \right]$$

$$\Rightarrow u = \pi - 2 \cos^{-1}(2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiate it with respect to x ,

$$\frac{du}{dx} = 0 - 2 \left(\frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-4x^2}} \quad (2)$$

$$\Rightarrow \frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \dots(iii)$$

from equation (ii),

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

$$\text{but, } x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$$

$$\therefore \frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\therefore \frac{du}{dv} = \frac{1}{x}$$

Q6.

Answer :

$$\text{Let, } u = \tan^{-1} \left(\frac{\sqrt{1+x^2}-1}{x} \right)$$

$$\text{put } x = \tan \theta$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\frac{1-\cos \theta}{\sin \theta} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\tan \frac{\theta}{2} \right) \quad \dots(i)$$

And,

$$v = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\Rightarrow v = \sin^{-1} \left(\frac{2 \tan \theta}{1+\tan^2 \theta} \right)$$

$$\Rightarrow v = \sin^{-1} (\sin 2\theta) \quad \dots(ii)$$

Here,

$$-1 < x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4} \quad \dots(A)$$

So, from equation (i)

$$u = \frac{\theta}{2} \quad \left[\text{Since, } \tan^{-1} (\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\Rightarrow u = \frac{1}{2} \tan^{-1} x \quad [\text{since, } x = \tan \theta]$$

$$\frac{du}{dx} = \frac{1}{2} \left(\frac{1}{1+x^2} \right)$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2(1+x^2)} \quad \dots(i)$$

Now, from equation (ii) and (A),

$$v = 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$\Rightarrow v = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

$$\frac{dv}{dx} = 2 \left(\frac{1}{1+x^2} \right) \quad \dots (iv)$$

dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2(1+x^2)} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = \frac{1}{4}$$

Q7.

Answer :

$$(i) \text{ Let, } u = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$$

$$\text{Put } x = \sin \theta$$

$$\Rightarrow u = \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right)$$

$$\Rightarrow u = \sin^{-1} (2 \sin \theta \cos \theta)$$

And,

$$\text{Let } v = \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow v = \sec^{-1} \left(\frac{1}{\sqrt{1-\sin^2 \theta}} \right)$$

$$\Rightarrow v = \sec^{-1} \left(\frac{1}{\cos \theta} \right)$$

$$\Rightarrow v = \sec^{-1} (\sec \theta)$$

$$\Rightarrow v = \cos^{-1} \left(\frac{1}{\frac{1}{\cos \theta}} \right)$$

$$\Rightarrow v = \cos^{-1} (\cos \theta) \quad \dots(ii)$$

Here,

$$x \in \left(0, \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \sin \theta \in \left(0, \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4} \right)$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\text{Let, } u = 2 \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2 \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \dots(iii)$$

And, from equation (ii),

$$v = \theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$\Rightarrow v = \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right]$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(iv)$$

dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\therefore \frac{du}{dv} = 2$$

$$(ii) \text{ Let, } u = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$$

$$\text{Put } x = \sin \theta$$

$$\Rightarrow u = \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right)$$

$$\Rightarrow u = \sin^{-1} (2 \sin \theta \cos \theta)$$

$$\Rightarrow u = \sin^{-1} (\sin 2\theta) \quad \dots(i)$$

And,

$$\text{Let, } v = \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow v = \sec^{-1} \left(\frac{1}{\sqrt{1 - \sin^2 \theta}} \right)$$

$$\Rightarrow v = \sec^{-1} \left(\frac{1}{\cos \theta} \right)$$

$$\Rightarrow v = \sec^{-1} (\sec \theta)$$

$$\Rightarrow v = \cos^{-1} \left(\frac{1}{\frac{1}{\cos \theta}} \right) \quad \left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right]$$

$$\Rightarrow v = \cos^{-1} (\cos \theta) \quad \dots(ii)$$

Here,

$$x \in \left(\frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \sin \theta \in \left(\frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2} \right)$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\text{Let, } u = 2 \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right]$$

$$\frac{du}{dx} = 2 \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \dots(iii)$$

And, from equation (ii),

$$v = \theta \quad [\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi]]$$

$$\Rightarrow v = \sin^{-1} x \quad [\text{Since, } x = \sin \theta]$$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(iv)$$

dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\therefore \frac{du}{dv} = 2$$

Q8.

Answer :

$$\text{Let, } u = (\cos x)^{\sin x}$$

Taking log on both sides,

$$\log u = \log (\cos x)^{\sin x}$$

$$\Rightarrow \log u = \sin x \log (\cos x)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{u} \frac{du}{dx} = \sin x \frac{d}{dx} (\log \cos x) + \log \cos x \frac{d}{dx} (\sin x) \quad [\text{using product rule}]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \sin x \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x (\cos x)$$

$$\Rightarrow \frac{du}{dx} = u [(\tan x) \times (-\sin x) + \log \cos x (\cos x)]$$

$$\Rightarrow \frac{du}{dx} = (\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x] \quad \dots(i)$$

$$\text{Let, } v = (\sin x)^{\cos x}$$

Taking log on both sides,

$$\log v = \log(\sin x)^{\cos x}$$

$$\Rightarrow \log v = \cos x \log(\sin x)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{v} \frac{dv}{dx} = \cos x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\cos x) \quad [\textit{using product rule}]$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \cos x \left(\frac{1}{\sin x} \right) \frac{d}{dx}(\sin x) + \log \sin x (-\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v [\cot x (\cos x) - \sin x \log \sin x]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x]$$

dividing equation (i) by (ii),

$$\therefore \frac{du}{dv} = \frac{(\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x]}{(\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x]}$$

Q9.

Answer :

$$\text{Let, } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\text{Put } x = \tan \theta$$

$$\Rightarrow u = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let } v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow v = \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$\Rightarrow v = \cos^{-1}(\cos 2\theta) \quad \dots(ii)$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow u = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \dots(iii)$$

from equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$\Rightarrow v = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = 1$$

Q10.

Answer :

$$\text{Let, } u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$$

$$\text{Put } ax = \tan \theta$$

$$\Rightarrow u = \tan^{-1}\left(\frac{1+\tan \theta}{1-\tan \theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left(\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \theta\right)\right]$$

$$\Rightarrow u = \frac{\pi}{4} + \theta$$

$$\Rightarrow u = \frac{\pi}{4} + \tan^{-1}(ax) \quad [\text{Since, } \tan \theta = ax]$$

$$\frac{du}{dx} = 0 + \frac{1}{1+(ax)^2} \frac{d}{dx}(ax)$$

$$\Rightarrow \frac{du}{dx} = \frac{a}{1+a^2x^2} \quad \dots(i)$$

Now,

$$\text{Let, } v = \sqrt{1+a^2x^2}$$

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}} \frac{d}{dx}(1+a^2x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}} (2a^2x)$$

$$\Rightarrow \frac{dv}{dx} = \frac{a^2x}{\sqrt{1+a^2x^2}} \quad \dots(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{a}{1+a^2x^2} \times \frac{\sqrt{1+a^2x^2}}{a^2x}$$

$$\frac{du}{dv} = \frac{1}{ax\sqrt{1+a^2x^2}}$$

Q11.

Answer :

$$\text{Let, } u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$\text{Put } x = \sin \theta$$

$$\Rightarrow u = \sin^{-1}\left(2 \sin \theta \sqrt{1 - \sin^2 \theta}\right)$$

$$\Rightarrow u = \sin^{-1}(2 \sin \theta \cos \theta)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let } v = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow v = \tan^{-1}\left(\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}\right)$$

$$\Rightarrow v = \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right)$$

$$\Rightarrow v = \tan^{-1}(\tan \theta) \quad \dots(ii)$$

$$\text{Here, } -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$\Rightarrow u = 2 \sin^{-1} x \quad [\text{Since, } x = \sin \theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \dots(iii)$$

from equation (ii),

$$v = \theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow v = \sin^{-1} x \quad [\text{Since, } x = \sin \theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\therefore \frac{du}{dv} = 2$$

Q12.

Answer :

$$\text{Let, } u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\text{Put } x = \tan \theta$$

$$\Rightarrow u = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$\Rightarrow u = \tan^{-1}(\tan 2\theta) \quad \dots(i)$$

$$\text{let, } v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow v = \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$\Rightarrow v = \cos^{-1}(\cos 2\theta) \quad \dots(ii)$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow u = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \dots(iii)$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$\Rightarrow v = 2 \tan^{-1} x \quad \left[\text{Since, } x = \tan \theta \right]$$

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = 1$$

Q13.

Answer :

$$\text{Let, } u = \tan^{-1} \left(\frac{x-1}{x+1} \right)$$

$$\text{Put } x = \tan \theta$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\tan \theta - 1}{\tan \theta + 1} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\tan \theta - \tan \frac{\pi}{4}}{1 + \tan \theta \tan \frac{\pi}{4}} \right)$$

$$\Rightarrow u = \tan^{-1} \left[\tan \left(\theta - \frac{\pi}{4} \right) \right] \quad \dots(i)$$

$$\text{Here, } -\frac{1}{2} < x < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} < \tan \theta < \frac{1}{2}$$

$$\Rightarrow -\tan^{-1} \left(\frac{1}{2} \right) < \theta < \tan^{-1} \left(\frac{1}{2} \right)$$

So, from equation (i),

$$u = \theta - \frac{\pi}{4} \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\Rightarrow u = \tan^{-1} x - \frac{\pi}{4} \quad \left[\text{Since, } x = \tan \theta \right]$$

differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$
$$\Rightarrow \frac{du}{dx} = \frac{1}{1+x^2} \quad \dots(ii)$$

And,

$$\text{Let, } v = \sin^{-1}(3x - 4x^3)$$

$$\text{Put } x = \sin \theta$$

$$\Rightarrow v = \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta)$$

$$\Rightarrow v = \sin^{-1}(\sin 3\theta) \quad \dots(iii)$$

$$\text{Now, } -\frac{1}{2} < x < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} < \sin \theta < \frac{1}{2}$$

$$\Rightarrow -\frac{1}{6} < \theta < \frac{\pi}{6}$$

So, from equation (iii),

$$v = 3\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$\Rightarrow v = 3 \sin^{-1} x \quad [\text{Since, } x = \sin \theta]$$

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\therefore \frac{du}{dv} = \frac{\sqrt{1-x^2}}{3(1+x^2)}$$

Q14.

Answer :

$$\text{Let, } u = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$$

$$\Rightarrow u = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\Rightarrow u = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{2}\right)$$

$$\frac{du}{dx} = -\frac{1}{2} \quad \dots(i)$$

$$\text{Let, } v = \sec^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}} \quad \dots(ii)$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2} \times \frac{x\sqrt{x^2-1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$$

Q15.

Answer :

$$\text{Let, } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\text{Put } x = \tan \theta \Rightarrow \theta = \tan^{-1} x,$$

$$\Rightarrow u = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$\Rightarrow u = \sin^{-1}(\sin 2\theta) \quad \dots(i)$$

$$\text{Let, } v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow v = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$\Rightarrow v = \tan^{-1}(\tan 2\theta) \quad \dots(ii)$$

$$\text{Here, } -1 < x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \tan \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$\Rightarrow u = 2 \tan^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \dots(iii)$$

from equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow v = 2 \tan^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\therefore \frac{du}{dv} = 1$$

Q16.**Answer :**

$$\text{Let, } u = \cos^{-1}(4x^3 - 3x)$$

$$\text{Put, } x = \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} x$$

$$\text{Now, } u = \cos^{-1}(4 \cos^3 \theta - 3 \cos \theta)$$

$$\Rightarrow u = \cos^{-1}(\cos 3\theta) \dots (i)$$

$$\text{Let, } v = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

$$\Rightarrow v = \tan^{-1}\left(\frac{\sqrt{1-\cos^2 \theta}}{\cos \theta}\right)$$

$$\Rightarrow v = \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right)$$

$$\Rightarrow v = \tan^{-1}(\tan \theta) \dots (ii)$$

Here,

$$\frac{1}{2} < x < 1$$

$$\Rightarrow \frac{1}{2} < \cos \theta < 1$$

So, from equation (i),

$$u = 3\theta \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$\Rightarrow u = 3 \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \dots (iii)$$

From equation (ii),

$$v = \theta \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow v = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-3}{\sqrt{1-x^2}} \right) \left(-\frac{\sqrt{1-x^2}}{1} \right)$$

$$\therefore \frac{du}{dv} = 3$$

Q17.

Answer :

$$\text{Let, } u = \tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$$

$$\text{Put } x = \sin \theta$$

$$\Rightarrow \theta = \sin^{-1} x$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\sin \theta}{\sqrt{1-\sin^2 \theta}} \right)$$

$$\Rightarrow u = \tan^{-1} \left(\frac{\sin \theta}{\cos \theta} \right)$$

$$\Rightarrow u = \tan^{-1} (\tan \theta) \quad \dots(i)$$

And

$$\text{Let, } v = \sin^{-1} (2x\sqrt{1-x^2})$$

$$v = \sin^{-1} (2 \sin \theta \sqrt{1-\sin^2 \theta})$$

$$v = \sin^{-1} (2 \sin \theta \cos \theta)$$

$$v = \sin^{-1} (\sin 2\theta) \quad \dots(ii)$$

Here,

$$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \tan^{-1} (\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$\Rightarrow u = \sin^{-1} x$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(iii)$$

from equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$\Rightarrow v = 2 \sin^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{1}{\sqrt{1-x^2}} \right) \left(\frac{\sqrt{1-x^2}}{2} \right)$$

$$\therefore \frac{du}{dv} = \frac{1}{2}$$

Q18.

Answer :

$$\text{Let, } u = \sin^{-1}(\sqrt{1-x^2})$$

$$\text{Put } x = \cos \theta$$

$$\Rightarrow \theta = \cos^{-1} x$$

$$\text{We get, } u = \sin^{-1}(\sin \theta) \quad \dots(i)$$

$$\text{Let, } v = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow v = \cot^{-1}\left(\frac{\cos \theta}{\sqrt{1-\cos^2 \theta}}\right)$$

$$\Rightarrow v = \cot^{-1}\left(\frac{\cos \theta}{\sin \theta}\right)$$

$$\Rightarrow v = \cot^{-1}(\cot \theta) \quad \dots(ii)$$

Here,

$$0 < x < 1$$

$$\Rightarrow 0 < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$u = \theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$\Rightarrow u = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iii)$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \cot^{-1}(\cot \theta) = \theta, \text{ if } \theta \in (0, \pi) \right]$$

$$\Rightarrow v = \cos^{-1} x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \dots(iv)$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-1}{\sqrt{1-x^2}} \right) \left(\frac{\sqrt{1-x^2}}{-1} \right)$$

$$\therefore \frac{du}{dv} = 1$$

Q19.

Answer :

$$\text{Let, } u = \sin^{-1} \left(2ax\sqrt{1-a^2x^2} \right)$$

$$\text{Put } ax = \sin \theta \Rightarrow \theta = \sin^{-1}(ax)$$

$$\therefore u = \sin^{-1} \left(2 \sin \theta \sqrt{1-\sin^2 \theta} \right)$$

$$\Rightarrow u = \sin^{-1} (2 \sin \theta \cos \theta)$$

$$\Rightarrow u = \sin^{-1} (\sin 2\theta) \quad \dots(i)$$

And

Let,

$$v = \sqrt{1-a^2x^2}$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx} (1-a^2x^2)$$

$$\Rightarrow \frac{dv}{dx} = \left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}} \right)$$

$$\Rightarrow \frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \quad \dots(ii)$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2 \times \frac{1}{\sqrt{1-(ax)^2}} \frac{d}{dx} (ax)$$

$$\Rightarrow \frac{du}{dx} = \frac{2}{1-a^2x^2} (a)$$

$$\Rightarrow \frac{du}{dx} = \frac{2a}{1-a^2x^2} \quad \dots(iii)$$

Dividing equation (iii) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{2a}{\sqrt{1-a^2x^2}} \right) \left(\frac{\sqrt{1-a^2x^2}}{-a^2x} \right)$$

$$\therefore \frac{du}{dv} = -\frac{2}{ax}$$

Q20.

Answer :

$$\text{Let, } u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$$

$$\text{Put } x = \tan \theta$$

$$\Rightarrow \theta = \tan^{-1} x$$

$$\Rightarrow u = \tan^{-1}\left(\frac{1 - \tan \theta}{1 + \tan \theta}\right)$$

$$\Rightarrow u = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right] \quad \dots(i)$$

Here,

$$-1 < x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} > -\theta > \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{4} < -\theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < \frac{\pi}{4} - \theta < \frac{\pi}{2}$$

So, from equation (i),

$$u = \frac{\pi}{4} - \theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$\Rightarrow u = \frac{\pi}{4} - \tan^{-1} x$$

$$\frac{du}{dx} = 0 - \left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \frac{du}{dx} = -\frac{1}{1+x^2} \quad \dots(ii)$$

$$\text{And let, } v = \sqrt{1-x^2}$$

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}(1-x^2)$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \quad \dots(iii)$$

Dividing equation (ii) by (iii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{-x}$$

$$\therefore \frac{du}{dv} = \frac{\sqrt{1-x^2}}{x(1+x^2)}$$