RD Sharma Solutions Class 12 Maths Chapter Ex 17.2

Increasing and Decreasing Functions Ex 17.2 Q1(i)

We have,

$f(x) = 10 - 6x - 2x^2$

$\therefore f'(x) = -6 - 4x$

Now,

$f'(x) = 0 \Rightarrow x = -\frac{3}{2}$

The point $x = -\frac{3}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{3}{2}\right)$ i.e., when $x < -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly increasing for $x < -\frac{3}{2}$.

In interval $\left(-\frac{3}{2}, \infty\right)$ i.e., when $x > -\frac{3}{2}$, $f'(x) = -6 - 4x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{3}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(ii)

We have,

$$f(x) = x^2 + 2x - 5$$

$$\therefore f'(x) = 2x + 2$$

Now,

$$f'(x) = 0 \Rightarrow x = -1$$

Point $x = -1$ divides the real line into two disjoint intervals i.e., $(-\infty, -1)$ and $(-1, \infty)$.

In interval $(-\infty, -1)$, $f'(x) = 2x + 2 < 0$.

$\therefore f$ is strictly decreasing in interval $(-\infty, -1)$.

Thus, f is strictly decreasing for $x < -1$.

In interval $(-1, \infty)$, $f'(x) = 2x + 2 > 0$.

$\therefore f$ is strictly increasing in interval $(-1, \infty)$.

Thus, f is strictly increasing for $x > -1$.

Increasing and Decreasing Functions Ex 17.2 Q1(iii)

We have,

$$f(x) = 6 - 9x - x^2$$

$$\therefore f'(x) = -9 - 2x$$

Now,

$$f'(x) = 0 \text{ gives } x = -\frac{9}{2}$$

The point $x = -\frac{9}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, -\frac{9}{2}\right)$ and $\left(-\frac{9}{2}, \infty\right)$.

In interval $\left(-\infty, -\frac{9}{2}\right)$ i.e., for $x < -\frac{9}{2}$, $f'(x) = -9 - 2x > 0$.

$\therefore f$ is strictly increasing for $x < -\frac{9}{2}$.

In interval $\left(-\frac{9}{2}, \infty\right)$ i.e., for $x > -\frac{9}{2}$, $f'(x) = -9 - 2x < 0$.

$\therefore f$ is strictly decreasing for $x > -\frac{9}{2}$.

Increasing and Decreasing Functions Ex 17.2 Q1(iv)

$f(x) = 2x^3 - 12x^2 + 18x + 15$

$$\begin{aligned} \therefore \quad f'(x) &= 6x^2 - 24x + 18 \\ &= 6(x^2 - 4x + 3) \\ &= 6(x-3)(x-1) \end{aligned}$$

Critical point

$$\begin{aligned} & f'(x) = 0 \\ \Rightarrow \quad & 6(x-3)(x-1) = 0 \\ \Rightarrow \quad & x = 3,\ 1 \end{aligned}$$

Clearly, $f(x) > 0$ if $x < 1$ and $x > 3$

and $f(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$, decreases on $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(v)

We have,

$$f(x) = 5 + 36x + 3x^2 - 2x^3$$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical point

$$f'(x) = 0$$

$\Rightarrow \quad 36 + 6x - 6x^2 = 0$

$\Rightarrow \quad -6\left(x^2 - x - 6\right) = 0$

$\Rightarrow \quad (x-3)(x+2) = 0$

$\therefore \quad x = 3, \ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

Also $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, increases if $x \in (-2, 3)$, decreases if $x \in (-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vi)

We have,

$$f(x) = 8 + 36x + 3x^2 - 2x^3$$

$\therefore \quad f'(x) = 36 + 6x - 6x^2$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 6\left(6 + x - x^2\right) = 0$

$\Rightarrow \quad (3-x)(2+x) = 0$

$\Rightarrow \quad x = 3, \ -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$

and $f'(x) < 0$ if $-\infty < x < -2$ and $3 < x < \infty$

Thus, increases in $(-2, 3)$, decreases in $(-\infty, -2) \cup (3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(vii)

We have,

$$f(x) = 5x^3 - 15x^2 - 120x + 3$$

$\therefore \quad f'(x) = 15x^2 - 30x - 120$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad 15\left(x^2 - 2x - 8\right) = 0$

$\Rightarrow \quad (x-4)(x+2) = 0$

$\Rightarrow \quad x = 4, \ -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$

and $f'(x) < 0$ if $-2 < x < 4$

Thus, increases in $(-\infty, -2) \cup (4, \infty)$, decreases in $(-2, 4)$

Increasing and Decreasing Functions Ex 17.2 Q1(viii)

$f(x) = x^3 - 6x^2 - 36x + 2$

$\therefore \quad f'(x) = 3x^2 - 12x - 36$

Critical point

$f'(x) = 0$

$\Rightarrow \quad 3(x^2 - 4x - 12) = 0$

$\Rightarrow \quad (x-6)(x+2) = 0$

$\Rightarrow \quad x = 6, \ -2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$

$f'(x) < 0$ if $-2x < x < 6$

Thus, increases in $(-\infty, -2) \cup (6, \infty)$, decreases in $(-2, 6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(ix)

We have,

$f(x) = 2x^3 - 15x^2 + 36x + 1$

$\therefore \quad f'(x) = 6x^2 - 30x + 36$

Critical points

$\Rightarrow \quad 6(x^2 - 5x + 6) = 0$

$\Rightarrow \quad (x-3)(x-2) = 0$

$\Rightarrow \quad x = 3, \ 2$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$

$f'(x) < 0$ if $2 < x < 3$

Thus, $f(x)$ increases in $(-\infty, 2) \cup (3, \infty)$, decreases in $(2, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(x)

We have,

$f(x) = 2x^3 + 9x^2 + 12x - 1$

$\therefore \quad f'(x) = 6x^2 + 18x + 12$

Critical ponts

$f'(x) = 0$

$\Rightarrow \quad 6(x^2 + 3x + 2) = 0$

$\Rightarrow \quad (x+2)(x+1) = 0$

$\Rightarrow \quad x = -2, \ -1$

Increasing and Decreasing Functions Ex 17.2 Q1(xi)

We have,

$$f(x) = 2x^3 - 9x^2 + 12x - 5$$

$\therefore$ $$f'(x) = 6x^2 - 18x + 12$$

Critical points

$$f'(x) = 0$$

$\Rightarrow$ $$6\left(x^2 - 3x + 2\right) = 0$$

$\Rightarrow$ $$(x-2)(x-1) = 0$$

$\Rightarrow$ $$x = 2,\ 1$$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 2$

$$f'(x) < 0 \text{ if } 1 < x < 2$$

Thus, $f(x)$ increases in $(-\infty, 1) \cup (2, \infty)$, decreases in $(1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xii)

We have,

$$f(x) = 6 + 12x + 3x^2 - 2x^3$$

$\therefore$ $$f'(x) = 12 + 6x - 6x^2$$

Critical points

$$f'(x) = 0$$

$\Rightarrow$ $$6\left(2 + x - x^2\right) = 0$$

$\Rightarrow$ $$(2-x)(1+x) = 0$$

$\Rightarrow$ $$x = 2,\ -1$$

Clearly, $f'(x) > 0$ if $-1 < x < 2$

$$f'(x) < 0 \text{ if } x < -1 \text{ and } x > 2.$$

Thus, $f(x)$ increases in $(-1, 2)$, decreases in $(-\infty, -1) \cup (2, \infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiii)

We have,

$$f(x) = 2x^3 - 24x + 107$$

$\therefore$ $$f'(x) = 6x^2 - 24$$

Critical points

$$f'(x) = 0$$

$\Rightarrow$ $$6\left(x^2 - 4\right) = 0$$

$\Rightarrow$ $$(x-2)(x+2) = 0$$

$\Rightarrow$ $$x = 2, -2$$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

$$f'(x) < 0 \text{ if } -2 < x < 2$$

Thus, $f(x)$ increases in $(-\infty, -2) \cup (2, \infty)$, decreases in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xiv)

We have

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$
$$f'(x) = -6x^2 - 18x - 12$$

Critical points

$$f'(x) = 0$$
$$-6x^2 - 18x - 12 = 0$$
$$x^2 + 3x + 2 = 0$$
$$(x+2)(x+1) = 0$$
$$x = -2, -1$$

Clearly, $f'(x) > 0$ if $x < -1$ and $x < -2$

$$f'(x) < 0 \text{ if } -2 < x < -1$$

Thus, $f(x)$ is increasing in $(-2,-1)$, decreasing in $(-\infty,-2)\cup(-1,\infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xv)

We have,

$$f(x) = (x-1)(x-2)^2$$
$$\therefore \quad f'(x) = (x-2)^2 + 2(x-1)(x-2)$$
$$f'(x) = (x-2)(x-2+2x-2)$$
$$\Rightarrow \quad f'(x) = (x-2)(3x-4)$$

Critical points

$$f'(x) = 0$$
$$\Rightarrow \quad (x-2)(3x-4) = 0$$
$$\Rightarrow \quad x = 2, \ \frac{4}{3}$$

Clearly, $f'(x) > 0$ if $x < \frac{4}{3}$ and $x > 2$

$$f'(x) < 0 \text{ if } \frac{4}{3} < x < 2$$

Thus, $f(x)$ increases in $\left(-\infty, \frac{4}{3}\right) \cup (2,\infty)$, decreases in $\left(\frac{4}{3}, 2\right)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvi)

We have,

$$f(x) = x^3 - 12x^2 + 36x + 17$$
$$\therefore \quad f'(x) = 3x^2 - 24x + 36$$

Critical points

$$f'(x) = 0$$
$$\Rightarrow \quad 3(x^2 - 8x + 12) = 0$$
$$\Rightarrow \quad (x-6)(x-2) = 0$$
$$\Rightarrow \quad x = 6, 2$$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 6$

$$f'(x) < 0 \text{ if } 2 < x < 6$$

Thus, $f(x)$ increases in $(-\infty, 2) \cup (6,\infty)$, decreases in $(2,6)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xvii)

We have

$$f(x) = 2x^3 - 24x + 7$$
$$f'(x) = 6x^2 - 24$$

Critical points

$$f'(x) = 0$$
$$6x^2 - 24 = 0$$
$$6x^2 = 24$$
$$x^2 = 4$$
$$x = 2, -2$$

Clearly, $f'(x) > 0$ if $x > 2$ and $x < -2$

$$f'(x) < 0 \text{ if } -2 \le x \le 2$$

Thus, $f(x)$ is increasing in $(-\infty, -2) \cup (2, \infty)$, decreasing in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xviii)

We have $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

$\therefore \quad f'(x) = \frac{3}{10}(4x^3) - \frac{4}{5}(3x^2) - 3(2x) + \frac{36}{5}$

$= \frac{6}{5}(x-1)(x+2)(x-3)$

Now $f'(x) = 0$

$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$

$\Rightarrow x = 1, -2$ or 3

The points $x = 1, -2$ and 3 divide the number line into four disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$.

Consider the interval $(-\infty, -2)$, i.e $-\infty < x < -2$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $-\infty < x < -2$

Thus, the function f is strictly decreasing in $(-\infty, -2)$

Consider the interval $(-2, 1)$, i.e $-2 < x < 1$

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) > 0$ when $-2 < x < 1$

Thus, the function f is strictly increasing in $(-2, 1)$

Now, consider the interval $(1, 3)$, i.e $1 < x < 3$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $1 < x < 3$

Thus, the function f is strictly decreasing in $(1, 3)$

Finally consider the interval $(3, \infty)$, i.e $3 < x < \infty$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

$\therefore \quad f'(x) > 0$ when $x > 3$

Thus, the function f is strictly increasing in $(3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(xix)

We have,

$$f(x) = x^4 - 4x$$

$$\therefore \quad f'(x) = 4x^3 - 4$$

Critical points,

$$f'(x) = 0$$

$$\Rightarrow \quad 4(x^3 - 1) = 0$$

$$\Rightarrow \quad x = 1$$

Clearly, $f'(x) > 0$ if $x > 1$

$f'(x) < 0$ if $x < 1$

Thus, $f(x)$ increases in $(1, \infty)$, decreases in $(-\infty, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xx)

$$f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

$$\therefore \quad f'(x) = x^3 + 2x^2 - 5x - 6$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad x^3 + 2x^2 - 5x - 6 = 0$$

$$\Rightarrow \quad (x+1)(x+3)(x-2) = 0$$

$$\Rightarrow \quad x = -1, \ -3, \ 2$$

Clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

$f'(x) < 0$ if $x < -3$ and $-1 < x < 2$

Thus, $f(x)$ increases in $(-3, -1) \cup (2, \infty)$, decreases in $(-\infty, -3) \cup (-1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxi)

$$f(x) = x^4 - 4x^3 + 4x^2 + 15$$

$$\therefore \quad f'(x) = 4x^3 - 12x^2 + 8x$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 4x(x^2 - 3x + 2) = 0$$

$$\Rightarrow \quad 4x(x-2)(x-1) = 0$$

$$\Rightarrow \quad x = 0, \ 2, \ 1$$

Clealry, $f'(x) > 0$ if $0 < x < 1$ and $x > 2$

$f'(x) < 0$ if $x < 0$ and $1 < x < 2$

Thus, $f(x)$ increases in $(0, 1) \cup (2, \infty)$, decreases in $(-\infty, 0) \cup (1, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxii)

We have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}};\ x > 0$$

$$\therefore \quad f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = 0$$

$$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}}(1 - x) = 0$$

$$\Rightarrow \quad x = 0,\ 1$$

Clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, $f(x)$ increases in $(0,1)$, decreases in $(1,\infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiii)

We have,

$$f(x) = x^8 + 6x^2$$

$$\therefore \quad f'(x) = 8x^7 + 12x$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 8x^7 + 12x = 0$$

$$\Rightarrow \quad 4x\left(2x^6 + 3\right) = 0$$

$$\Rightarrow \quad x = 0$$

Clearly, $f'(x) > 0$ if $x > 0$

$f'(x) < 0$ if $x < 0$

Thus, $f(x)$ increases in $(0,\infty)$, decreases in $(-\infty,0)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiv)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 9$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 3\left(x^2 - 4x + 3\right) = 0$$

$$\Rightarrow \quad (x - 3)(x - 1) = 0$$

$$\Rightarrow \quad x = 3,\ 1$$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

$f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases in $(-\infty,1) \cup (3,\infty)$, decreases in $(1,3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxv)

We have,

$$y=[x(x-2)]^2=[x^2-2x]^2$$

$$\therefore \frac{dy}{dx}=y'=2(x^2-2x)(2x-2)=4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx}=0 \Rightarrow x=0, x=2, x=1.$$

The points $x=0$, $x=1$, and $x=2$ divide the real line into four disjoint intervals i.e., $(-\infty,0)$, $(0,1)$ $(1,2)$, and $(2,\infty)$.

In intervals $(-\infty,0)$ and $(1,2)$, $\frac{dy}{dx}<0$.

$\therefore y$ is strictly decreasing in intervals $(-\infty,0)$ and $(1,2)$.

However, in intervals $(0, 1)$ and $(2, \infty)$, $\frac{dy}{dx}>0$.

$\therefore y$ is strictly increasing in intervals $(0, 1)$ and $(2, \infty)$.

$\therefore y$ is strictly increasing for $0<x<1$ and $x>2$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxvi)

Consider the given function

$f(x)=3x^4-4x^3-12x^2+5$

$\Rightarrow f'(x)=12x^3-12x^2-24x$

$\Rightarrow f'(x)=12x(x^2-x-2)$

$\Rightarrow f'(x)=12x(x+1)(x-2)$

For $f(x)$ to be increasing, we must have,

$f'(x)>0$

$\Rightarrow 12x(x+1)(x-2)>0$

$\Rightarrow x(x+1)(x-2)>0$

$\Rightarrow -1<x<0$ or $2<x<\infty$

$\Rightarrow x\in(-1,0)\cup(2,\infty)$

So, $f(x)$ *is* increasing in $(-1,0)\cup(2,\infty)$

For $f(x)$ to be decreasing, we must have,

$f'(x)<0$

$\Rightarrow 12x(x+1)(x-2)<0$

$\Rightarrow x(x+1)(x-2)<0$

$\Rightarrow -\infty<x<-1$ or $0<x<2$

$\Rightarrow x\in(-\infty,-1)\cup(0,2)$

So, $f(x)$ *is decreasing* in $(-\infty,-1)\cup(0,2)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxvii)

Consider the given function

$$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$$

$$\Rightarrow f'(x) = 4 \times \frac{3}{2}x^3 - 12x^2 - 90x$$

$$\Rightarrow f'(x) = 6x^3 - 12x^2 - 90x$$

$$\Rightarrow f'(x) = 6x\left(x^2 - 2x - 15\right)$$

$$\Rightarrow f'(x) = 6x(x+3)(x-5)$$

For $f(x)$ to be increasing, we must have,

$$f'(x) > 0$$

$$\Rightarrow 6x(x+3)(x-5) > 0$$

$$\Rightarrow x(x+3)(x-5) > 0$$

$$\Rightarrow -3 < x < 0 \text{ or } 5 < x < \infty$$

$$\Rightarrow x \in (-3, 0) \cup (5, \infty)$$

So, $f(x)$ *i*s *increa*sing in $(-3, 0) \cup (5, \infty)$

For $f(x)$ to be decreasing, we must have,

$$f'(x) < 0$$

$$\Rightarrow 6x(x+3)(x-5) < 0$$

$$\Rightarrow x(x+3)(x-5) < 0$$

$$\Rightarrow -\infty < x < -3 \text{ or } 0 < x < 5$$

$$\Rightarrow x \in (-\infty, -3) \cup (0, 5)$$

So, $f(x)$ *i*s *decrea*sing in $(-\infty, -3) \cup (0, 5)$

Increasing and Decreasing Functions Ex 17.2 Q1(xxviii)

Consider the given function

$$f(x) = \log(2+x) - \frac{2x}{2+x}, x \in R$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{2+x-4}{(2+x)^2}$$

$$\Rightarrow f'(x) = \frac{x-2}{(2+x)^2}$$

For f(x) to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow x - 2 > 0$

$\Rightarrow 2 < x < \infty$

$\Rightarrow x \in (2, \infty)$

So, f(x) *i* sin*crea* sing in $(2, \infty)$

For f(x) to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow x - 2 < 0$

$\Rightarrow -\infty < x < 2$

$\Rightarrow x \in (-\infty, 2)$

So, f(x) is decreasing in $(-\infty, 2)$

Increasing and Decreasing Functions Ex 17.2 Q2

We have,

$$f(x) = x^2 - 6x + 9$$

$$\therefore \quad f'(x) = 2x - 6$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 2(x-3) = 0$$

$$\Rightarrow \quad x = 3$$

Clearly, $f'(x) > 0$ if $x > 3$

$$f'(x) < 0 \text{ if } x < 3$$

Thus, $f(x)$ increases in $(3, \infty)$, decreases in $(-\infty, 3)$

IInd part

The given equation of curves

$$y = x^2 - 6x + 9 \qquad \text{---(i)}$$

$$y = x + 5 \qquad \text{---(ii)}$$

Slope of (i)

$$m_1 = \frac{dy}{dx} = 2x - 6$$

Slope of (ii)

$$m_2 = 1$$

Given that slope of normal to (i) is parallelt to (ii)

$$\therefore \quad \frac{-1}{2x-6} = 1$$

$$\Rightarrow \quad 2x - 6 = -1$$

$$\Rightarrow \quad x = \frac{5}{2}$$

From (i)

$$y = \frac{25}{4} - 15 + 9$$

$$= \frac{25}{4} - 6$$

$$= \frac{1}{4}$$

Thus, the required point is $\left(\frac{5}{2}, \frac{1}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q3

We have,

$$f(x) = \sin x - \cos x, \quad 0 < x < 2\pi$$

$$\therefore \quad f'(x) = \cos x + \sin x$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \cos x + \sin x = 0$$

$$\Rightarrow \quad \tan x = -1$$

$$\Rightarrow \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

$$f'(x) < 0 \text{ if } \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

Thus, $f(x)$ increases in $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$, decreases in $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q4

We have,

$$f(x) = e^{2x}$$

$$\therefore \quad f'(x) = 2e^{2x}$$

We know that

$$f(x) \text{ is increasing if } f'(x) > 0$$

$$\Rightarrow \quad 2e^{2x} > 0$$

$$\Rightarrow \quad e^{2x} > 0$$

Since, the value of e lies between 2 and 3

So, any power of e will be greater than zero.

Thus, $f(x)$ is increasing on R.

Increasing and Decreasing Functions Ex 17.2 Q5

We have,

$$f(x) = e^{\frac{1}{x}}, \qquad x \neq 0$$

$$f'(x) = e^{\frac{1}{x}} \times \left(\frac{-1}{x^2}\right)$$

$$\therefore \qquad f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$$

Now,

$$x \in R,\ x \neq 0$$

$$\Rightarrow \qquad \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$$

$$\Rightarrow \qquad \frac{e^{\frac{1}{x}}}{x^2} > 0$$

$$\Rightarrow \qquad -\frac{e^{\frac{1}{x}}}{x^2} < 0$$

$$\Rightarrow \qquad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for all $x \neq 0$.

Increasing and Decreasing Functions Ex 17.2 Q6

We have,

$$f(x) = \log_a x,\ \ 0 < a < 1$$

$$\Rightarrow \qquad f'(x) = \frac{1}{x \log a}$$

$$\therefore \qquad 0 < a < 1$$

$$\Rightarrow \qquad \log a < 0$$

Now,

$$x > 0$$

$$\Rightarrow \qquad \frac{1}{x} > 0$$

$$\Rightarrow \qquad \frac{1}{x \log a} < 0$$

$$\Rightarrow \qquad f'(x) < 0$$

Thus, $f(x)$ is a decreasing function for $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q7

The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right)$, $\cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$, we have $f'(x) < 0$.

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q8

We have,

$f(x) = \log \sin x$

$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f'(x) = \cot x > 0$.

$\therefore f$ is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $f'(x) = \cot x < 0$.

$\therefore f$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

Increasing and Decreasing Functions Ex 17.2 Q9

We have,

$$f(x) = x - \sin x$$

$$\therefore \quad f'(x) = 1 - \cos x$$

Now,

$$x \in R$$

$$\Rightarrow \quad -1 < \cos x < 1$$

$$\Rightarrow \quad -1 > \cos x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is increasing for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q10

We have,

$$f(x) = x^3 - 15x^2 + 75x - 50$$

$$\therefore \quad f'(x) = 3x^2 - 30x + 75$$

$$\Rightarrow \quad f'(x) = 3\left(x^2 - 10x + 25\right)$$

$$= 3(x-5)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad (x-5)^2 > 0$$

$$\Rightarrow \quad 3(x-5)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q11

We have,

$$f(x) = \cos^2 x$$

$$\therefore \quad f'(x) = 2\cos x(-\sin x)$$

$$\Rightarrow \quad f'(x) = -2\sin x \cos x$$

$$\Rightarrow \quad f'(x) = -\sin 2x$$

Now,

$$x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \quad 2x \in (0, \pi)$$

$$\Rightarrow \quad \sin 2x > 0 \text{ when } 2x \in (0, \pi)$$

$$\Rightarrow \quad -\sin 2x < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q12

We have

$$f(x) = \sin x$$
$$f'(x) = \cos x$$

Now,

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
$$\Rightarrow \cos x > 0$$
$$\Rightarrow f'(x) > 0$$

Therefore, $f(x) = \sin x$ is an increasing function on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q13

We have,

$$f(x) = \cos x$$
$$\therefore \quad f'(x) = -\sin x$$

Now,

$$\text{If } x \in (0, \pi)$$
$$\Rightarrow \quad \sin x > 0$$
$$\Rightarrow \quad -\sin x < 0$$

Hence, $f(x)$ is decreasing function on $(0, \pi)$

If $x \in (-\pi, 0)$

$$\Rightarrow \quad \sin x < 0 \qquad [\because \sin(-\theta) = -\sin\theta]$$
$$\Rightarrow \quad -\sin x > 0$$

Hence, $f(x)$ is increasing function on $(-\pi, 0)$

If $x \in (-\pi, \pi)$

Thus, $\sin x > 0$ for $x \in (0, \pi)$

and $\quad \sin x < 0$ for $x \in (-\pi, 0)$

$\Rightarrow \quad -\sin x < 0$ for $x \in (0, \pi)$

and $\quad -\sin x > 0$ for $x \in (-\pi, 0)$

Hence, $f(x)$ is neither increasing nor decreasing on $(-\pi, \pi)$.

Increasing and Decreasing Functions Ex 17.2 Q14

We have,

$$f(x) = \tan x$$
$$\therefore \quad f'(x) = \sec^2 x$$

Now,

$$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$
$$\Rightarrow \quad \sec^2 x > 0$$
$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is increasing function on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q15

We have,

$$f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\therefore \quad f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$= \frac{\cos x - \sin x}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now,

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \cos x - \sin x < 0$$

$$\Rightarrow \quad \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0 \qquad [\because 2(1 + \sin x \cos x) > 0]$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q16

We have,

$$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\therefore \quad f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\therefore \quad f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

Now,

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi < 2x < \frac{\pi}{4} < \frac{3\pi}{2}$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} \text{ lies in IIIrd quadrant}$$

$$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad 2\cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q17

We have,

$$f(x) = \tan x - 4x$$

$$\therefore \quad f'(x) = \sec^2 x - 4$$

$$= \frac{1 - 4\cos^2 x}{\cos^2 x}$$

$$= \frac{(1+2\cos x)(1-2\cos x)}{\cos^2 x}$$

$$= 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right)$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos x > \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad 4\sec^2 x\left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q18

We have,

$$f(x) = (x-1)e^x + 1$$

$$\therefore \quad f'(x) = e^x + (x-1)e^x$$

$$\Rightarrow \quad f'(x) = e^x(1 + x - 1) = xe^x$$

Now,

$$x > 0$$

$$\Rightarrow \quad e^x > 0$$

$$\Rightarrow \quad xe^x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all $x > 0$.

Increasing and Decreasing Functions Ex 17.2 Q19

We have,

$$f(x) = x^2 - x + 1$$

$$\therefore \quad f'x = 2x - 1$$

Now,

$$x \in (0,1)$$

$$\Rightarrow \quad 2x - 1 > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad 2x - 1 < 0 \text{ if } x < \frac{1}{2}$$

$$\Rightarrow \quad f'(x) > 0 \text{ if } x > \frac{1}{2}$$

$$\text{and} \quad f'(x) < 0 \text{ if } x < \frac{1}{2}$$

Thus, $f(x)$ is neither increasing nor decreasing on $(0,1)$.

Increasing and Decreasing Functions Ex 17.2 Q20

We have,

$$f(x) = x^9 + 4x^7 + 11$$

$$f'(x) = 9x^8 + 28x^6$$

$$= x^6\left(9x^2 + 28\right)$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^6 > 0 \text{ and } 9x^2 + 28 > 0$$

$$\Rightarrow \quad x^6\left(9x^2 + 28\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q21

We have,

$$f(x) = x^3 - 6x^2 + 12x - 18$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 12$$

$$= 3\left(x^2 - 4x + 4\right)$$

$$= 3(x-2)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad (x-2)^2 > 0$$

$$\Rightarrow \quad 3(x-2)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Thus, $f(x)$ is on increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q22

A function $f(x)$ is said to be increasing on $[a,b]$ if $f(x) > 0$

Now, we have,

$$f(x) = x^2 - 6x + 3$$

$$\therefore \quad f'(x) = 2x - 6$$

$$= 2(x-3)$$

Again,

$$x \in [4,6]$$

$$\Rightarrow \quad 4 \le x \le 6$$

$$\Rightarrow \quad 1 \le x - 3 \le 3$$

$$\Rightarrow \quad (x-3) > 0$$

$$\Rightarrow \quad 2(x-3) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in [4,6]$.

Increasing and Decreasing Functions Ex 17.2 Q23

We have,

$$f(x) = \sin x - \cos x$$

$$\therefore \quad f'(x) = \cos x + \sin x$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\frac{\sin \pi}{4}\cos x + \frac{\cos \pi}{4}\sin x\right)$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\Rightarrow \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \quad \sin 0° < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$$

$$\Rightarrow \quad 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$$

$$\Rightarrow \quad \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q24

We have,

$$f(x) = \tan^{-1} x - x$$

$$\therefore \quad f'(x) = \frac{1}{1+x^2} - 1$$

$$= \frac{-x^2}{1+x^2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^2 > 0 \text{ and } 1+x^2 > 0$$

$$\Rightarrow \quad \frac{x^2}{1+x^2} > 0$$

$$\Rightarrow \quad \frac{-x^2}{1+x^2} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q25

We have,

$$f(x) = -\frac{x}{2} + \sin x$$

$$\therefore \quad f'(x) = -\frac{1}{2} + \cos x$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \cos\frac{\pi}{3} < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow \quad -\frac{1}{2} + \cos x + 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q26

We have,

$$f(x) = \log(1+x) - \frac{x}{1+x}$$

$$\therefore \quad f'(x) = \frac{1}{1+x} - \left(\frac{(1+x)-x}{(1+x)^2}\right)$$

$$= \frac{1}{1+x} - \frac{1}{(1+x)^2}$$

$$= \frac{x}{(1+x)^2}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{x}{(1+x)^2} = 0$$

$$\Rightarrow \quad x = 0, \ -1$$

Clearly, $f'(x) > 0$ if $x > 0$

and $f'(x) < 0$ if $-1 < x < 0$ or $x < -1$

Hence, $f(x)$ increases in $(0,\infty)$, decreases in $(-\infty,-1) \cup (-1,0)$.

Increasing and Decreasing Functions Ex 17.2 Q27

We have,

$$f(x) = (x+2)e^{-x}$$

$$\therefore \quad f'(x) = e^{-x} - e^{-x}(x+2)$$

$$= e^{-x}(1-x-2)$$

$$= -e^{-x}(x+1)$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad -e^{-x}(x+1) = 0$$

$$\Rightarrow \quad x = -1$$

Clearly, $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence, $f(x)$ increases in $(-\infty,-1)$, decreases in $(-1,\infty)$

.

Increasing and Decreasing Functions Ex 17.2 Q28

We have,

$$f(x) = 10^x$$

$$\therefore \quad f'(x) = 10^x \times \log 10$$

Now,

$$x \in R$$

$$\Rightarrow \quad 10^x > 0$$

$$\Rightarrow \quad 10^x \log 10 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ in an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q29

We have,

$$f(x) = x - [x]$$

$$\therefore \quad f'(x) = 1 > 0$$

$\therefore$ $f(x)$ in an increasing function on $(0, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q30

We have,

$$f(x) = 3x^5 + 40x^3 + 240x$$

$$\therefore \quad f'(x) = 15x^4 + 120x^2 + 240$$

$$= 15(x^4 + 8x^2 + 16)$$

$$= 15(x^2 + 4)^2$$

Now,

$$x \in R$$

$$\Rightarrow \quad (x^2 + 4)^2 > 0$$

$$\Rightarrow \quad 15(x^2 + 4)^2 > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for all x.

Increasing and Decreasing Functions Ex 17.2 Q31

We have,

$$f(x) = \log \cos x$$

$$\therefore f'(x) = \frac{1}{\cos x}(-\sin x) = -\tan x$$

In interval $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0 \Rightarrow -\tan x < 0.$

$$\therefore f'(x) < 0 \text{ on } \left(0, \frac{\pi}{2}\right)$$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0.$

$$\therefore f'(x) > 0 \text{ on } \left(\frac{\pi}{2}, \pi\right)$$

Increasing and Decreasing Functions Ex 17.2 Q32

Given $f(x) = x^3 - 3x^2 + 4x$

$$\therefore \quad f'(x) = 3x^2 - 6x + 4$$
$$= 3\left(x^2 - 2x + 1\right) + 1$$
$$= 3(x-1)^2 + 1 > 0, \text{ for all } x \in \mathbf{R}$$

Hence, f is strictly increasing on **R**.

Increasing and Decreasing Functions Ex 17.2 Q33

Given $f(x) = \cos x$

$$\therefore \quad f'(x) = -\sin x$$

(i) Since for each $x \in (0, \pi)$, $\sin x > 0$

$$\Rightarrow \quad f'(x) < 0$$

So f is strictly decreasing in $(0, \pi)$

(ii) Since for each $x \in (\pi, 2\pi)$, $\sin x < 0$

$$\Rightarrow \quad f'(x) > 0$$

So f is strictly increasing in $(\pi, 2\pi)$

(iii) Clearly from (i) & (ii) above, f is neither increasing nor decreasing in $(0, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q34

We have,

$$f(x) = x^2 - x\sin x$$

$\therefore \quad f'(x) = 2x - \sin x - x\cos x$

Now,

$$x \in \left(0, \frac{\pi}{2}\right)$$

$\Rightarrow \quad 0 \le \sin x \le 1, \quad 0 \le \cos x \le 1$

$\Rightarrow \quad 2x - \sin x - x\cos x > 0$

$\Rightarrow \quad f'(x) \ge 0$

Hence, $f(x)$ is an increasing function on $\left(0, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q35

We have,

$$f(x) = x^3 - ax$$

$\therefore \quad f'(x) = 3x^2 - a$

Given that $f(x)$ is on increasing function

$\therefore \quad f'(x) > 0 \qquad$ for all $x \in R$

$\Rightarrow \quad 3x^2 - a > 0 \qquad$ for all $x \in R$

$\Rightarrow \quad a < 3x^2 \qquad$ for all $x \in R$

But the last value of $3x^2 = 0$ for $x = 0$

$\therefore \quad a \le 0$

Increasing and Decreasing Functions Ex 17.2 Q36

We have,

$$f(x) = \sin x - bx + c$$

$\therefore \quad f'(x) = \cos x - b$

Given that $f(x)$ is a decreasing function on R

$\therefore \quad f'(x) < 0 \qquad$ for all $x \in R$

$\Rightarrow \quad \cos x - b < 0 \qquad$ for all $x \in R$

$\Rightarrow \quad b > \cos x \qquad$ for all $x \in R$

But man value of $\cos x$ in 1

$\therefore \quad b \ge 1$

Increasing and Decreasing Functions Ex 17.2 Q37

We have,

$$f(x) = x + \cos x - a$$

$$\therefore \quad f'(x) = 1 - \sin x = \frac{2\cos^2 x}{2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad \frac{\cos^2 x}{2} > 0$$

$$\Rightarrow \quad \frac{2\cos^2 x}{2} > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q38

As $f(0) = f(1)$ and f is differentiable, hence by Rolles theorem:

$f'(c) = 0$ for some $c \in [0, 1]$

Let us now apply LMVT (as function is twice differentiable) for point c and $x \in [0, 1]$, hence

$$\frac{|f'(x) - f(c)|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x) - 0|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)|}{x - c} = f''(d)$$

As given that $|f'(d)| \leq 1$ for $x \in [0, 1]$

$$\Rightarrow \frac{|f'(x)|}{x - c} \leq 1$$

$$\Rightarrow |f'(x)| \leq x - c$$

Now as both x and c lie in $[0, 1]$, hence $x - c \in (0, 1)$

$$\Rightarrow |f'(x)| < 1 \text{ for all } x \in [0, 1]$$

Increasing and Decreasing Functions Ex 17.2 Q39(i)

Consider the given function,

$f(x) = x|x|, \ x \in R$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

$\Rightarrow f'(x) > 0$, for values of x

Therefore, $f(x)$ is an increasing function for all real values.

Increasing and Decreasing Functions Ex 17.2 Q39(ii)

Consider the function

$f(x) = \sin x + |\sin x|,\ 0 < x \leq 2\pi$

$$\Rightarrow f(x) = \begin{cases} 2\sin x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2\cos x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

The function 2cosx will be positive between $\left(0, \frac{\pi}{2}\right)$.

Hence the function $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$.

The function 2cosx will be negative between $\left(\frac{\pi}{2}, \pi\right)$.

Hence the function $f(x)$ is decreasing in the interval $\left(\frac{\pi}{2}, \pi\right)$.

The value of $f'(x) = 0$, when $\pi \leq x < 2\pi$.

Therefore, the function $f(x)$ is neither increasing nor decreasing in the interval $(\pi, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q39(iii)

Consider the function,

$$f(x) = \sin x(1 + \cos x),\ 0 < x < \frac{\pi}{2}$$

$$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$$

$$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$$

$$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$$

$$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$$

$$\Rightarrow f'(x) = 2\cos^2 x + \cos x - 1$$

$$\Rightarrow f'(x) = 2\cos^2 x + 2\cos x - \cos x - 1$$

$$\Rightarrow f'(x) = 2\cos x(\cos x + 1) - 1(\cos x + 1)$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$$

For f(x) to be increasing, we must have,

$$f'(x) > 0$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) > 0$$

$$\Rightarrow 0 < x < \frac{\pi}{3}$$

$$\Rightarrow x \in \left(0, \frac{\pi}{3}\right)$$

So, f(x) is increasing in $\left(0, \frac{\pi}{3}\right)$

For f(x) to be decreasing, we must have,

$$f'(x) < 0$$

$$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) < 0$$

$$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$$

$$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

So, f(x) is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$