RD Sharma
Solutions
Class 12 Maths
Chapter 15
Ex 15.1

Differentiating it with respect to x, $f'(x) = \frac{2}{3} \times \frac{1}{(x-2)^{\frac{1}{3}}}$

Mean Value Theorems Ex 15.1 Q1(i)

 $f(x) = 3 + (x - 2)^{\frac{2}{3}}$ on [1, 3]

Clearly,
$$\lim_{x \to 2} = \frac{2}{3} \times \frac{1}{(x-2)^{\frac{1}{3}}}$$

Hene, Rolle's theorem is not applicable for
$$f(x)$$
 in $x \in [1,3]$.

Mean Value Theorems Ex 15.1 O1(ii)

Mean Value Theorems Ex 15.1 Q1(ii)

Here,
$$f(x) = [x]$$
 and $x \in [-1, 1]$, at $n = 1$

$$LHL = \lim_{x \to (1-h)} [x]$$

LHL =
$$\lim_{x \to (1-h)} [x]$$
=
$$\lim_{h \to 0} [1-h]$$

$$= \lim_{h \to 0} [1 - h]$$

$$= 0$$
RHL = $\lim_{h \to 0} [x]$

$$= 0$$
RHL = $\lim_{x \to (1+h)} [x]$

$$= \lim_{x \to 1} [1+h]$$

LHL ≠ RHL

= 1

$$= 0$$

$$= \lim_{x \to (1+h)} [x]$$

$$\lim_{x \to \infty} [x \to h]$$

So, f(x) is not continuous at $1 \in [-1,1]$

Mean Value Theorems Ex 15.1 Q1(iii)

$$= \lim_{x \to (1+h)} [x]$$

$$= \lim_{h \to 0} [1+h]$$

$$\lim_{x \to (1+h)} [x]$$

$$\lim_{x \to (1+h)} [1+h]$$

Hence, rolle's theorem is not applicable on f(x) in [-1,1].

Thus, f(x) is not differentiable at $x = 2 \in (1,3)$

Here,
$$f(x) = \sin\left(\frac{1}{x}\right)$$
, $x \in [-1,1]$, at $n = 0$

LHS
$$= \lim_{x \to (0-h)} \sin\left(\frac{1}{x}\right)$$

$$= \lim_{h \to 0} \sin\left(\frac{1}{0-h}\right)$$

$$= \lim_{h \to 0} \sin\left(\frac{-1}{h}\right)$$

$$= -\lim_{h \to 0} \sin\left(\frac{1}{h}\right)$$

$$= -k \qquad \qquad \left[\text{Let } \lim_{h \to 0} \sin\left(\frac{1}{h}\right) = k \text{ as } k \in [-1, 1] \right]$$

RHS =
$$\lim_{x \to (0+h)} \sin\left(\frac{1}{x}\right)$$

= $\lim_{h \to 0} \sin\left(\frac{1}{h}\right)$
= k

- ⇒ LHS≠RHS
- \Rightarrow f(x) is not continuous at n = 0

So, rolle's theorem is not applicable on f(x) in [-1,1]

Mean Value Theorems Ex 15.1 Q1(iv)

Here, $f(x) = 2x^2 - 5x + 3$ on [1, 3]

f(x) is continuous in [1,3] and f(x) is differentiable is (1,3) since it is a polynomial function.

Now,

$$f(x) = 2x^{2} - 5x + 3$$

$$f(1) = 3(1)^{2} - 5(1) + 3$$

$$= 2 - 5 + 3$$

$$f(1) = 0 ---(i)$$

$$f(3) = 2(3)^{2} - 5(3) + 3$$

$$= 18 - 15 + 3$$

$$f(3) = 6 ---(ii)$$

From equation (i) and (ii),

$$f(1) \neq f(3)$$

So, rolle's theorem is not applicable on f(x) in [1,3].

Mean Value Theorems Ex 15.1 Q1(v)

LHS =
$$\lim_{x \to (1-h)} (-4x + 5)$$

= $\lim_{h \to 0} [-4(1-h) + 5]$
= $-4 + 5$
LHS = 1
RHS = $\lim_{x \to (1+h)} (2x - 3)$
= $\lim_{h \to 0} [2(1+h) - 3]$
= $2 - 3$
RHS = -1

f(x) is not continuous at $x = 1 \in [0,2]$

Rolle's theorem is not applicable on f(x) in [0,2].

LHS ≠ RHS

Mean Value Theorems Ex 15.1 Q2(i)

Here, $f(x) = x^{\frac{2}{3}}$ on [-1,1]

 $f'(x) = \frac{2}{\frac{1}{3x^{\frac{3}{3}}}}$

 $f'(0) = \frac{2}{3(0)^{\frac{1}{3}}}$

So, f'(x) does not exist at $x = 0 \in (-1, 1)$

Mean Value Theorems Ex 15.1 Q1(vi)

Here, $f(x) = \begin{cases} -4x + 5, & 0 \le x \le 1 \\ 2x - 3, & 1 < x \le 2 \end{cases}$

For n = 1

So.

 \Rightarrow

 \Rightarrow

f(x) is not differentiatable in $x \in (-1,1)$

So, rolle's theorem is not applicable on f(x) in [-1,1].

 $f'(0) = \infty$

$$f(x) = x^2 - 8x + 12$$
 on [2,6]

f(x) is continuous is [2,6] and differentiable is (2,6) as it is a polynomial function

And
$$f(2) = (2)^2 - 8(2) + 12 = 0$$

 $f(6) = (6)^2 - 8(6) + 12 = 0$
 $\Rightarrow f(2) = f(6)$

So, Rolle's theorem is applicable, therefore we show have f'(c) = 0 such that $c \in (2,6)$

So,
$$f(x) = x^2 - 8x + 12$$

 $\Rightarrow f'(x) = 2x - 8$

So,
$$f'(c) = 0$$

 $2c - 8 = 0$
 $c = 4 \in (2,6)$

Therefore, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(ii)

The given function is $f(x) = x^2 - 4x + 3$

f, being a polynomial function, is continuous in [1, 4] and is differentiable in (1, 4) whose derivative is 2x - 4.

$$f(1) = 1^{2} - 4 \times 1 + 3 = 0, f(4) = 4^{2} - 4 \times 4 + 3 = 3$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{f(4) - f(1)}{4 - 1} = \frac{3 - (0)}{3} = \frac{3}{3} = 1$$

Mean Value Theorem states that there is a point $c \in (1, 4)$ such that f'(c) = 1

$$f'(c) = 1$$

 $\Rightarrow 2c - 4 = 1$
 $\Rightarrow c = \frac{5}{2}$, where $c = \frac{5}{2} \in (1, 4)$

Hence, Mean Value Theorem is verified for the given function

Mean Value Theorems Ex 15.1 Q2(iii)

So, Rolle's theorem is applicable on f(x) in $\lceil 1,2 \rceil$, therefore, there exist a $c \in (1,2)$ such that f'(c) = 0

 $f(x) = (x-1)(x-2)^2$

f'(x) = (x - 2)(3x - 4)

 $f(2) = (2-1)(2-2)^2 = 0$

it is a polynomial function.

 \Rightarrow f(1) = f(2)

So, f'(c) = 0

And $f(1) = (1-1)(1-2)^2 = 0$

 $f(x) = (x-1)(x-2)^2$ on (1,2)

f(x) is cantinuous is [1,2] and differentiable is (1,2) since

Here.

Now.

 $\Rightarrow c = 2 \text{ or } c = \frac{4}{3} \in (1,2)$

(c-2)(3c-4)=0

 $f'(x) = (x-1) \times 2(x-2) + (x-2)^2$

 $f(x) = x(x-1)^2$ on [0,1]f(x) is continuous on [0,1] and differentiable on (0,1) as

Here,

Now,

$$f(0) = 0(0-1)^2 = 0$$

 $f(1) = 1(1-1)^2 = 0$
 $\Rightarrow f(0) = f(1)$

So, Rolle's theorem is applicable on f(x) in [0,1] therefore, we should show that there exist a $c \in (0,1)$ such that f'(c) = 0

Now,

$$f(x) = x(x-1)^{2}$$

$$f'(x) = (x-1)^{2} + x \times 2(x-1)$$

$$= (x-1)(x-1+2x)$$

$$f'(x) = (x-1)(3x-1)$$
So, $f'(c) = 0$

Thus, Rolle's theorem is verified.

(c-1)(3c-1)=0

 $\Rightarrow c = 1 \text{ or } c = \frac{1}{3} \in (0, 1)$

Mean Value Theorems Ex 15.1 Q2(v)

 $f(x) = (x^2 - 1)(x - 2)$ on [-1, 2]f(x) is continuous is [-1,2] and differentiable in (-1,2) as

it is a polynomial functions.

Here,

Now,

$$f(-1) = (1-1)(-1-2) = 0$$

$$f(2) = (4-1)(2-2) = 0$$

$$\Rightarrow f(-1) = f(2)$$

So, Rolle's theorem is applicable on f(x) is [-1,2] therefore, we have to show that there exist a $c \in (-1,2)$ such that f'(c) = 0

Now,

$$f(x) = (x^{2} - 1)(x - 2)$$

$$f'(x) = 2x(x - 2) + (x^{2} - 1)$$

$$= 2x^{2} - 4 + x^{2} - 1$$

$$f'(x) = 3x^{2} - 5$$

$$\Rightarrow \qquad x = -\sqrt{\frac{5}{3}} \text{ or } x = \sqrt{\frac{5}{3}} \in (-1, 2)$$

f'(c) = 0

 \Rightarrow $3x^2 - 5 = 0$

Thus, Rolle's theorem is verified.

Here, $f(x) = x(x-4)^2$ on [0,4]f(x) is continuous is [0,4] and differentiable is (0,4) since f(x) is a polynomial function.

---(i)

---(ii)

Now,

$$f(0) = 0(0 - 4)^{2}$$

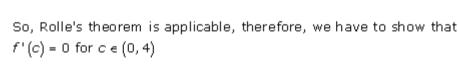
$$f(0) = 0$$

$$f(4) = 4(4 - 4)^{2}$$

$$f(4) = 0$$

 $f(x) = x(x-4)^2$

From equation (i) and (ii),
$$f(0) = f(4)$$



 $f'(x) = x \times 2(x-4) + (x-4)^2$

$$f'(x) = x \times 2(x-4) + (x-4)^{-1}$$
$$= 2x^{2} - 8x + x^{2} + 16 - 8x$$
So,
$$f'(c) = 3c^{2} - 16c + 16$$

So,
$$f'(c) = 3c^2 - 16c + 16$$

 $0 = 3c^2 - 12c - 4c + 16$
 $0 = 3c(c - 4) - 4(c - 4)$

 $c = 4 \text{ or } c = \frac{4}{3} \in (0,4)$

0 = (c - 4)(3c - 4)

And $f(0) = 0(0-2)^2 = 0$ $f(2) = 2(2-2)^2 = 0$ \Rightarrow f(0) = f(2)

f(x) is continuous is [0,2] and differentiable is (0,2)

Here, $f(x) = x(x-2)^2$ on [0,2]

as it is a polynomial function.

So, Rolle's theorem is applicable on
$$f(x)$$
 is $[0,2]$, therefore, we have to show that $f'(c) = 0$ as $c \in (0,2)$

$$f(x) = x(x-2)^{2}$$

$$f'(x) = x \times 2(x-2) + (x-2)$$

$$f'(x) = 2x(x-2) + (x-2)$$

$$f'(x) = 2x(x-2) + (x-2)$$

$$\Rightarrow f'(c) = 0$$

$$2c(c-2) + (c-2) = 0$$

$$(c-2)(2c+1) = 0$$

⇒
$$c = 2 \in (0,2)$$

So, Rolle's theorem is verified.

 $c = 2 \text{ or } c = -\frac{1}{2}$

Mean Value Theorems Ex 15.1 Q2(viii)

since it is a polynomial function. Now, $f(x) = x^2 + 5x + 6$

Here, $f(x) = x^2 + 5x + 6$ on [-3, -2]

f(x) is continuous is $\begin{bmatrix} -3,-2 \end{bmatrix}$ and f(x) is differentiable is $\begin{pmatrix} -3,-2 \end{pmatrix}$

$$f(-3) = (-3)^{2} + 5(-3) + 6$$

$$= 9 - 15 + 6$$

$$f(-3) = 0$$

$$f(-2) = (-2)^{2} + 5(-2) + 6$$

$$f(-2) = (-2)^{2} + 5(-2) + 6$$
$$= 4 - 10 + 6$$
$$f(-2) = 20$$

From equation (i) and (ii), f(-3) = f(-2)

So, Rolle's theorem is applicable is
$$[-3,-2]$$
, we have to show that $f'(c) = 0$ as $c \in (-3,-2)$.

Now, $f(x) = x^2 + 5x + 6$

$$f'(x) = x^{-} + 5x$$
$$f'(x) = 2x + 5$$
$$\Rightarrow f'(c) = 0$$

2c + 5 = 0

So, Rolle's theorem is verified.

 $c=\frac{-5}{2}\in\left(-3,-2\right)$

--- (ii)

--- (i)

Mean Value Theorems Ex 15.1 Q3(i)

Here.

 $f(x) = \cos 2\left(x - \frac{\pi}{4}\right)$ on $\left[0, \frac{\pi}{2}\right]$

 $f(0) = \cos 2\left(0 - \frac{\pi}{4}\right) = 0$

 $f\left(\frac{\pi}{2}\right) = \cos 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 0$

So, Rolle's theorem is applicable.

 $f'(x) = -\sin 2\left(x - \frac{\pi}{4}\right) \times 2$

 $f'(x) = -2\sin\left(2x - \frac{\pi}{2}\right)$

 $\Rightarrow -2\sin\left(2c - \frac{\pi}{2}\right) = 0$

 \Rightarrow $\sin\left(2c - \frac{\pi}{2}\right) = \sin 0$

 $C = \frac{\pi}{4} \in \left[0, \frac{\pi}{2}\right]$

Hence, Rolle's theorem is verified. Mean Value Theorems Ex 15.1 Q3(ii)

 $\Rightarrow 2c - \frac{\pi}{2} = 0$

Hence, there must exists a $c \in \left(0, \frac{\pi}{2}\right)$ such that f'(c) = 0.

We know that cosine function is continuous and differentiable every where, so f(x) is continuous is $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left[0, \frac{\pi}{2}\right]$.

Now,

Now,

 \Rightarrow $f(0) = f\left(\frac{\pi}{2}\right)$

 $f(x) = \sin 2x$ on $\left[0, \frac{\pi}{2}\right]$

 $f(0) = \sin 0 = 0$

 $f\left(\frac{\pi}{2}\right) = \sin \pi = 0$

 $f'(x) = 2\cos 2x$ $f'(c) = 2 \cos 2c = 0$

Thus, Rolle's theorem verified.

Mean Value Theorems Ex 15.1 Q3(iii)

So, Rolle's theorem is applicable, so, there must exist a $c \in \left(0, \frac{\pi}{2}\right)$

 $\Rightarrow f(0) = f\left(\frac{\pi}{2}\right)$

such that f'(c) = 0

⇒ cos 2c = 0

 $\Rightarrow \qquad C = \frac{\pi}{4} \in \left(0, \frac{\pi}{4}\right)$

 \Rightarrow $2c = \frac{\pi}{2}$

We know that $\sin x$ is a continuous and differentiable every where. So, f(x) is continuous in $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

Now,

We know that cos x is a continuous and differentiable every where. So,

such that f'(c) = 0

 $\sin 2C = 0$

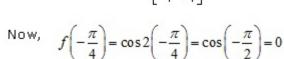
 $f'(x) = 2 \sin 2x$ $f'(c) = 2\sin 2C = 0$

 $f(x) = \cos 2x$ on $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

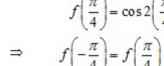
 $f\left(\frac{\pi}{4}\right) = \cos 2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$

$$f(x)$$
 is continuous in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ and differentiable is $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$.

So, Rolle's theorem is applicable, so, there must exist a $c \in \left[0, \frac{\pi}{2}\right]$











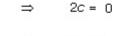


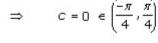






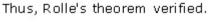








Now,





Mean Value Theorems Ex 15.1 Q3(iv)









Here, $f(x) = e^x \times \sin x \quad \text{on } [0, \pi]$

We know that since and expential function are continuous and differentiable every where so, f(x) is continuous is $[0,\pi]$ and differentiable is $(0,\pi)$.

Now,

$$f(0) = e^{0} \sin 0 = 0$$

$$f(\pi) = e^{\pi} \sin \pi = 0$$

 $\Rightarrow f(0) = f(\pi)$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that f'(c) = 0.

Now,
$$f\left(x\right) =e^{x}\sin x$$

 $f'(x) = e^x \cos x + e^x \sin x$ Now, f'(c) = 0 $e^c (\cos c + \sin c) = 0$

$$e^{c}$$
 (cosc + sinc) = 0
 $\Rightarrow e^{c}$ = 0 or cosc = - sinc

$$\Rightarrow e^{c} = 0 \text{ gives no value of } c \text{ or } \tan c = -1$$

$$\Rightarrow \tan c = \tan \left(\pi - \frac{\pi}{4} \right)$$

 $c = \frac{3\pi}{4} \in (0, \pi)$

Mean Value Theorems Ex 15.1 Q3(v)

Here.

 $f(x) = e^x \cos x$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

We know that expontial and cosine function are continuous and differentiable every where so,
$$f(x)$$
 is continuous is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and differentiable is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

So, Rolle's theorem is applicable, so there must exist a point $c \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now,

Now,

 $f\left(-\frac{\pi}{2}\right) = e^{\frac{\pi}{2}}\cos\left(-\frac{\pi}{2}\right) = 0$

 $f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}}\cos\left(\frac{\pi}{2}\right) = 0$

 \Rightarrow $f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$

such that f'(c) = 0.

So, f'(c) = 0

 \Rightarrow tan c = 1

⇒ -sinc+cosc = 0

 $C = \frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

 $f(x) = e^x \cos x$

 $f'(x) = -e^x \sin x + e^x \cos x$

 e^c (- sinc + cosc) = 0 \Rightarrow $e^c = 0$ gives no value of c

Hence, Rolle's theorem is verified. Mean Value Theorems Ex 15.1 Q3(vi)

 $f(x) = \cos 2x$ on $[0, \pi]$

We know that, ∞ sine function is contiuous and differentiable every where, so f(x) is continuous is $[0,\pi]$ and differentiable is $(0,\pi)$.

Now,

$$f(0) = \cos 0 = 1$$

$$f(\pi) = \cos (2\pi) = 1$$

$$\Rightarrow f(0) = f(\pi)$$

Here,

Now,

 \Rightarrow

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that f'(c) = 0.

$$f(x) = \cos 2x$$

$$f'(x) = -2\sin 2x$$
So,
$$f'(c) = 0$$

$$\Rightarrow -2\sin 2c = 0$$

$$\Rightarrow \sin 2c = 0$$

$$\Rightarrow 2c = 0 or 2c = \pi$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(vii)

C=0 or $C=\frac{\pi}{2}\in (0,\pi)$

$$f(x) = \frac{\sin x}{e^x}$$
 on $x \in [0, \pi]$
We know that, exponential and sine both functions are continuous and differentiable every where, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $[0, \pi]$

Now,

$$f(0) = \frac{\sin 0}{e^0} = 0$$

$$f(\pi) = \frac{\sin \pi}{e^{\pi}} = 0$$

$$\Rightarrow$$
 f(0) = f(π)
Since Rolle's theorem applicable, therefore there must exist a point c \in [0, π]

such that
$$f'(c) = 0$$

Now,
 $f(x) = \frac{\sin x}{2}$

$$f(x) = \frac{\sin x}{e^x}$$

$$\Rightarrow f'(x) = \frac{e^{x}(\cos x) - e^{x}(\sin x)}{(e^{x})^{2}}$$
Now,

$$e^{x}$$

$$\Rightarrow f'(x) = \frac{e^{x}(\cos x) - e^{x}(\sin x)}{(-x)^{2}}$$

f'(c) = 0 \Rightarrow e^c(cosc – sinc) = 0 \Rightarrow e^c \neq 0 and cosc – sinc = 0

⇒
$$e^{c}(\cos c - \sin c) = 0$$

⇒ $e^{c} \neq 0$ and $\csc - \sin c = 0$
⇒ $\tan c = 1$

$$c = \frac{\pi}{4} \in [0, \pi]$$

 $f(x) = \sin 3x$ on $[0, \pi]$

We know that, sine function is continuous and differentiable every where. So, f(x) is continuous is $(0,\pi)$ and differentiable is $(0,\pi)$.

Now,

$$f(0) = \sin 0 = 0$$
$$f(\pi) = \sin 3\pi = 0$$

 $c \in (0,\pi)$ such that f'(c) = 0.

 \Rightarrow $f(0) = f(\pi)$

So, Rolle's theorem is applicable, so there must exists a point

Now,

 $f(x) = \sin 3x$ $f'(x) = 3\cos 3x$

Now,

f'(c) = 0 \Rightarrow 3 cos 3x = 0

 \Rightarrow $\cos 3x = 0$ $3x = \frac{\pi}{2}$

Hence, Rolle's theorem is verified.

 $X = \frac{\pi}{6} \in (0, \pi)$

Mean Value Theorems Ex 15.1 Q3(ix)

 $f(x) = e^{1-x^2}$ on [-1, 1]

Here,

Now.

We know that, exponential function is continuous and differentiable every where. So, f(x) is continuous is [-1,1] and differentiable is (-1,1).

Now, $f(-1) = e^{1-1} = 1$ $f(1) = e^{1-1} = 1$ \Rightarrow f(-1) = 1

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1,1)$ such that f'(c) = 0.

 $f(x) = e^{1-x^2}$ $f'(x) = e^{1-x^2} \left(-2x\right)$ Now,

f'(c) = 0 $-2ce^{1-c^2}=0$ \Rightarrow c = 0 or $e^{1-c^2} = 0$ \Rightarrow $C = 0 \in (-1, 1)$

Hence, Rolle's theorem is verified. Mean Value Theorems Ex 15.1 Q3(x)

Now, $f(-1) = \log(1+2) - \log 3 = 0$ $f(1) = \log(1+2) - \log 3 = 0$

 \Rightarrow f(-1) = f(1)

 $f(x) = \log(x^2 + 2) - \log 3 \text{ on } [-1,1]$

Here,

So, Rolle's theorem is applicable, so there must exist a point
$$c \in (-1,1)$$
 such that $f'(c) = 0$.

We know that, logarithmic function is continuous and differentiable

is its domain, so f(x) is continuous is [-1,1] and differentiable is (-1,1).

Now,
$$f(x) = \log(x^2 + 2) - \log 3$$
$$f'(x) = \frac{(2x)}{x^2 + 2}$$
 Now,

f'(c) = 0

 $\frac{2c}{c^2+2}=0$

 \Rightarrow $c = 0 \in (-1, 1)$

Hence, Rolle's theorem is verified.

 $f(x) = \sin x + \cos x$ on $\left[0, \frac{\pi}{2}\right]$

We know that $\sin x$ and $\cos x$ are continuous and differentiable every where, so f(x) is continuous is $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

 $\Rightarrow f(0) = f\left(\frac{\pi}{2}\right)$

that f'(c) = 0.

Now,

Now,

 $f(0) = \sin 0 + \cos c0 = 1$

 $f\left(\frac{\pi}{2}\right) = \frac{\sin \pi}{2} + \frac{\cos \pi}{2} = 1$

 $f(x) = \sin x + \cos x$ $f'(x) = \cos x - \sin x$

f'(c) = 0

tanc = 1

 $\Rightarrow \qquad C = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$

 $\cos c - \sin c = 0$

Hence, Rolle's theorem is verified. Mean Value Theorems Ex 15.1 Q3(xii)

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{2}\right)$ such

$$f(x) = 2\sin x + \sin 2x$$
 on $[0, \pi]$

We know that sine function is continuous and differentiable every where, so f(x) is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = 2 \sin 0 + \sin 0 = 0$$
$$f(\pi) = 2 \sin \pi + \sin 2\pi = 0$$
$$\Rightarrow f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0,\pi)$ such that f'(c) = 0.

Now,

$$f(x) = 2\sin x + \sin 2x$$
$$f'(x) = 2\cos x + 2\cos 2x$$

Now,

$$f'(c) = 0$$

2 cos $c + 2$ cos $2c = 0$

$$\Rightarrow 2\left(\cos c + 2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \qquad \left(2\cos^2 + 2\cos c - \cos c - 1\right) = 0$$

$$\Rightarrow (2\cos c - 1)(\cos c + 1) = 0$$

$$\Rightarrow$$
 $\cos c = \frac{1}{2}, \cos c = -1$

$$\Rightarrow$$
 tan $c = 1$

$$C = \frac{\pi}{3} \in \left(0, \pi\right), \ C = \pi$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xiii)

Here,

$$f(x) = \frac{x}{2} - \sin \frac{\pi x}{6}$$
 on $[-1, 0]$

We know that sine function is continuous and differentiable every where, so f(x) is continuous is [-1,0] and differentiable is (-1,0).

Now,

And

$$f(-1) = \frac{-1}{2} - \sin\left(-\frac{\pi}{6}\right)$$

$$= -\frac{1}{2} + \sin\frac{\pi}{6}$$

$$= -\frac{1}{2} + \frac{1}{2}$$

$$f(-1) = 0 \qquad ---(i)$$

$$f(0) = 0 - \sin 0$$

$$f(0) = 0 \qquad ---(ii)$$

From equation (i) and (ii),

$$f(-1) = f\begin{pmatrix} 0 \end{pmatrix}$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1,0)$ such that f'(c) = 0.

since, $\cos^{-1} x \in [-1,1]$

$$f(x) = \frac{x}{2} - \sin\left(\frac{\pi x}{6}\right)$$
$$f'(x) = \frac{1}{2} - \frac{\pi}{6}\cos\left(\frac{\pi x}{6}\right)$$

Now,

$$f'(c) = 0$$

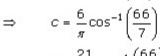
$$\frac{1}{2} - \frac{\pi}{6} \cos\left(\frac{\pi c}{6}\right) = 0$$

$$\Rightarrow -\frac{\pi}{6}\cos\left(\frac{\pi c}{6}\right) = -\frac{1}{2}$$

$$\Rightarrow \qquad \cos\left(\frac{\pi c}{6}\right) = 3\pi$$

$$\frac{\pi c}{6} = \cos^{-1}\left(\frac{66}{6}\right)$$

$$\Rightarrow \frac{\pi c}{6} = \cos^{-1}\left(\frac{66}{7}\right)$$



$$\Rightarrow c = \frac{0}{\pi} \cos^{-1} \left(\frac{66}{7} \right)$$

$$\Rightarrow c = \frac{21}{11} \cos^{-1} \left(\frac{66}{7} \right)$$

 \Rightarrow $c \in (-1.9, 1.9)$

$$\Rightarrow c \in \left(-\frac{21}{11}, \frac{21}{11}\right)$$

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x \text{ on } \left[0, \frac{\pi}{6}\right]$$

We know that sine and its square function is continuous and differentiable every where, f(x) is continuous is $\left[0, \frac{\pi}{6}\right]$ and differentiable is $\left(0, \frac{\pi}{6}\right)$.

Now,

$$f(0) = 0 - 0 = 0$$

$$f\left(\frac{\pi}{6}\right) = 1 - 1 = 0$$

$$\Rightarrow$$
 $f(0) = f\left(\frac{\pi}{6}\right)$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{6}\right)$ such that f'(c) = 0.

Now,

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x$$
$$f'(x) = \frac{6}{\pi} - 8\sin x \cos x$$
$$f'(x) = \frac{6}{\pi} - 4\sin 2x$$

Now,

$$f'(c) = 0$$

$$\frac{6}{\pi} - 4\sin 2c = 0$$

$$\Rightarrow 4 \sin 2c = \frac{6}{\pi}$$

$$\Rightarrow \sin 2c = \frac{3}{2\pi}$$

$$\Rightarrow \qquad 2c = \sin^{-1}\left(\frac{21}{44}\right)$$

$$\Rightarrow c = \frac{1}{2} \sin^{-1} \left(\frac{21}{44} \right)$$

$$\Rightarrow$$
 $c \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ [since, $\sin^{-1} x \in [-1, 1]$]

$$\Rightarrow \qquad c \in \left(0, \frac{11}{21}\right)$$

$$\Rightarrow$$
 $c \in \left(0, \frac{\pi}{6}\right)$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xv)

$$f(x) = 4^{\sin x}$$
 on $[0, \pi]$

We know that exponential and $\sin x$ both are continuous and differentiable, so f(x) is continuous is $[0,\pi]$ and differentiable is $(0,\pi)$.

Now,

$$f(0) = 4^{\sin 0} = 4^0 = 1$$

$$f(\pi) = 4^{\sin x} = 4^0 = 1$$

$$\Rightarrow$$
 $f(0) = f(\pi)$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that f'(c) = 0.

Now,

$$f(x) = 4^{\sin x}$$

$$f'(x) = 4^{\sin x} \log 4 \times \cos x$$

Now,

$$f'(c) = 0$$

$$4^{\sin c} \times \cos x c \log 4 = 0$$

$$\Rightarrow$$
 cosc = 0

$$\Rightarrow$$
 $C = \frac{\pi}{2} \in (0, \pi)$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xvi)

Here,

$$f(x) = x^2 - 5x + 4$$
 on $[1, 4]$

f(x) is continuous and differentiable as it is a polynomial function.

Now,

$$f(1) = (1)^2 - 5(1) + 4 = 0$$

$$f(4) = (4)^2 - 5(4) + 4 = 0$$

$$\Rightarrow f(1) = f(4)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (1,4)$ such that f'(c) = 0.

Now,

$$f(x) = x^2 - 5x + 4$$

$$f'(x) = 2x - 5$$

So,

$$f'(c) = 0$$

$$\Rightarrow$$
 2c - 5 = 0

$$\Rightarrow \qquad c = \frac{5}{2} \in (1, 4)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xvii)

Here. $f(x) = \sin^4 x + \cos^4 x$ on $\left[0, \frac{\pi}{2}\right]$

We know that sine and cosine function are differentiable and continuous.

So, f(x) is continuous is $\left[0, \frac{\pi}{2}\right]$ and it is differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,
$$f(0) = \sin^4(0) + \cos^4(0) = 1$$

$$f\left(\frac{\pi}{2}\right) = \sin^4\left(\frac{\pi}{2}\right) + \cos^4\left(\frac{\pi}{2}\right) = 1$$

$$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) = f\left(0\right) = f\left(\frac{\pi}{2}\right)$$

 \Rightarrow $f(0) = f\left(\frac{\pi}{2}\right)$

So, Rolle's theorem is applicable, so there must exist a point
$$c \in \left(0, \frac{\pi}{2}\right)$$
 such

that f'(c) = 0.

 $f(x) = \sin^4 x + \cos^4 x$ $f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$ $= -2 \left(2 \sin x \cos x\right) \left(\cos^2 x - \cos^2 x\right)$

f'(c) = 0 $-\sin 4x = 0$ $\sin 4x = 0$

Now,

$$\Rightarrow x = 0 \qquad \text{or} \qquad x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Mean Value Theorems Ex 15.1 Q3(xvii)

4x = 0 or $4x = \pi$

Since trigonometric functions are differentiable and continuous, the given function, $f(x) = \sin x - \sin 2x$ is also continuous and differentiable.

Now
$$f(0) = \sin 0 - \sin 2 \times 0 = 0$$

and

and
$$f(\pi) = \sin \pi - \sin 2x \pi = 0$$

$$f(\pi) = \sin \pi - \sin 2 \times \pi = 0$$

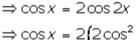
$$\Rightarrow f(0) = f(\pi)$$

Therefore, there exists
$$c \in [0, \pi]$$
 such that $f'(c) = 0$

Now $f(x) = \sin x - \sin 2x$

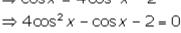
$$\cos 2x = 0$$

$$\Rightarrow f'(x) = \cos x - 2\cos 2x = 0$$
$$\Rightarrow \cos x = 2\cos 2x$$



$$\Rightarrow \cos x = 2(2\cos^2 x - 1)$$





$$\Rightarrow \cos x = \frac{1 \pm \sqrt{33}}{8} = 0.8431 \text{ or } -0.5931$$

$$\Rightarrow \cos x = \frac{11\sqrt{33}}{8} = 0.8431 \text{ or } -0.593$$

$$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } \cos^{-1}(-0.5931)$$

$$\Rightarrow$$
 x=cos⁻¹(0.8431) or 180°

$$\Rightarrow x = \cos^{-1}(0.8431) \text{ or } 180^{\circ} - \cos^{-1}(0.5931) \quad \left[\because \cos^{-1}(-x) = \pi - \cos^{-1}(x)\right]$$
$$\Rightarrow x = 32^{\circ}32' \text{ or } x = 126^{\circ}23'$$

Both 32°32' and 126°23'
$$\in$$
 [0, π] such that $f(c) = 0$.
Hence Rolle's Theorem is verified.

$$\Rightarrow$$
 f(0) = f(π)
Thus, f(\times) satisfies conditions of the Rolle's Theorem on [0, π].

Since trigonometric functions are differentiable and continuous, the given function, $f(x) = \sin x - \sin 2x$ is also continuous and differentiable.

Now
$$f(0) = \sin 0 - \sin 2 \times 0 = 0$$

and

$$f(\pi) = \sin \pi - \sin 2 \times \pi = 0$$

$$\Rightarrow$$
 f(0) = f(π)

Thus, f(x) satisfies conditions of the Rolle's Theorem on $[0,\pi]$.

Therefore, there exists $c \in [0, \pi]$ such that f(c) = 0

Now
$$f(x) = \sin x - \sin 2x$$

$$\Rightarrow f'(x) = \cos x - 2\cos 2x = 0$$

$$\Rightarrow \cos x = 2\cos 2x$$

$$\Rightarrow \cos x = 2(2\cos^2 x - 1)$$

$$\Rightarrow \cos x = 4\cos^2 x - 2$$

$$\Rightarrow 4\cos^2 x - \cos x - 2 = 0$$

$$\Rightarrow \cos x = \frac{1 \pm \sqrt{33}}{8} = 0.8431 \text{ or } -0.5931$$

$$\Rightarrow \times = \cos^{-1}(0.8431) \text{ or } \cos^{-1}(-0.5931)$$

$$\Rightarrow \times = \cos^{-1}(0.8431) \text{ or } 180^{\circ} - \cos^{-1}(0.5931) \quad [\because \cos^{-1}(-x) = \pi - \cos^{-1}(x)]$$

$$\Rightarrow x = 32^{\circ}32' \text{ or } x = 126^{\circ}23'$$

Both 32°32' and 126°23' $\in [0, \pi]$ such that f(c) = 0.

Hence Rolle's Theorem is verified.

Mean Value Theorems Ex 15.1 Q7

Let
$$f(x) = 16 - x^2$$
, then $f'(x) = -2x$

f(x) is continuous on [-1,1] because it is a polynomial function.

Also
$$f(-1) = 16 - (-1)^2 = 15$$

$$f(1)=16-(1)^2=15$$

 $f(-1)=f(1)$

There exists a $c \in [-1,1]$ such that f'(c) = 0

$$\Rightarrow -2c = 0$$

$$\Rightarrow c=0$$

Thus, at $0 \in [-1, 1]$ the tangent is parallel to the x-axis.

Mean Value Theorems Ex 15.1 Q8(i)

Let $f(x) = x^2$, then f'(x) = 2x

f(x) is continuous on [-2,2] because it is a polynomial function.

f(x) is differentiable on (-2,2) as it is a polynomial function.

Also
$$f(-2) = (-2)^2 = 4$$

$$f(2) = 2^2 = 4$$

$$\Rightarrow f(-2) = f(2)$$

:. There exists $c \in (-2,2)$ such that f'(c) = 0

$$\Rightarrow c = 0$$

Thus, at $0 \in [-2,2]$ the tangent is parallel to the x-axis.

$$x = 0$$
, then $y = 0$

Therefore, the point is (0, 0)

Mean Value Theorems Ex 15.1 Q8(ii)

Let $f(x) = e^{1-x^2}$ on [-1,1]

Since, f(x) is a composition of two continuous functions, it is continuous on [-1,1]

Also
$$f(x) = -2xe^{1-x^2}$$

 $f(2) = 2^2 = 4$

:
$$f'(x)$$
 exists for every value of x in $(-1,1)$

$$\Rightarrow$$
 $f(x)$ is differentiable on $(-1,1)$

By rolle's theorem, there exists $c \in (-1,1)$ such that f'(c) = 0

$$\Rightarrow -2ce^{1-c^2} = 0$$

$$\Rightarrow$$
 $c = 0$

Thus, at $c = 0 \in [-1,1]$ the tangent is parallel to the x-axis.

$$x = 0$$
, then $y = e$

Therefore, the point is (0, e)

Mean Value Theorems Ex 15.1 Q8(iii)

Let
$$f(x) = 12(x+1)(x-2)$$

Since, f(x) is a polynomial function, it is continuous on $\begin{bmatrix} -1,2 \end{bmatrix}$ and differentiable on $\begin{pmatrix} -1,2 \end{pmatrix}$

Also
$$f'(x) = 12[(x-2)+(x+1)] = 12[2x-1]$$

By rolle's theorem, there exists $c \in (-1,2)$ such that f'(c) = 0

$$\Rightarrow 12(2c-1)=0$$

$$\Rightarrow$$
 $c = \frac{1}{2}$

Thus, at $c = \frac{1}{2} \in (-1,2)$ the tangent to y = 12(x+1)(x-2) is parallel to x-axis

Mean Value Theorems Ex 15.1 Q9

It is given that $f:[-5,5] \to \mathbf{R}$ is a differentiable function.

Since every differentiable function is a continuous function, we obtain

- (a) f is continuous on [-5, 5].
- (b) f is differentiable on (-5, 5).

Therefore, by the Mean Value Theorem, there exists $c \square (-5, 5)$ such that

$$f'(c) = \frac{f(5) - f(-5)}{5 - (-5)}$$

$$\Rightarrow 10f'(c) = f(5) - f(-5)$$

It is also given that f'(x) does not vanish anywhere.

$$\therefore f'(c) \neq 0$$

$$\Rightarrow 10f'(c) \neq 0$$

$$\Rightarrow f(5) - f(-5) \neq 0$$

$$\Rightarrow f(5) \neq f(-5)$$

Hence, proved.

Mean Value Theorems Ex 15.1 Q10

By Rolle's Theorem, for a function $f:[a, b] \to \mathbb{R}$, if

(a)
$$f$$
 is continuous on $[a, b]$

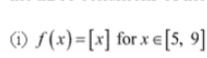
(b) f is differentiable on (a, b)

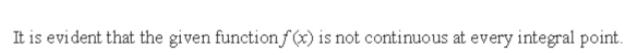
$$(c) f(a) = f(b)$$

then, there exists some
$$c \in (a, b)$$
 such that $f'(c) = 0$

Therefore, Rolle's Theorem is not applicable to those functions that do not satisfy any

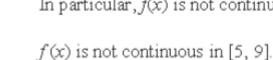
the three conditions of the hypothesis.











Also,
$$f(5) = [5] = 5$$
 and $f(9) = [9] = 9$
 $f(5) \neq f(9)$

The differentiability of f in (5, 9) is checked as follows.

Let n be an integer such that $n \in (5, 9)$.

The left hand limit of f at x = n is,

$$\lim_{h \to 0^{-}} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^{-}} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^{-}} \frac{n - 1 - n}{h} = \lim_{h \to 0^{-}} \frac{-1}{h} = \infty$$

The right hand limit of f at x = n is,

$$\lim_{h \to 0^+} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^+} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^+} \frac{n-n}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at x = n are not equal, f is not differentiable at x = n

f is not differentiable in (5, 9).

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for f(x) = [x] for $x \in [5, 9]$.

(ii)
$$f(x) = [x]$$
 for $x \in [-2, 2]$

It is evident that the given function f(x) is not continuous at every integral point.

In particular, f(x) is not continuous at x = -2 and x = 2

f(x) is not continuous in [-2, 2].

Also,
$$f(-2) = [-2] = -2$$
 and $f(2) = [2] = 2$
 $\therefore f(-2) \neq f(2)$

The differentiability of f in (-2, 2) is checked as follows.

Let n be an integer such that $n \in (-2, 2)$.

The left hand limit of f at x = n is,

$$\lim_{h \to 0^{-}} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^{-}} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^{-}} \frac{n - 1 - n}{h} = \lim_{h \to 0^{-}} \frac{-1}{h} = \infty$$

The right hand limit of f at x = n is,

$$\lim_{h \to 0^{\circ}} \frac{f(n+h) - f(n)}{h} = \lim_{h \to 0^{\circ}} \frac{[n+h] - [n]}{h} = \lim_{h \to 0^{\circ}} \frac{n-n}{h} = \lim_{h \to 0^{\circ}} 0 = 0$$

Since the left and right hand limits of f at x = n are not equal, f is not differentiable at x = n

f is not differentiable in (-2, 2).

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for f(x) = [x] for $x \in [-2, 2]$.