# RD Sharma Solutions Class 12 Maths Chapter 15 Ex 15.2

## Mean Value Theorems Ex 15.2 Q1(i)

Here,

$$f(x) = x^2 - 1 \text{ on } [2,3]$$

It is a polynomial function so it is continuous in $[2,3]$ and differentiable in $(2,3)$. So, both conditions of Lagrange's mean value theorem are satisfied.

Therefore, there exist a point $c \in (2,3)$ such that

$$f'(c) = \frac{f(3) - f(2)}{3 - 2}$$

$$2c = \frac{\left((3)^2 - 1\right) - \left((2)^2 - 1\right)}{1}$$

$$2c = (8 - 3)$$

$$c = \frac{5}{2} \in (2,3)$$

Hence, Lagrange's mean value theorem is verified.

## Mean Value Theorems Ex 15.2 Q1(ii)

Here,

$$f(x) = x^3 - 2x^2 - x + 3 \text{ on } [0,1]$$

Since, $f(x)$ is a polynomial function. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = \frac{\left[(1)^3 - 2(1)^2 - (1) + 3\right] - 3}{1}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = 1 - 3$$

$$\Rightarrow \quad 3c^2 - 4c + 1 = 0$$

$$\Rightarrow \quad 3c^2 - 3c - c + 1 = 0$$

$$\Rightarrow \quad 3c(c - 1) - 1(c - 1) = 0$$

$$\Rightarrow \quad (3c - 1)(c - 1) = 0$$

$$\Rightarrow \quad c = \frac{1}{3} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(iii)**

Here,

$$f(x) = x(x - 1)$$

$$f(x) = x^2 - x \text{ on } [1,2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,2]$ and $f(x)$ is differentiable in $(1,2)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,2)$ such that

$$f'(c) = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 2c - 1 = \frac{(4 - 2) - (1 - 1)}{1}$$

$$\Rightarrow \quad 2c - 1 = \frac{2 - 0}{1}$$

$$\Rightarrow \quad 2c = 3$$

$$\Rightarrow \quad c = \frac{3}{2} \in (1,2)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(iv)**

Here,

$$f(x) = x^2 - 3x + 2 \text{ on } [-1,2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[-1,2]$ and differentiable in $(-1,2)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (-1,2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2+1}$$

$$\Rightarrow \quad 2c - 3 = \frac{(4-6+2)-(1+3+2)}{3}$$

$$\Rightarrow \quad 2c - 3 = -\frac{6}{3}$$

$$\Rightarrow \quad 2c = 1$$

$$\Rightarrow \quad c = \frac{1}{2} \in (-1,2)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(v)**

Here,

$$f(x) = 2x^2 - 3x + 1 \text{ on } [1,3]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,3]$ and $f(x)$ is differentiable in $(1,3)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3-1}$$

$$\Rightarrow \quad 4c - 3 = \frac{\left(2(3)^2 - 3(3) + 1\right) - (2-3+1)}{3-1}$$

$$\Rightarrow \quad 4c - 3 = \frac{10}{2}$$

$$\Rightarrow \quad 4c = 5 + 3$$

$$\Rightarrow \quad 4c = 8$$

$$\Rightarrow \quad c = 2 \in (1,3)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(vi)**

Here,

$$f(x) = x^2 - 2x + 4 \text{ on } [1,5]$$

We know that, polynomial is always continuous and differentiable. So, $f(x)$ is continuous in $[1,5]$ and it is differentiable in $(1,5)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$\Rightarrow \quad 2c - 2 = \frac{\left((5)^2 - 2(5) + 4\right) - (1 - 2 + 4)}{4}$$

$$\Rightarrow \quad 2c - 2 = \frac{19 - 3}{4}$$

$$\Rightarrow \quad 2c - 2 = 4$$

$$\Rightarrow \quad 2c = 6$$

$$\Rightarrow \quad c = 3 \in (1,5)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(vii)**

Here,

$$f(x) = 2x - x^2 \text{ on } [0,1]$$

We know that, polynomial is continuous and differentiable. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 2 - 2c = \frac{\left(2(1) - (1)^2\right) - (0)}{1}$$

$$\Rightarrow \quad 2 - 2c = 1$$

$$\Rightarrow \quad 1 = 2c$$

$$\Rightarrow \quad c = \frac{1}{2} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(viii)**

$$f(x) = (x-1)(x-2)(x-3) \text{ on } [0,4]$$

We know that, polynomial is continuous and differentiable every where. So, $f(x)$ is continuous in $[0,4]$ and differentiable in $(0,4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4-0}$$

$$\Rightarrow \quad (c-1)(c-2) + (c-2)(c-3) + (c-1)(c-3) = \frac{(3)(2)(1) - (-1)(-2)(-3)}{4-0}$$

$$\Rightarrow \quad c^2 - 3c + 2 + c^2 + 5c + 6 + c^2 - 4c + 3 = \frac{6+6}{4}$$

$$\Rightarrow \quad 3c^2 - 12c + 11 = 3$$

$$\Rightarrow \quad 3c^2 = 12c + 8 = 0$$

$$\Rightarrow \quad c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{6}$$

$$\Rightarrow \quad c = \frac{12 \pm \sqrt{48}}{6}$$

$$\Rightarrow \quad c = 2 \pm \frac{2\sqrt{3}}{3} \in (0,4)$$

$$\Rightarrow \quad c = 2 \pm \frac{2}{\sqrt{3}} \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(ix)**

Here,

$$f(x) = \sqrt{25 - x^2} \text{ on } [-3, 4]$$

Given function is continuous as it has unique value for each $x \in [-3, 4]$ and

$$f'(x) = \frac{-2x}{2\sqrt{25 - x^2}}$$

$$f'(x) = \frac{-x}{\sqrt{25 - x^2}}$$

So, $f'(x)$ exists for all values for $x \in (-3, 4)$ so, $f(x)$ is differentiable in $(-3, 4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (-3, 4)$ such that

$$f'(c) = \frac{f(4) - f(-3)}{4 + 3}$$

$$\Rightarrow \quad \frac{-2c}{2\sqrt{25 - c^2}} = \frac{\sqrt{9} - \sqrt{16}}{7}$$

$$\Rightarrow \quad -7c = -\sqrt{25 - c^2}$$

Squaring both the sides,

$$49c^2 = 25 - c^2$$

$$c^2 = \frac{1}{2}$$

$$c = \pm\frac{1}{\sqrt{2}} \in (-3, 4)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(x)**

Here,

$$f(x) = \tan^{-1} x \text{ on } [0, 1]$$

We know that, $\tan^{-1} x$ has unique value in $[0,1]$ so, it is continuous in $[0,1]$

$$f'(x) = \frac{1}{1+x^2}$$

So, $f'(x)$ exists for each $x \in (0,1)$

So, $f'(x)$ is differentiable in $(0,1)$, thus Lagrange's mean value theorem is applicable, so there exist a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad \frac{1}{1+c^2} = \frac{\tan^{-1}(1) - \tan^{-1}(0)}{1}$$

$$\Rightarrow \quad \frac{1}{1+c^2} = \frac{\frac{\pi}{4} - 0}{1}$$

$$\Rightarrow \quad \frac{4}{\pi} = 1 + c^2$$

$$\Rightarrow \quad c = \sqrt{\frac{4}{\pi} - 1}$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(xi)**

Here,

$$f(x) = x + \frac{1}{x} \text{ on } [1, 3]$$

$f(x)$ attiams a unique value for each $x \in [1,3]$, so it is continuous

$f'(x) = 1 - \frac{1}{x^2}$ is definded for each $x \in (1,3)$

$\Rightarrow$ $f(x)$ is differentiable in $(1,3)$, so Lagrange's mean value theorem is a applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\left(3 + \frac{1}{3} - (1+1)\right)}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\frac{10}{3} - 2}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{4}{3 \times 2}$$

$$\Rightarrow \quad 1 - \frac{2}{3} = \frac{1}{c^2}$$

$$\Rightarrow \quad c^2 = 3$$

$$\Rightarrow \quad c = \sqrt{3} \in (1,3)$$

So, Lagrange's mean value theorem is verified.

## Mean Value Theorems Ex 15.2 Q1(xii)

Here,

$$f(x) = x(x+4)^2 \text{ on } [0,4]$$

We know that every polynomial function is continuous and differentiable every wher, so, $f(x)$ is continuous in $[0,4]$ and differentiable in $(0,4)$, so, Lagrange's mean value theorem is applicable, thus there exist a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow \quad 3c^2 + 16c + 16 = \frac{4 \times (8)^2 - 0}{4}$$

$$\Rightarrow \quad 3c^2 + 16c + 16 = 64$$

$$\Rightarrow \quad 3c^2 + 16c - 48 = 0$$

$$\Rightarrow \quad c = \frac{-16 \pm \sqrt{256 + 576}}{6}$$

$$\Rightarrow \quad = \frac{-16 \pm \sqrt{832}}{6}$$

$$\Rightarrow \quad = \frac{-16 \pm 8\sqrt{13}}{6}$$

$$\Rightarrow \quad c = \frac{-8 \pm 4\sqrt{13}}{3}$$

$$c = \frac{-8 + 4\sqrt{13}}{3} \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

## Mean Value Theorems Ex 15.2 Q1(xiii)

Here,

$$f(x) = x\sqrt{x^2 - 4} \text{ on } [2,4]$$

$f(x)$ is continuous at it attains a unique value for each $x \in [2,4]$ and

$$f'(x) = \frac{2x}{2\sqrt{x^2 - 4}}$$

$$f'(x) = \frac{x}{\sqrt{x^2 - 4}}$$

$\Rightarrow$ $f'(x)$ exists for each $x \in (2,4)$

$\Rightarrow$ $f(x)$ is differentiable in $(2,4)$, so

Lagrange's mean value theorem is applicable, so there exist a $c \in (2,4)$ such that

$$f'(c) = \frac{f(4) - f(2)}{4 - 2}$$

$$\Rightarrow \quad \frac{c}{\sqrt{c^2 - 4}} = \frac{\sqrt{12} - 0}{2}$$

Squarintg both the sides,

$$\Rightarrow \quad \frac{c^2}{c^2 - 4} = \frac{12}{4}$$

$$\Rightarrow \quad 4c^2 = 12c^2 - 48$$

$$\Rightarrow \quad 8c^2 = 48$$

$$\Rightarrow \quad c^2 = 6$$

$$\Rightarrow \quad c = \sqrt{6} \in (2,4)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(xiv)**

Here,

$$f(x) = x^2 + x - 1 \text{ on } [0,4]$$

$f(x)$ is polynomial, so it is continuous is $[0,4]$ and differentiable in $(0,4)$ as every polynomial is continuous and differentiable every where. So,

Lagrange's mean value theorem is applicable, so there exists a point $c \in [0,4]$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow \quad 2c + 1 = \frac{\left((4)^2 + 4 - 1\right) - (0 - 1)}{4}$$

$$\Rightarrow \quad 2c + 1 = \frac{19 + 1}{4}$$

$$\Rightarrow \quad 2c + 1 = 5$$

$$\Rightarrow \quad c = 2 \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(xv)**

Here,

$$f(x) = \sin x - \sin 2x - x \text{ on } [0, \pi]$$

We know that $\sin x$ and polynomial is continuous and differentiable every where so, $f(x)$ is continuous in $[0, \pi]$ and differentiable in $[0, \pi]$. So, Lagrange's mean value theorem is applicable. So, there exist a point $c \in (0, \pi)$ such that

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$\Rightarrow \quad \cos c - 2\cos 2c - 1 = \frac{(\sin \pi - \sin 2\pi - \pi) - (0)}{\pi}$$

$$\Rightarrow \quad \cos c - 2\cos 2c = -1 + 1$$

$$\Rightarrow \quad \cos c - 2\left(2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \quad 4\cos^2 c - \cos c - 2 = 0$$

$$\Rightarrow \quad \cos c - \frac{-(-1) \pm \sqrt{1 - 4 \times 4 \times (-2)}}{8}$$

$$\Rightarrow \quad \cos c = \frac{1 \pm \sqrt{33}}{8}$$

$$\Rightarrow \quad c = \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right) \in (0, \pi)$$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q1(xvi)**

The given function is $f(x) = x^3 - 5x^2 - 3x$, $f$ being a polynomial function, is continuous in [1,3] and is differentiable in [1,3] whose derivative is $3x^2 - 10x - 3$.

$$f(1) = 1^3 - 5(1)^2 - 3(1) = -7$$

$$f(3) = 3^3 - 5(3)^2 - 3(3) = 27 - 45 - 9 = -27$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} = \frac{-27 + 7}{2} = -10$$

Mean value theorem states that there is a point $c(1,3)$ such that $f'(c) = 3c^2 - 10c - 3$

$$f'(c) = -10$$

$$3c^2 - 10c - 3 = -10$$

$$3c^2 - 10c + 7 = 0$$

$$3c^2 - 3c - 7c + 7 = 0$$

$$c = \frac{7}{3}, \text{ where } c = \frac{7}{3} \in (1,3)$$

Hence, Mean value theorem is verified for the given function.

**Mean Value Theorems Ex 15.2 Q2**

Here,

$$f(x) = |x| \text{ on } [-1, 1]$$

$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

For differentiability at $x = 0$

$$\text{LHD} = \lim_{x \to 0^-} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{-(0-h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$\text{LHD} = -1$$

$$\text{RHD} = \lim_{x \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

$\therefore$ LHD $\neq$ RHD

$\Rightarrow$ $f(x)$ is not differentiable at $x = 0 \in (-1, 1)$

Hence, Lagrange's mean value theorem is verified.

**Mean Value Theorems Ex 15.2 Q3**

Here,

$$f(x) = \frac{1}{x} \text{ on } [-1, 1]$$

$$f'(x) = -\frac{1}{x^2}$$

$\Rightarrow$ $f'(x)$ doesnot exist at $x = 0 \in (-1, 1)$

$\Rightarrow$ $f(x)$ is not differentiable in $(-1, 1)$

Hence, LMVT is verified

**Mean Value Theorems Ex 15.2 Q4**

Here,

$$f(x)=\frac{1}{4x-1}, x\in[1,4]$$

f(x) attain unique value for each $x\in[1,4]$, so f(x) is continuous in [1, 4].

$$f'(x)=-\frac{4}{(4x-1)^2}$$

$\Rightarrow f'(x)$ exists for each $x\in(1,4)$

$\Rightarrow f'(x)$ is differentiable in $(1,4)$

So, Lagranges mean value theroem is applicable.

So, there exist a point $c\in(1,4)$ such that,

$$f'(c)=\frac{f(4)-f(1)}{4-1}$$

$$\Rightarrow -\frac{4}{(4x-1)^2}=\frac{\frac{1}{15}-\frac{1}{3}}{3}$$

$$\Rightarrow -\frac{4}{(4x-1)^2}=-\frac{4}{45}$$

$$\Rightarrow (4x-1)^2=45$$

$$\Rightarrow 4x-1=\pm 3\sqrt{5}$$

$$\Rightarrow x=\frac{3\sqrt{5}+1}{4}\in[1,4]$$

**Mean Value Theorems Ex 15.2 Q5**

Here,

curve is $y=(x-4)^2$

Since, it a polynomial function so it is differentiable and continuous. So, it Lagrange's mean value theorem is applicable, so, there exist a point $c$ such that,

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow \quad 2(c-4)=\frac{f(5)-f(4)}{5-4}$$

$$\Rightarrow \quad 2c-8=\frac{1-0}{1}$$

$$\Rightarrow \quad 2c=9$$

$$\Rightarrow \quad c=\frac{9}{2}$$

$$\Rightarrow \quad y=\left(\frac{9}{2}-4\right)^2$$

$$y=\frac{1}{4}$$

Thus, $(c,y)=\left(\frac{9}{2},\frac{1}{4}\right)$ is required point.

**Mean Value Theorems Ex 15.2 Q6**

Here,

$$y = x^2 + x$$

Since, $y$ is a polynomial function, so it continuous differentiable,

$\Rightarrow$ Lagrange's mean value theorem is applicable, so, there exist a point $c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 2c + 1 = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 2c + 1 = 2$$

$$\Rightarrow \quad c = \frac{1}{2}$$

$$\Rightarrow \quad y = \left(\frac{1}{2}\right)^2 + \frac{1}{2}$$

$$\Rightarrow \quad y = \frac{3}{4}$$

So, $(c, y) = \left(\frac{1}{2}, \frac{3}{4}\right)$ is the required point.

**Mean Value Theorems Ex 15.2 Q7**

Here,

$$y = (x - 3)^2$$

Since, $y$ is a polynomial function, so it continuous differentiable,

$\Rightarrow$ Lagrange's mean value theorem is applicable

$\Rightarrow$ There exist a point $c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 2(c - 3) = \frac{f(4) - f(3)}{4 - 3}$$

$$\Rightarrow \quad 2c - 6 = \frac{1 - 0}{1}$$

$$\Rightarrow \quad 2c = 7$$

$$\Rightarrow \quad c = \frac{7}{2}$$

$$\Rightarrow \quad y = \left(\frac{7}{2} - 3\right)^2$$

$$\Rightarrow \quad y = \frac{1}{4}$$

So, $(c, y) = \left(\frac{7}{2}, \frac{1}{4}\right)$ is the required point.

**Mean Value Theorems Ex 15.2 Q8**

Here,

$$y = x^3 - 3x$$

$y$ is a polynomial function, so it is continuous differentiable, so
Lagrange's mean value theorem is applicable thus there exists a point $c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{2 + 2}{1}$$

$$\Rightarrow \quad 3c^2 = 7$$

$$\Rightarrow \quad c = \pm\sqrt{\frac{7}{3}}$$

$$\Rightarrow \quad y = \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

So, $(c, y) = \left(\pm\sqrt{\frac{7}{3}}, \mp\frac{2}{3}\sqrt{\frac{7}{3}}\right)$ is the required point.

**Mean Value Theorems Ex 15.2 Q9**

Here,

$$y = x^3 + 1$$

It is a polynomial function, so it is continuous differentiable.

$\Rightarrow$ Lagrange's mean value theorem is applicable, so there exists a point $c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 3c^2 = \frac{28 - 2}{2}$$

$$\Rightarrow \quad c^2 = \frac{13}{3}$$

$$\Rightarrow \quad c = \sqrt{\frac{13}{3}}$$

$$\Rightarrow \quad y = \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1$$

So, $(c, y) = \left(\sqrt{\frac{13}{3}}, \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1\right)$ is the required point.

**Mean Value Theorems Ex 15.2 Q10**

Trigonometric functions are continuous and differentiable.

Thus, the curve C is continuous between the points $(a,0)$ and $(0,a)$and is differentiable on $[a,a]$

Therefore, by Lagrange's Mean Value Theorem, there exists a real number $c \in (a,a)$ such that

$$f'(c) = \frac{a-0}{0-a} = -1$$

Now consider the parametric functions of the given function

$x = a\cos^3\theta$

and

$y = a\sin^3\theta$

$$\Rightarrow \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

and

$$\Rightarrow \frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3a\sin^2\theta(\cos\theta)}{3a\cos^2\theta(-\sin\theta)}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

Slope of the chord joining the points $(a,0)$ and $(0,a)$

=Slope of the tangent at $(c, f(c))$, where c lies on the curve

$$\Rightarrow \frac{a-0}{0-a} = -\tan\theta$$

$$\Rightarrow -1 = -\tan\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Now substituting $\theta = \frac{\pi}{4}$, in the parametric representations, we have,

$x = a\cos^3\theta, y = a\sin^3\theta$

$$\Rightarrow x = a\cos^3\left(\frac{\pi}{4}\right), y = a\sin^3\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x = \frac{a}{2\sqrt{2}}, y = \frac{a}{2\sqrt{2}}$$

Thus, $P\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ is a point on C, where the tangent is parallel to the chord joining the points $(a,0)$ and $(0,a)$.

**Mean Value Theorems Ex 15.2 Q11**

Consider the function as

$$f(x) = \tan x, \qquad \left\{x \in [a,b] \text{ such that } 0 < a < b < \frac{\pi}{2}\right\}$$

We know that $\tan x$ is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$, so, Lagrange's mean value theorem is applicable on $(a,b)$, so there exists a point $c$ such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad \sec^2 c = \frac{\tan b - \tan a}{b - a} \qquad \text{---(i)}$$

Now,

$$c \in (a,b)$$

$$\Rightarrow \quad a < c < b$$

$$\Rightarrow \quad \sec^2 a < \sec^2 c < \sec^2 b$$

$$\Rightarrow \quad \sec^2 a < \left(\frac{\tan b - \tan a}{b - a}\right) < \sec^2 b$$

Using equation (i),

$$\Rightarrow \quad (b - a)\sec^2 a < (\tan b - \tan a) < (b - a)\sec^2 b$$