

RD Sharma
Solutions
Class 12 Maths
Chapter 16
Ex 16.2

Tangents and Normals Ex 16.2 Q1

The given equation of the curve is

$$\sqrt{x} + \sqrt{y} = a \quad \text{---(i)}$$

Differentiating with respect to x , we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore m = \left(\frac{dy}{dx} \right)_{\left(\frac{a^2}{4}, \frac{a^2}{4} \right)} = -\frac{\frac{a}{2}}{\frac{a}{2}} = -1$$

Thus,

the equation of tangent is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - \frac{a^2}{4} = (-1) \left(x - \frac{a^2}{4} \right)$$

$$\Rightarrow x + y = \frac{a^2}{4} + \frac{a^2}{4}$$

$$\Rightarrow x + y = \frac{a^2}{2}$$

Tangents and Normals Ex 16.2 Q2

The equation of the curve is

$$y = 2x^3 - x^2 + 3 \quad \text{---(i)}$$

$$\text{Slope} = m = \frac{dy}{dx} = 6x^2 - 2x$$

$$\therefore m = \left(\frac{dy}{dx} \right)_{(1,4)} = 4$$

Now,

The equation of normal is (i) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$\Rightarrow (y - 4) = \frac{-1}{4}(x - 1)$$

$$\Rightarrow x + 4y = 16 + 1$$

$$\Rightarrow x + 4y = 17$$

Tangents and Normals Ex 16.2 Q3(i)

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x , we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left. \frac{dy}{dx} \right|_{(0, 5)} = -10$$

Thus, the slope of the tangent at $(0, 5)$ is -10 . The equation of the tangent is given as:

$$y - 5 = -10(x - 0)$$

$$\Rightarrow y - 5 = -10x$$

$$\Rightarrow 10x + y = 5$$

The slope of the normal at $(0, 5)$ is $\frac{-1}{\text{Slope of the tangent at } (0, 5)} = \frac{1}{10}$.

Therefore, the equation of the normal at $(0, 5)$ is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$

$$\Rightarrow 10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x , we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left. \frac{dy}{dx} \right|_{(1, 3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at $(1, 3)$ is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$

$$\Rightarrow y - 3 = 2x - 2$$

$$\Rightarrow y = 2x + 1$$

The slope of the normal at $(1, 3)$ is $\frac{-1}{\text{Slope of the tangent at } (1, 3)} = \frac{-1}{2}$.

Therefore, the equation of the normal at $(1, 3)$ is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$\Rightarrow 2y - 6 = -x + 1$$

$$\Rightarrow x + 2y - 7 = 0$$

The equation of the curve is $y = x^2$.

On differentiating with respect to x , we get:

$$\frac{dy}{dx} = 2x$$

$$\left. \frac{dy}{dx} \right|_{(0, 0)} = 0$$

Thus, the slope of the tangent at $(0, 0)$ is 0 and the equation of the tangent is given as:

$$y - 0 = 0(x - 0)$$

$$\Rightarrow y = 0$$

The slope of the normal at $(0, 0)$ is $\frac{-1}{\text{Slope of the tangent at } (0, 0)} = -\frac{1}{0}$, which is not defined.

Therefore, the equation of the normal at $(x_0, y_0) = (0, 0)$ is given by

$$x = x_0 = 0.$$

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$y = 2x^2 - 3x - 1 \quad P = (1, -2)$$

$$\text{Slope } m = \frac{dy}{dx} = 4x - 3$$

$$m = \left(\frac{dy}{dx} \right)_P = 1$$

\therefore equation of tangent from (A)

$$(y + 2) = 1(x - 1)$$

$$\Rightarrow x - y = 3$$

And equation of normal from (B)

$$(y + 2) = -1(x - 1)$$

$$\Rightarrow x + y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(v)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = \frac{x^3}{4-x} \quad P = (2, -2)$$

Differentiating with respect to x , we get

$$2y \frac{dy}{dx} = \frac{3x^2(4-x) + x^3}{(4-x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2(4-x) + x^3}{2y(4-x)^2}$$

$$\begin{aligned} \therefore \text{Slope } m &= \left(\frac{dy}{dx} \right)_P = \frac{3 \times 4(4-2) + 8}{-2 \times 2(4-2)^2} \\ &= \frac{32}{-16} = -2 \end{aligned}$$

From (A)

Equation of tangent is

$$(y + 2) = -2(x - 2)$$

$$\Rightarrow 2x + y = 2$$

From (B)

Equation of Normal is

$$(y + 2) = \frac{1}{2}(x - 2)$$

$$\Rightarrow x - 2y = 6$$

Tangents and Normals Ex 16.2 Q3(vi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$y = x^2 + 4x + 1 \quad \text{and} \quad P = (x = 3)$$

$$\text{Slope} = \frac{dy}{dx} = 2x + 4$$

$$\therefore m = \left(\frac{dy}{dx} \right)_P = 10$$

From (A)

Equation of tangent is

$$(y - 22) = 10(x - 3)$$

$$\Rightarrow 10x - y = 8$$

From (B)

Equation of normal is

$$(y - 22) = \frac{-1}{10}(x - 3)$$

$$\Rightarrow x + 10y = 223$$

Tangents and Normals Ex 16.2 Q3(vii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad P = (a \cos \theta, b \sin \theta)$$

Differentiating with respect to x , we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\begin{aligned} \therefore \text{Slope } m &= \left(\frac{dy}{dx} \right)_P = \frac{-a \cos \theta b^2}{b \sin \theta a^2} \\ &= \frac{-b}{a} \cot \theta \end{aligned}$$

From (A)

Equation of tangent is,

$$(y - b \sin \theta) = \frac{-b}{a} \cot \theta (x - a \cos \theta)$$

$$\Rightarrow \frac{b}{a} x \cot \theta + y = b \sin \theta + b \cot \theta \times \cos \theta$$

$$\Rightarrow \frac{x}{a} \cot \theta + \frac{y}{b} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta}$$

$$\Rightarrow \frac{x}{a} \cot \theta + \frac{y}{b} = \frac{1}{\sin \theta}$$

$$\Rightarrow \frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

From (B)

Equation of normal is

$$(y - b \sin \theta) = \frac{a \sin \theta}{b \cos \theta} (x - a \cos \theta)$$

$$\Rightarrow \frac{a}{b} x \tan \theta - y = \frac{a^2}{b} \sin \theta - b \sin \theta$$

$$\Rightarrow \frac{a}{b} x \tan \theta - y = \frac{a^2 - b^2}{b} \sin \theta$$

$$\Rightarrow \frac{a}{b} x \sec \theta - y \operatorname{cosec} \theta = \frac{a^2 - b^2}{b}$$

$$\Rightarrow ax \sec \theta - by \operatorname{cosec} \theta = a^2 - b^2$$

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad P = (a \sec \theta, b \tan \theta)$$

Differentiating with respect to x , we get

$$\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$\begin{aligned} \therefore \text{Slope } m &= \left(\frac{dy}{dx} \right)_P = \frac{a \sec \theta b^2}{b \tan \theta a^2} \\ &= \frac{b}{a \sin \theta} \end{aligned}$$

From (A)

Equation of tangent is,

$$(y - b \tan \theta) = \frac{b}{a \sin \theta} (x - a \sec \theta)$$

$$\Rightarrow \frac{b}{a} \frac{x}{\sin \theta} - y = \frac{b \sec \theta}{\sin \theta} - b \tan \theta$$

$$\Rightarrow \frac{bx}{a \sin \theta} - y = \frac{b \sec \theta}{\sin \theta} (1 - \sin^2 \theta)$$

$$\Rightarrow \frac{x}{a} - \frac{y}{b} \sin \theta = \cos \theta$$

$$\Rightarrow \frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$$

From (B)

Equation of normal is

$$y - b \tan \theta = \frac{-a \sin \theta}{b} (x - a \sec \theta)$$

$$\Rightarrow ax \sin \theta + by = b^2 \tan \theta + a^2 \tan \theta$$

$$\Rightarrow ax \cos \theta + by \cot \theta = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q3(ix)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = 4ax \quad P\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Differentiating with respect to x , we get

$$2y \frac{dy}{dx} = 4a$$

$$\Rightarrow \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx}\right)_P = m$$

From (A)

Equation of tangent is

$$\left(y - \frac{2a}{m}\right) = m\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow m^2x - my = 2a - a$$

$$\Rightarrow m^2x - my = a$$

From (B)

Equation of normal is

$$\left(y - \frac{2a}{m}\right) = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow (my - 2a) = \frac{-m^2x + a}{m^2}$$

$$\Rightarrow m^2x + m^3y = 2am^2 + a$$

$$\Rightarrow m^2x + m^3y - 2am^2 - a = 0$$

Tangents and Normals Ex 16.2 Q3(x)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$c^2(x^2 + y^2) = x^2y^2 \quad \rho = \left(\frac{c}{\cos \theta}, \frac{c}{\sin \theta} \right)$$

Differentiating with respect to x , we get

$$c^2 \left(2x + 2y \frac{dy}{dx} \right) = 2xy^2 + 2x^2y \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} (2yc^2 - 2x^2y) = 2xy^2 - 2xc^2$$

$$\therefore \frac{dy}{dx} = \frac{x(y^2 - c^2)}{y(c^2 - x^2)}$$

$$\begin{aligned} \therefore \text{Slope } m &= \left(\frac{dy}{dx} \right)_\rho = \frac{\frac{c}{\cos \theta} \left(\frac{c^2}{\sin^2 \theta} - c^2 \right)}{\frac{c}{\sin \theta} \left(c^2 - \frac{c^2}{\cos^2 \theta} \right)} \\ &= \frac{c^2 \tan \theta (1 - \sin^2 \theta)}{c^2 \tan^2 \theta (\cos^2 \theta - 1)} \\ &= \frac{1}{-\tan \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{-\cos^3 \theta}{\sin^3 \theta} \end{aligned}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{\sin \theta} \right) = \frac{-\cos^3 \theta}{\sin^3 \theta} \left(x - \frac{c}{\cos \theta} \right)$$

$$\Rightarrow x \cos^3 \theta + y \sin^3 \theta = c \sin^2 \theta + c \cos^2 \theta$$

$$\Rightarrow x \cos^3 \theta + y \sin^3 \theta = c$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{\sin \theta}\right) = \frac{\sin^3 \theta}{\cos^3 \theta} \left(x - \frac{c}{\cos \theta}\right)$$

$$\Rightarrow x \sin^3 \theta - y \cos^3 \theta = \frac{c \sin^3 \theta}{\cos \theta} - \frac{c \cos^3 \theta}{\sin \theta}$$

$$\begin{aligned}\Rightarrow x \sin^3 \theta - y \cos^3 \theta &= \frac{c(\sin^4 \theta - \cos^4 \theta)}{\cos \theta \times \sin \theta} \\ &= \frac{c(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)}{\frac{1}{2} \sin 2\theta} \\ &= \frac{-2c \cos 2\theta}{\sin 2\theta} = -2c \cot 2\theta\end{aligned}$$

$$\therefore x \sin^3 \theta - y \cos^3 \theta + 2c \cot 2\theta = 0$$

Tangents and Normals Ex 16.2 Q3(xi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$xy = c^2 \quad P = \left(ct, \frac{c}{t}\right)$$

Differentiating with respect to x , we get

$$y + x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{-c}{ct} = \frac{-1}{t^2}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{t}\right) = \frac{-1}{t^2}(x - ct)$$

$$\Rightarrow x + t^2 y = tc + ct$$

$$\Rightarrow x + t^2 y = 2ct$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{t}\right) = t^2(x - ct)$$

$$\Rightarrow xt^3 - ty = ct^3 \times t - c$$

$$\Rightarrow xt^3 - ty = ct^4 - c$$

Tangents and Normals Ex 16.2 Q3(xii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---(i)} \quad P = (x_1, y_1)$$

Differentiating with respect to x , we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\frac{x_1b^2}{y_1a^2}$$

From (A)

Equation of tangent is

$$(y - y_1) = -\frac{x_1b^2}{y_1a^2}(x - x_1)$$

$$\Rightarrow xx_1b^2 + yy_1a^2 = x_1^2b^2 + y_1^2a^2$$

Divide by a^2b^2 both side

$$\Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$
$$= 1$$

[$\because (x_1, y_1)$ lies on (i)]

$$\therefore \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

From (B)

Equation of normal is

$$(y - y_1) = \frac{y_1a^2}{x_1b^2}(x - x_1)$$

$$xy_1a^2 - yx_1b^2 = x_1y_1a^2 - y_1x_1b^2$$

Dividing by x_1y_1 both side

$$\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(xiii)

Differentiating $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with respect to x , we have:

$$\begin{aligned}\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{2y}{b^2} \frac{dy}{dx} &= \frac{2x}{a^2} \\ \Rightarrow \frac{dy}{dx} &= \frac{b^2x}{a^2y}\end{aligned}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left. \frac{dy}{dx} \right|_{(x_0, y_0)} = \frac{b^2x_0}{a^2y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$\begin{aligned}y - y_0 &= \frac{b^2x_0}{a^2y_0}(x - x_0) \\ \Rightarrow a^2yy_0 - a^2y_0^2 &= b^2xx_0 - b^2x_0^2 \\ \Rightarrow b^2xx_0 - a^2yy_0 - b^2x_0^2 + a^2y_0^2 &= 0\end{aligned}$$

Tangents and Normals Ex 16.2 Q3(xiv)

Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ with respect to x , we get

$$\begin{aligned}\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{dy}{dx} &= -\left(\frac{y}{x}\right)^{\frac{1}{3}}\end{aligned}$$

Therefore, the slope of the tangent at $(1, 1)$ is $\left. \frac{dy}{dx} \right|_{(1,1)} = -1$

So, the equation of the tangent at $(1, 1)$ is

$$\begin{aligned}y - 1 &= -1(x - 1) \\ \Rightarrow y + x - 2 &= 0\end{aligned}$$

Also, the slope of the normal at $(1, 1)$ is given by $\frac{-1}{\text{slope of tangent at } (1, 1)} = 1$

\therefore the equation of the normal at $(1, 1)$ is

$$\begin{aligned}y - 1 &= 1(x - 1) \\ \Rightarrow y - x &= 0\end{aligned}$$

Tangents and Normals Ex 16.2 Q3(xv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \quad (\text{A}) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad (\text{B}) \quad \text{Normal}$$

Where m is the slope

We have,

$$x^2 = 4y \quad P = (2, 1)$$

$$\therefore 2x = \frac{4dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx} \right)_P = 1$$

From (A)

Equation of tangent is

$$y - 1 = 1(x - 2)$$

$$\Rightarrow x - y = 1$$

From (B)

Equation of normal is

$$(y - 1) = -1(x - 2)$$

$$\Rightarrow x + y = 3$$

Tangents and Normals Ex 16.2 Q3(vi)

The equation of the given curve is $y^2 = 4x$.

Differentiating with respect to x , we have:

$$2y \frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$

$$\therefore \left. \frac{dy}{dx} \right|_{(1,2)} = \frac{2}{2} = 1$$

Now, the slope at point $(1, 2)$ is $\left. \frac{dy}{dx} \right|_{(1,2)} = \frac{-1}{1} = -1$.

\therefore Equation of the tangent at $(1, 2)$ is $y - 2 = -1(x - 1)$.

$$\Rightarrow y - 2 = -x + 1$$

$$\Rightarrow x + y - 3 = 0$$

Equation of the normal is,

$$y - 2 = -(-1)(x - 1)$$

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(xix)

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$\Rightarrow y^2 = \frac{b^2}{a^2} x^2 - b^2$$

$$2y \frac{dy}{dx} = \frac{b^2}{a^2} 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2}{a^2} \frac{x}{y}$$

Differentiating the above function w.r.t. x , we get,

$$\Rightarrow \left[\frac{dy}{dx} \right]_{(\sqrt{2}a, b)} = \frac{b^2}{a^2} \frac{\sqrt{2}a}{b} = \frac{\sqrt{2}b}{a}$$

$$\text{Slope of the tangent } m = \frac{\sqrt{2}b}{a}$$

Equation of the tangent is

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - b) = \frac{\sqrt{2}b}{a}(x - \sqrt{2}a)$$

$$\Rightarrow a(y - b) = \sqrt{2}b(x - \sqrt{2}a)$$

$$\Rightarrow \sqrt{2}bx - ay + ab - 2ab = 0$$

$$\Rightarrow \sqrt{2}bx - ay - ab = 0$$

$$\text{Slope of the normal is } -\frac{1}{\frac{\sqrt{2}b}{a}} = -\frac{a}{b\sqrt{2}}$$

Equation of the normal is

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$

$$\Rightarrow \sqrt{2}b(y - b) = -a(x - \sqrt{2}a)$$

$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}b^2 - \sqrt{2}a^2 = 0$$

$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}(a^2 + b^2) = 0$$

The given equations are,
 $x = \theta + \sin \theta$, $y = 1 + \cos \theta$
 $\frac{dx}{d\theta} = 1 + \cos \theta$, $\frac{dy}{d\theta} = -\sin \theta$

$$\therefore \frac{dx}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin \theta}{1 + \cos \theta}$$

Slope,

$$m = \left(\frac{dy}{dx} \right)_{\theta = \frac{\pi}{4}} = \frac{-\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}}$$

$$= -1 + \frac{1}{\sqrt{2}}$$

Thus, equation of tangent is,

$$y - y_1 = m(x - x_1)$$

$$y - \left(1 + \frac{1}{\sqrt{2}}\right) = \left(-1 + \frac{1}{\sqrt{2}}\right) \left(x - \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)\right)$$

$$y - 1 - \frac{1}{\sqrt{2}} = \left(-1 + \frac{1}{\sqrt{2}}\right) \left(x - \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$$

Tangents and Normals Ex 16.2 Q5(i)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$$x = \theta + \sin \theta, \quad y = 1 + \cos \theta, \quad \theta = \frac{\pi}{2}$$

$$\therefore P = \left[\left(\frac{\pi}{2} + 1 \right), 1 \right]$$

$$\text{and } \frac{dx}{d\theta} = 1 + \cos \theta, \quad \frac{dy}{d\theta} = -\sin \theta$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx} \right)_P = \left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \right) = \frac{-1}{+1} = -1$$

Equation of tangent from (A)

$$(y - 1) = -1 \left(x - \left(\frac{\pi}{2} + 1 \right) \right)$$

$$\Rightarrow x + y = \frac{\pi}{2} + 1 + 1$$

$$\Rightarrow 2(x + y) = \pi + 4$$

From (B)

Equation of normal is

$$(y - 1) = 1 \left(x - \left(\frac{\pi}{2} + 1 \right) \right)$$

$$\Rightarrow 2(x - y) = \pi$$

Tangents and Normals Ex 16.2 Q5(ii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$$x = \frac{2at^2}{1+t^2}, \quad y = \frac{2at^3}{1+t^2}, \quad t = \frac{1}{2}$$

$$\therefore P = \left(x = \frac{a}{2 + \frac{1}{2}} = \frac{2a}{5}, y = \frac{a}{4+1} = \frac{a}{5} \right)$$

Now,

$$\begin{aligned} \frac{dx}{dt} &= \frac{4a + (1+t^2) - 2at^2(2t)}{(1+t^2)^2} \\ &= \frac{4at}{(1+t^2)^2} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{6at^2(1+t^2) - (2at^3)(2t)}{(1+t^2)^2} \\ &= \frac{6at^2 - 2at^4}{(1+t^2)^2} \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at^2 + 2at^4}{4at}$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx} \right)_P = \frac{\frac{3a}{2} + \frac{a}{8}}{2a} = \frac{13}{16}$$

From (A)

Equation of tangent is,

$$\begin{aligned} \left(y - \frac{a}{5} \right) &= \frac{13}{16} \left(x - \frac{2a}{5} \right) \\ 16y - \frac{16a}{5} &= 13x - \frac{26a}{5} \end{aligned}$$

$$13x - 16y - 2a = 0$$

Equation of normal is,

$$\begin{aligned} \left(y - \frac{a}{5} \right) &= -\frac{16}{13} \left(x - \frac{2a}{5} \right) \\ 13y - \frac{13a}{5} &= -16x + \frac{32a}{5} \end{aligned}$$

$$16x + 13y - 9a = 0$$

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$$x = at^2, \quad y = 2at, \quad t = 1$$

$$\therefore P = (a, 2a)$$

and

$$\frac{dx}{dt} = 2at, \quad \frac{dy}{dt} = 2a$$

$$\therefore \text{Slope } m = \left(\frac{dy}{dx} \right)_P = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2a} = 1$$

From (A)

Equation of tangent is

$$(y - 2a) = 1(x - a)$$

$$\Rightarrow x - y + a = 0$$

From (B)

Equation of normal is

$$(y - 2a) = -1(x - a)$$

$$\Rightarrow x + y = 3a$$

Tangents and Normals Ex 16.2 Q5(iv)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$$x = a \sec t, \quad y = b \tan t, \quad t = t$$

$$\therefore \frac{dx}{dt} = a \sec t \times \tan t$$

and

$$\frac{dy}{dt} = b \sec^2 t$$

$$\therefore \text{Slope } m = \frac{dy}{dx} = \frac{b \sec^2 t}{a \sec t \times \tan t} \\ = \frac{b}{a} \operatorname{cosec} t$$

From (A)

Equation of tangent

$$(y - b \tan t) = \frac{b}{a} \operatorname{cosec} t (x - a \sec t)$$

$$\Rightarrow bx \operatorname{cosec} t - ay = ab \operatorname{cosec} t \times \sec t - ab \tan t \\ = \frac{ab [1 - \sin^2 t]}{\sin t \times \cos t} \\ = \frac{ab \cos t}{\sin t}$$

$$\Rightarrow bx \sec t - ay \tan t = ab$$

From (B)

Equation of normal is

$$(y - b \tan t) = \frac{-a \sin t}{b} (x - a \sec t)$$

$$\Rightarrow ax \sin t + by = a^2 \tan t + b^2 \tan t$$

$$\Rightarrow ax \cos t + by \cot t = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q5(v)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$$

$$\frac{dx}{d\theta} = a(1 + \cos\theta), \frac{dy}{d\theta} = a \sin\theta$$

$$\therefore \text{Slope } m = \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta} = \frac{\frac{2 \sin\theta}{2} \times \frac{\cos\theta}{2}}{\frac{2 \cos^2\theta}{2}} = \frac{\tan\theta}{2}$$

Now,

From (A)

Equation of tangent

$$y - a(1 - \cos\theta) = \frac{\tan\theta}{2}(x - a(\theta + \sin\theta))$$

$$\Rightarrow \frac{x \tan\theta}{2} - y = a(\theta + \sin\theta) \frac{\tan\theta}{2} - a(1 - \cos\theta)$$

From (B)

Equation of normal is

$$y - a(1 - \cos\theta) = \frac{-\cot\theta}{2}(x - a(\theta + \sin\theta))$$

$$\Rightarrow (y - 2a) \frac{\tan\theta}{2} + x - a\theta = 0$$

Tangents and Normals Ex 16.2 Q5(vi)

$$x = 3\cos\theta - \cos^3\theta, y = 3\sin\theta - \sin^3\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta \sin\theta \text{ and } \frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta \cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta - 3\sin^2\theta \cos\theta}{-3\sin\theta + 3\cos^2\theta \sin\theta} = \frac{\cos\theta(1 - \sin^2\theta)}{-\sin\theta(1 - \cos^2\theta)} = \frac{\cos^3\theta}{-\sin^3\theta} = -\tan^3\theta$$

So equation of the tangent at θ is

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + \cos^3\theta)$$

$$\Rightarrow 4(y \cos^3\theta - x \sin^3\theta) = 3\sin 4\theta$$

So equation of normal at θ is

$$y - 3\sin\theta + \sin^3\theta = \frac{1}{\tan^3\theta}(x - 3\cos\theta + \cos^3\theta)$$

$$\Rightarrow y \cos^3\theta - x \sin^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

$$\Rightarrow y \sin^3\theta - x \cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

Tangents and Normals Ex 16.2 Q6

The given equation of curve is

$$x^2 + 2y^2 - 4x - 6y + 8 = 0 \quad \text{---(i) at } x = 2$$

Differentiating with respect to x , we get

$$2x + 4y \frac{dy}{dx} - 4 - 6 \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [4y - 6] = 4 - 2x$$

$$\therefore \frac{dy}{dx} = \frac{2 - x}{2y - 3}$$

Now,

From (i) at $x = 2$

$$4 + 2y^2 - 8 - 6y + 8 = 0$$

$$\Rightarrow 2y^2 - 6y + 4 = 0$$

$$\Rightarrow y^2 - 3y + 2 = 0$$

$$\Rightarrow (y - 2)(y - 1) = 0$$

$$\Rightarrow y = 2, 1$$

Thus,

$$\text{Slope } m_1 = \left(\frac{dy}{dx} \right)_{(2,2)} = 0$$

$$m_2 = \left(\frac{dy}{dx} \right)_{(2,1)} = 0$$

Thus, the equation of normal is

$$(y - y_1) = \frac{-1}{0}(x - 2)$$

$$\Rightarrow x = 2$$

Tangents and Normals Ex 16.2 Q7

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to x , we have:

$$2ay \frac{dy}{dx} = 3x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at (x_0, y_0) is $\left. \frac{dy}{dx} \right|_{(x_0, y_0)}$.

\Rightarrow The slope of the tangent to the given curve at (am^2, am^3) is

$$\left. \frac{dy}{dx} \right|_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

\therefore Slope of normal at (am^2, am^3)

$$= \frac{-1}{\text{slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$$

Hence, the equation of the normal at (am^2, am^3) is given by,

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$

$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$$

Tangents and Normals Ex 16.2 Q8

The given equations are

$$y^2 = ax^3 + b \quad \text{---(i)}$$

$$y = 4x - 5 \quad \text{---(ii)} \quad P = (2, 3)$$

Differentiating (i) with respect to x , we get

$$2y \frac{dy}{dx} = 3ax^2$$

$$\therefore \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$$\therefore m_1 = \left(\frac{dy}{dx} \right)_P = \frac{12a}{6} = 2a$$

$$m_2 = \text{slope of (ii)} = 4$$

According to the question

$$m_1 = m_2 \Rightarrow 2a = 4 \Rightarrow a = 2$$

From (i)

$$y^2 = 2 \times 2^3 + b$$

$$\Rightarrow 9 = 16 + b$$

$$\Rightarrow b = -7$$

Thus,

$$a = 2, b = -7$$

Tangents and Normals Ex 16.2 Q9

The given equations are,

$$y = x^2 + 4x - 16 \quad \text{--- (i)}$$

$$3x - y + 1 = 0 \quad \text{--- (ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 2x + 4$$

Slope m_2 of (ii)

$$m_2 = 3$$

As per question

$$m_1 = m_2$$

$$\Rightarrow 2x + 4 = 3$$

$$\Rightarrow x = \frac{-1}{2}$$

From (i)

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

$$\therefore P = \left(\frac{-1}{2}, \frac{-71}{4} \right)$$

Thus, the equation of tangent

$$\left(y + \frac{71}{4} \right) = 3 \left(x + \frac{1}{2} \right)$$

$$\Rightarrow 3x - y = \frac{71}{4} - \frac{3}{2}$$

$$\Rightarrow 3x - y = \frac{65}{4}$$

$$\Rightarrow 12x - 4y - 65 = 0$$

Tangents and Normals Ex 16.2 Q10

The given equation is

$$y = x^3 + 2x + 6 \quad \text{---(i)}$$

$$x + 14y + 4 = 0 \quad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 3x^2 + 2$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{14}$$

∴ Slope of normal to (i) is

$$\frac{-1}{m_1} = \frac{-1}{3x^2 + 2}$$

According to the question

$$\frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$$\Rightarrow 3x^2 + 2 = 14$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

From (i)

$$y = 8 + 4 + 6 \quad \text{or} \quad -8 - 4 + 6$$

$$= 18 \quad \text{or} \quad -6$$

so, $P = (2, 18)$ and $Q = (-2, -6)$

Thus, the equation of normal is

$$(y - 18) = \frac{-1}{14}(x - 2) \Rightarrow x + 14y + 86 = 0$$

$$\text{or} \quad (y + 6) = \frac{-1}{14}(x + 2) \Rightarrow x + 14y - 254 = 0$$

The given equations are,

$$y = 4x^3 - 3x + 5 \quad \text{--- (i)}$$

$$9y + x + 3 = 0 \quad \text{--- (ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 12x^2 - 3$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{9}$$

According to the question

$$m_1 \times m_2 = -1$$

$$\Rightarrow (12x^2 - 3) \left(-\frac{1}{9}\right) = -1$$

$$\Rightarrow 4x^2 - 1 = 3$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

From (i)

$$y = 4 - 3 + 5 \quad \text{or} \quad -4 + 3 + 5$$
$$= 6 \quad \text{or} \quad 4$$

$$\therefore P = (1, 6) \text{ or } Q = (-1, 4)$$

Thus, the equation of tangent is

$$(y - 6) = 9(x - 1) \quad \Rightarrow \quad 9x - y - 3 = 0$$

$$(y - 4) = 9(x + 1) \quad \Rightarrow \quad 9x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q12

The given equations are,

$$y = x \log_e x \quad \text{--- (i)}$$

$$2x - 2y + 3 = 0 \quad \text{--- (ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = \log_e x + 1$$

slope m_2 of (ii)

$$m_2 = 1$$

Tangents and Normals Ex 16.2 Q13

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x , we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is $2x - y + 9 = 0$.

$$2x - y + 9 = 0 \Rightarrow y = 2x + 9$$

This is of the form $y = mx + c$.

\therefore Slope of the line = 2

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$$2 = 2x - 2$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

Now, $x = 2$

$$\Rightarrow y = 4 - 4 + 7 = 7$$

Thus, the equation of the tangent passing through $(2, 7)$ is given by,

$$y - 7 = 2(x - 2)$$

$$\Rightarrow y - 2x - 3 = 0$$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$$

This is of the form $y = mx + c$.

\therefore Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent

$$\text{is } \frac{-1}{\text{slope of the line}} = \frac{-1}{3}.$$

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$

$$\Rightarrow 2x = \frac{5}{3}$$

$$\Rightarrow x = \frac{5}{6}$$

$$\text{Now, } x = \frac{5}{6}$$

$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$

$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$

$$\Rightarrow 36y - 217 = -2(6x - 5)$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Tangents and Normals Ex 16.2 Q14

The equation of the given curve is $y = \frac{1}{x-3}$, $x \neq 3$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$

$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

Hence, there is no tangent to the given curve having slope 2.

Tangents and Normals Ex 16.2 Q15

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-(2x-2)}{(x^2-2x+3)^2} = \frac{-2(x-1)}{(x^2-2x+3)^2}$$

If the slope of the tangent is 0, then we have:

$$\frac{-2(x-1)}{(x^2-2x+3)^2} = 0$$

$$\Rightarrow -2(x-1) = 0$$

$$\Rightarrow x = 1$$

$$\text{When } x = 1, y = \frac{1}{1-2+3} = \frac{1}{2}.$$

∴ The equation of the tangent through $\left(1, \frac{1}{2}\right)$ is given by,

$$y - \frac{1}{2} = 0(x-1)$$

$$\Rightarrow y - \frac{1}{2} = 0$$

$$\Rightarrow y = \frac{1}{2}$$

Hence, the equation of the required line is $y = \frac{1}{2}$.

The equation of the given curve is $y = \sqrt{3x-2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is $4x - 2y + 5 = 0$.

$$4x - 2y + 5 = 0 \Rightarrow y = 2x + \frac{5}{2} \text{ (which is of the form } y = mx + c\text{)}$$

\therefore Slope of the line = 2

Now, the tangent to the given curve is parallel to the line $4x - 2y - 5 = 0$ if the slope of the tangent is equal to the slope of the line.

$$\begin{aligned}\frac{3}{2\sqrt{3x-2}} &= 2 \\ \Rightarrow \sqrt{3x-2} &= \frac{3}{4} \\ \Rightarrow 3x-2 &= \frac{9}{16} \\ \Rightarrow 3x &= \frac{9}{16} + 2 = \frac{41}{16} \\ \Rightarrow x &= \frac{41}{48}\end{aligned}$$

$$\text{When } x = \frac{41}{48}, y = \sqrt{3\left(\frac{41}{48}\right) - 2} = \sqrt{\frac{41}{16} - 2} = \sqrt{\frac{41-32}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

\therefore Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$\begin{aligned}y - \frac{3}{4} &= 2\left(x - \frac{41}{48}\right) \\ \Rightarrow \frac{4y-3}{4} &= 2\left(\frac{48x-41}{48}\right) \\ \Rightarrow 4y-3 &= \frac{48x-41}{6} \\ \Rightarrow 24y-18 &= 48x-41 \\ \Rightarrow 48x-24y &= 23\end{aligned}$$

Hence, the equation of the required tangent is $48x - 24y = 23$

The given equations are,

$$x^2 + 3y - 3 = 0 \quad \text{--- (i)}$$

$$y = 4x - 5 \quad \text{--- (ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = -\frac{2x}{3}$$

Slope m_2 of (ii)

$$m_2 = 4$$

According to the question

$$m_1 = m_2$$

$$\Rightarrow -\frac{2x}{3} = 4$$

$$\Rightarrow x = -6$$

From (i)

$$36 + 3y - 3 = 0$$

$$\Rightarrow 3y = -33$$

$$\therefore y = -11$$

So, $P = (-6, -11)$

Thus, the equation of tangent is

$$(y + 11) = 4(x + 6)$$

$$\Rightarrow 4x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q18

The equations are

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \quad \text{---(i)}$$

$$\frac{x}{a} + \frac{y}{b} = 2 \quad \text{---(ii)}$$

$$P = (a, b)$$

We need to prove (ii) is the tangent to (i)

Differentiating (i) with respect to x , we get

$$n\left(\frac{x}{a}\right)^{n-1} \times \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \times \frac{1}{b} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x^{n-1}}{a^n} + \frac{y^{n-1}}{b^n} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$

$$\begin{aligned} \therefore \text{Slope } m &= \left(\frac{dy}{dx}\right)_P = -\left(\frac{a}{b}\right)^{n-1} \times \left(\frac{b}{a}\right)^n \\ &= -\frac{b}{a} \end{aligned}$$

Thus, the equation of tangent is

$$(y - b) = -\frac{b}{a}(x - a)$$

$$\Rightarrow bx + ay = ab + ab$$

$$\Rightarrow bx + ay = 2ab$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 2$$

Tangents and Normals Ex 16.2 Q19

We have,

$$x = \sin 3t, \quad y = \cos 2t, \quad t = \frac{\pi}{4}$$

$$\therefore P = \left(x = \frac{1}{\sqrt{2}}, y = 0\right)$$

Now,

$$\frac{dx}{dt} = 3 \cos 3t, \quad \frac{dy}{dt} = -2 \sin 2t$$

$$\therefore \text{Slope } m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2 \sin 2t}{3 \cos 3t}$$

$$\begin{aligned} &= \frac{-2}{-3 \times \frac{1}{\sqrt{2}}} \\ &= \frac{+2\sqrt{2}}{3} \end{aligned}$$

Thus, equation of tangent is

$$(y - 0) = \frac{+2\sqrt{2}}{3} \left(x - \frac{1}{\sqrt{2}}\right)$$

$$2\sqrt{2}x - 3y = 2$$