

RD Sharma
Solutions
Class 12 Maths
Chapter 18
Ex 18.2

Maxima and Minima Ex 18.2 Q1

$$f(x) = (x - 5)^4$$

$$\therefore f'(x) = 4(x - 5)^3$$

For local maxima and minima

$$f'(x) = 0$$

$$\Rightarrow 4(x - 5)^3 = 0$$

$$\Rightarrow x - 5 = 0$$

$$\Rightarrow x = 5$$

$f'(x)$ changes from -ve to +ve as passes through 5.

So, $x = 5$ is the point of local minima

Thus, local minimum value is $f(5) = 0$

Maxima and Minima Ex 18.2 Q2

$$g(x) = x^3 - 3x$$

$$\therefore g'(x) = 3x^2 - 3$$

Now,

$$g'(x) = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$$

$$g''(x) = 6x$$

$$g''(1) = 6 > 0$$

$$g''(-1) = -6 < 0$$

By second derivative test, $x = 1$ is a point of local minima and local minimum value of g at $x = 1$ is $g(1) = 1^3 - 3 = 1 - 3 = -2$. However,

$x = -1$ is a point of local maxima and local maximum value of g at

$$x = -1 \text{ is } g(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2.$$

Maxima and Minima Ex 18.2 Q3

$$f(x) = x^3(x-1)^2$$

$$\begin{aligned}\therefore f'(x) &= 3x^2(x-1)^2 + 2x^3(x-1) \\ &= (x-1)(3x^2(x-1) + 2x^3) \\ &= (x-1)(3x^3 - 3x^2 + 2x^3) \\ &= (x-1)(5x^3 - 3x^2) \\ &= x^2(x-1)(5x-3)\end{aligned}$$

For all maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow x^2(x-1)(5x-3) = 0$$

$$\Rightarrow x = 0, 1, \frac{3}{5}$$

At $x = \frac{3}{5}$ $f'(x)$ changes from +ve to -ve

$\therefore x = \frac{3}{5}$ is point of minima.

At $x = 1$ $f'(x)$ changes from -ve to +ve

$\therefore x = 1$ is point of maxima

Maxima and Minima Ex 18.2 Q4

$$f(x) = (x-1)(x+2)^2$$

$$\begin{aligned}\therefore f'(x) &= (x+2)^2 + 2(x-1)(x+2) \\ &= (x+2)(x+2+2x-2) \\ &= (x+2)(3x)\end{aligned}$$

For point of maxima and minima

$$f'(x) = 0$$

$$\Rightarrow (x+2) \times 3x = 0$$

$$\Rightarrow x = 0, -2$$

At $x = -2$ $f'(x)$ changes from +ve to -ve

$\therefore x = -2$ is point of local maxima

At $x = 0$ $f'(x)$ changes from -ve to +ve

$\therefore x = 0$ is point of local minima

Thus, local min value = $f(0) = -4$

local max value = $f(-2) = 0$.

Maxima and Minima Ex 18.2 Q5

$$f(x) = (x-1)^3(x+1)^2$$

$$\begin{aligned}\therefore f'(x) &= 3(x-1)^2(x+1)^2 + 2(x-1)^3(x+1) \\ &= (x-1)^2(x+1)(3(x+1) + 2(x-1)) \\ &= (x-1)^2(x+1)(5x+1)\end{aligned}$$

For the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow (x-1)^2(x+1)(5x+1) = 0$$

$$\Rightarrow x = 1, -1, -\frac{1}{5}$$

Here,

At $x = -1$ $f'(x)$ changes from +ve to -ve so $x = -1$ is point of maxima.

At $x = -\frac{1}{5}$, $f'(x)$ changes from -ve to +ve so $x = -\frac{1}{5}$ is point of minima

Hence, local max value = 0

$$\text{local min value} = -\frac{3456}{3125}$$

Maxima and Minima Ex 18.2 Q6

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\begin{aligned}\therefore f'(x) &= 3x^2 - 12x + 9 \\ &= 3(x^2 - 4x + 3) \\ &= 3(x - 3)(x - 1)\end{aligned}$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3(x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3, 1$$

At $x = 1$, $f'(x)$ changes from +ve to -ve

$\therefore x = 1$ is point of local maxima

At $x = 3$, $f'(x)$ changes from -ve to +ve

$\therefore x = 3$ is point of local minima

Hence, local max value = $f(1) = 19$

local min value = $f(3) = 15$.

Maxima and Minima Ex 18.2 Q7

$$f(x) = \sin 2x, \quad 0 < x, \pi$$

$$\therefore f'(x) = 2 \cos 2x$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$$

At $x = \frac{\pi}{4}$, $f'(x)$ changes from +ve to -ve

$\therefore x = \frac{\pi}{4}$ is point of local maxima

At $x = \frac{3\pi}{4}$, $f'(x)$ changes from -ve to +ve

$\therefore x = \frac{3\pi}{4}$ is point of local minima,

Hence, local max value = $f\left(\frac{\pi}{4}\right) = 1$

local min value = $f\left(\frac{3\pi}{4}\right) = -1$.

Maxima and Minima Ex 18.2 Q8

$$f(x) = \sin x - \cos x, 0 < x < 2\pi$$

$$\therefore f'(x) = \cos x + \sin x$$

$$f'(x) = 0 \Rightarrow \cos x = -\sin x \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4} \in (0, 2\pi)$$

$$f''(x) = -\sin x + \cos x$$

$$f''\left(\frac{3\pi}{4}\right) = -\sin \frac{3\pi}{4} + \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} < 0$$

$$f''\left(\frac{7\pi}{4}\right) = -\sin \frac{7\pi}{4} + \cos \frac{7\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$$

Therefore, by second derivative test, $x = \frac{3\pi}{4}$ is a point of local maxima and the local maximum value of f at $x = \frac{3\pi}{4}$ is

$$f\left(\frac{3\pi}{4}\right) = \sin \frac{3\pi}{4} - \cos \frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}. \text{ However, } x = \frac{7\pi}{4} \text{ is a point of local minima and the}$$

$$\text{local minimum value of } f \text{ at } x = \frac{7\pi}{4} \text{ is } f\left(\frac{7\pi}{4}\right) = \sin \frac{7\pi}{4} - \cos \frac{7\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}.$$

Maxima and Minima Ex 18.2 Q9

$$f(x) = \cos x, 0 < x < \pi$$

$$\therefore f'(x) = -\sin x$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow -\sin x = 0$$

$$\Rightarrow x = 0, \text{ and } \pi$$

But, these two points lies outside the interval $(0, \pi)$

So, no local maxima and minima will exist in the interval $(0, \pi)$.

Maxima and Minima Ex 18.2 Q10

$$\therefore f'(x) = 2 \cos 2x - 1$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 2 \cos 2x - 1 = 0$$

$$\Rightarrow \cos 2x = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow 2x = \frac{\pi}{3}, -\frac{\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{6}, -\frac{\pi}{6}$$

At $x = -\frac{\pi}{6}$, $f'(x)$ changes from -ve to +ve

$\therefore x = -\frac{\pi}{6}$ is point of local minima

At $x = \frac{\pi}{6}$, $f'(x)$ changes from +ve to -ve

$\therefore x = \frac{\pi}{6}$ is point of local maxima

$$\text{Hence, local max value} = f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$\text{local min value} = f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$$

Maxima and Minima Ex 18.2 Q11

$$f(x) = 2 \sin x - x, \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

For checking the minima and maxima, we have

$$f'(x) = 2 \cos x - 1 = 0$$

$$\Rightarrow \cos x = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{3}$$

At $x = -\frac{\pi}{3}$, $f(x)$ changes from -ve to +ve

$$\Rightarrow x = -\frac{\pi}{3} \text{ is point of local minima with value} = -\sqrt{3} - \frac{\pi}{3}$$

At $x = \frac{\pi}{3}$, $f(x)$ changes from +ve to +ve

$$\Rightarrow x = \frac{\pi}{3} \text{ is point of local maxima with value} = \sqrt{3} - \frac{\pi}{3}$$

Maxima and Minima Ex 18.2 Q12

$$\begin{aligned}\therefore f'(x) &= \sqrt{1-x} + x \cdot \frac{1}{2\sqrt{1-x}}(-1) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} \\ &= \frac{2(1-x) - x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}\end{aligned}$$

$$f'(x) = 0 \Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0 \Rightarrow 2-3x = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{1}{2} \left[\frac{\sqrt{1-x}(-3) - (2-3x)\left(\frac{-1}{2\sqrt{1-x}}\right)}{1-x} \right]$$

$$= \frac{\sqrt{1-x}(-3) + (2-3x)\left(\frac{1}{2\sqrt{1-x}}\right)}{2(1-x)}$$

$$= \frac{-6(1-x) + (2-3x)}{4(1-x)^{\frac{3}{2}}}$$

$$= \frac{3x-4}{4(1-x)^{\frac{3}{2}}}$$

$$f''\left(\frac{2}{3}\right) = \frac{3\left(\frac{2}{3}\right) - 4}{4\left(1 - \frac{2}{3}\right)^{\frac{3}{2}}} = \frac{2-4}{4\left(\frac{1}{3}\right)^{\frac{3}{2}}} = \frac{-1}{2\left(\frac{1}{3}\right)^{\frac{3}{2}}} < 0$$

Therefore, by second derivative test, $x = \frac{2}{3}$ is a point of local maxima and the local maximum

value of f at $x = \frac{2}{3}$ is

$$f\left(\frac{2}{3}\right) = \frac{2}{3} \sqrt{1 - \frac{2}{3}} = \frac{2}{3} \sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}.$$

We have,

$$f(x) = x^3(2x - 1)^3$$

$$\begin{aligned}\therefore f'(x) &= 3x^2(2x - 1)^3 + 3x^3(2x - 1)^2 \times 2 \\ &= 3x^2(2x - 1)^2(2x - 1 + 2x) \\ &= 3x^2(4x - 1)\end{aligned}$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3x^2(4x - 1) = 0$$

$$\Rightarrow x = 0, \frac{1}{4}$$

At $x = \frac{1}{4}$, $f'(x)$ changes from -ve to +ve

$\therefore x = \frac{1}{4}$ is the point of local minima,

$$\therefore \text{local min value} = f\left(\frac{1}{4}\right) = \frac{-1}{512}.$$

Maxima and Minima Ex 18.2 Q14

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, x > 0$$

$$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

For the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x = \sqrt{4}, -\sqrt{4}$$

$$\Rightarrow x = 2, -2$$

At $x = 2$, $f'(x)$ changes from -ve to +ve

$\therefore x = 2$ is point of local minima,

$$\therefore \text{local min value} = f(2) = 2.$$

Maxima and Minima Ex 18.2 Q15

$$g(x) = \frac{1}{x^2 + 2}$$

$$\therefore g'(x) = \frac{-(2x)}{(x^2 + 2)^2}$$

$$g'(x) = 0 \Rightarrow \frac{-2x}{(x^2 + 2)^2} = 0 \Rightarrow x = 0$$

Now, for values close to $x = 0$ and to the left of 0, $g'(x) > 0$. Also, for values close to $x = 0$ and to the right of 0, $g'(x) < 0$.

Therefore, by first derivative test, $x = 0$ is a point of local maxima and the local maximum value of $g(0)$ is $\frac{1}{0+2} = \frac{1}{2}$.