

RD Sharma
Solutions
Class 12 Maths
Chapter 18
Ex 18.3

Maxima and Minima 18.3 Q1(i)

$$f(x) = x^4 - 62x^2 + 120x + 9$$

$$\therefore f'(x) = 4x^3 - 124x + 120 = 4(x^3 - 31x + 30)$$

$$f''(x) = 12x^2 - 124 = 4(3x^2 - 31)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 4(x^3 - 31x + 30) = 0$$

$$\Rightarrow 4(x^3 - 31x + 30) = 0$$

$$\Rightarrow x = 5, 1, -6$$

Now,

$$f''(5) = 176 > 0$$

$\Rightarrow x = 5$ is point of local minima

$$f''(1) = -112 < 0$$

$\Rightarrow x = 1$ is point of local maxima

$$f''(-6) = 308 > 0$$

$\Rightarrow x = -6$ is point of local minima

$$\therefore \text{local max value} = f(1) = 68$$

$$\text{local min value} = f(5) = -316$$

$$\text{and } = f(-6) = -1647.$$

Maxima and Minima 18.3 Q1(ii)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\begin{aligned}\therefore f'(x) &= 3x^2 - 12x + 9 \\ &= 3(x^2 - 4x + 3) \\ f''(x) &= 6x - 12 \\ &= 6(x - 2)\end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow 3(x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3, 1$$

Now,

$$f''(3) = 6 > 0$$

$\therefore x = 3$ is point of local minima

$$f''(1) = -6 < 0$$

$\therefore x = 1$ is point of local maxima

\therefore local max value = $f(1) = 19$

local min value = $f(3) = 15$.

Maxima and Minima 18.3 Q1(iii)

We have,

$$f(x) = (x - 1)(x + 2)^2$$

$$\begin{aligned}\therefore f'(x) &= (x + 2)^2 + 2(x - 1)(x + 2) \\ &= (x + 2)(x + 2 + 2x - 2) \\ &= (x + 2)(3x)\end{aligned}$$

$$\begin{aligned}\text{and, } f''(x) &= 3(x + 2) + 3x \\ &= 6x + 6\end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3x(x + 2) = 0$$

$$\Rightarrow x = 0, -2$$

Now,

$$f''(0) = 6 > 0$$

$\therefore x = 0$ is point of local minima

$$f''(-2) = -6 < 0$$

$\therefore x = -2$ is point of local maxima

\therefore local max value = $f(-2) = 0$

local min value = $f(0) = -4$.

Maxima and Minima 18.3 Q1(iv)

We have,

$$f(x) = \frac{2}{x} - \frac{2}{x^2}, \quad x > 0$$

$$\therefore f'(x) = \frac{-2}{x^2} + \frac{4}{x^3}$$

$$\text{and, } f''(x) = \frac{+4}{x^3} - \frac{12}{x^4}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{-2}{x^2} + \frac{4}{x^3} = 0$$

$$\Rightarrow \frac{-2(x-2)}{x^3} = 0$$

$$\Rightarrow x = 2$$

Now,

$$f''(2) = \frac{4}{8} - \frac{12}{6} = \frac{1}{2} - \frac{3}{4} = \frac{-1}{4} < 0$$

$\therefore x = 2$ is point of local maxima

$$\text{local max value} = f(2) = \frac{1}{2}.$$

Maxima and Minima 18.3 Q1(v)

We have,

$$f(x) = xe^x$$

$$\therefore f'(x) = e^x + xe^x = e^x(x+1)$$

$$\begin{aligned} f''(x) &= e^x(x+1) + e^x \\ &= e^x(x+2) \end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow e^x(x+1) = 0$$

$$\Rightarrow x = -1$$

Now,

$$f''(-1) = e^{-1} = \frac{1}{e} > 0$$

$\therefore x = -1$ is point of local minima

Hence,

$$\text{local min value} = f(-1) = \frac{-1}{e}.$$

Maxima and Minima 18.3 Q1(vi)

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, \quad x > 0$$

$$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$$\text{and, } f''(x) = \frac{4}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \frac{x^2 - 4}{2x^2} = 0$$

$$\Rightarrow x = 2, -2$$

Now,

$$f''(2) = \frac{1}{2} > 0$$

$\therefore x = 2$ is point of minima

We will not consider $x = -2$ as $x > 0$

\therefore local min value = $f(2) = 2$.

Maxima and Minima 18.3 Q1(vii)

We have,

$$f(x) = (x+1)(x+2)^{\frac{1}{3}}, \quad x \geq -2$$

$$\therefore f'(x) = (x+2)^{\frac{1}{3}} + \frac{1}{3}(x+1)(x+2)^{-\frac{2}{3}}$$

$$= (x+2)^{-\frac{2}{3}} \left(x+2 + \frac{1}{3}(x+1) \right)$$

$$= \frac{1}{3}(x+2)^{-\frac{2}{3}}(4x+7)$$

$$\text{and, } f''(x) = -\frac{2}{9}(x+2)^{-\frac{5}{3}}(4x+7) + \frac{1}{3}(x+2)^{-\frac{2}{3}} \times 4$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1}{3}(x+2)^{-\frac{2}{3}}(4x+7) = 0$$

$$\Rightarrow x = -\frac{7}{4}$$

Now,

$$f''\left(-\frac{7}{4}\right) = \frac{4}{3}\left(-\frac{7}{4}+2\right)^{-\frac{2}{3}}$$

$\therefore x = -\frac{7}{4}$ is point of minima

\therefore local min value = $f\left(-\frac{7}{4}\right) = \frac{-3}{4^{\frac{2}{3}}}$.

Maxima and Minima 18.3 Q1(viii)

We have,

$$f(x) = x\sqrt{32-x^2}, -5 \leq x \leq 5$$

$$\begin{aligned}\therefore f'(x) &= \sqrt{32-x^2} + \frac{x}{2\sqrt{32-x^2}} \times (-2x) \\ &= \frac{2(32-x^2) - 2x^2}{2\sqrt{32-x^2}} \\ &= \frac{64-4x^2}{2\sqrt{32-x^2}}\end{aligned}$$

$$\begin{aligned}\text{and, } f''(x) &= \frac{2\sqrt{32-x^2} \times (-8x) - \frac{-2(64-4x^2)}{2\sqrt{32-x^2}} \times (-2x)}{4(32-x^2)} \\ &= \frac{-4(32-x^2) \times 8x + 4x(64-x^2)}{8(32-x^2)^{\frac{3}{2}}}\end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{4(16-x^2)}{2\sqrt{32-x^2}} = 0$$

$$\Rightarrow x = \pm 4$$

Now,

$$f''(4) = \frac{4 \times 4(64-16-8 \times 32+8 \times 16)}{8(32-16)^{\frac{3}{2}}} < 0$$

$\therefore x = 4$ is point of maxima

Maxima and Minima 18.3 Q1(ix)

Local Maximum value = $f(4)$

$$\begin{aligned}&= 4\sqrt{32-4^2} \\ &= 4\sqrt{32-16} \\ &= 4\sqrt{16} \\ &= 16\end{aligned}$$

Local minimum at $x = -4$;

Local Minimum value = $f(-4)$

$$\begin{aligned}&= -4\sqrt{32-(-4)^2} \\ &= -4\sqrt{32-16} \\ &= -4\sqrt{16} \\ &= -16\end{aligned}$$

Maxima and Minima 18.3 Q1(x)

$$f(x) = x + \frac{a^2}{x}$$

$$\therefore f'(x) = 1 - \frac{a^2}{x^2}$$

$$f''(x) = \frac{2a^2}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 1 - \frac{a^2}{x^2} = 0$$

$$\Rightarrow x^2 - a^2 = 0$$

$$\Rightarrow x = \pm a$$

Now,

$$f''(a) = \frac{2}{a} > 0 \text{ as } a > 0$$

$\therefore x = a$ is point of minima

$$f''(-a) = \frac{-2}{a} < 0 \text{ as } a > 0$$

$\therefore x = -a$ is point of maxima

Hence,

$$\text{local max value} = f(-a) = -2a$$

$$\text{local min value} = f(a) = 2a.$$

Maxima and Minima 18.3 Q1(xi)

$$f(x) = x\sqrt{2-x^2}$$

$$\begin{aligned}\therefore f'(x) &= \sqrt{2-x^2} - \frac{2x^2}{2\sqrt{2-x^2}} \\ &= \frac{2(2-x^2) - 2x^2}{2\sqrt{2-x^2}} \\ &= \frac{2-2x^2}{\sqrt{2-x^2}} \\ f''(x) &= \frac{\sqrt{2-x^2}(-4x) + \frac{(2-2x^2)2x}{\sqrt{2-x^2}}}{(\sqrt{2-x^2})^2} \\ &= \frac{-(2-x^2)4x + 4x - 4x^3}{(2-x^2)^{\frac{3}{2}}}\end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{2(1-x^2)}{\sqrt{2-x^2}} = 0$$

$$\Rightarrow x = \pm 1$$

Now,

$$f''(1) < 0$$

$\Rightarrow x = 1$ is point of local maxima

$$f''(-1) > 0$$

$\Rightarrow x = -1$ is point of local minima

Hence,

$$\text{local max value} = f(1) = 1$$

$$\text{local min value} = f(-1) = -1.$$

Maxima and Minima 18.3 Q1(xii)

$$f(x) = x + \sqrt{1-x}$$

$$\therefore f'(x) = 1 - \frac{1}{2\sqrt{1-x}} = \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}}$$

$$\therefore f'(x) = \frac{2\sqrt{1-x} \left(\frac{-1}{\sqrt{1-x}} \right) + \frac{(2\sqrt{1-x} - 1)}{\sqrt{1-x}}}{4(1-x)}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \sqrt{1-x} = \frac{1}{2}$$

$$\Rightarrow x = 1 - \frac{1}{4} = \frac{3}{4}$$

Now,

$$f''\left(\frac{3}{4}\right) < 0$$

$$\Rightarrow x = \frac{3}{4} \text{ is point of local maxima}$$

Hence,

$$\text{local max value} = f\left(\frac{3}{4}\right) = \frac{5}{4}.$$

Maxima and Minima 18.3 Q2(i)

$$f(x) = (x-1)(x-2)^2$$

$$\begin{aligned}\therefore f'(x) &= (x-2)^2 + 2(x-1)(x-2) \\ &= (x-2)(x-2+2x-2) \\ &= (x-2)(3x-4)\end{aligned}$$

$$f''(x) = (3x-4) + 3(x-2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow (x-2)(3x-4) = 0$$

$$\Rightarrow x = 2, \frac{4}{3}$$

Now,

$$f''(2) > 0$$

$$\therefore x = 2 \text{ is local minima}$$

$$f''\left(\frac{4}{3}\right) = -2 < 0$$

$$\therefore x = \frac{4}{3} \text{ is point of local maxima}$$

$$\therefore \text{local max value} = f\left(\frac{4}{3}\right) = \frac{4}{27}$$

$$\text{local min value} = f(2) = 0.$$

Maxima and Minima 18.3 Q2(ii)

$$f(x) = x\sqrt{1-x}$$

$$\begin{aligned}\therefore f'(x) &= \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1) \\ &= \frac{2(1-x) - x}{2\sqrt{1-x}} \\ &= \frac{2-3x}{2\sqrt{1-x}} \\ f''(x) &= \frac{2\sqrt{1-x}(-3) + \frac{(2-3x)}{\sqrt{1-x}}}{4(1-x)}\end{aligned}$$

For maximum and minimum,

$$f'(x) = 0$$

$$\Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0$$

$$\Rightarrow x = \frac{2}{3}$$

Now,

$$f''\left(\frac{2}{3}\right) < 0$$

$$\therefore x = \frac{2}{3} \text{ is point of maxima}$$

$$\therefore \text{local max value} = f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}$$

Maxima and Minima 18.3 Q2(iii)

$$f(x) = -(x-1)^3(x+1)^2$$

$$\begin{aligned} \therefore f'(x) &= -3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1) \\ &= -(x-1)^2(x+1)(3x+3+2x-2) \\ &= -(x-1)^2(x+1)(5x+1) \end{aligned}$$

$$\therefore f''(x) = -2(x-1)(x+1)(5x+1) - (x-1)^2(5x+1) - 5(x-1)^2(x+1)$$

For maximum and minimum value,

$$f'(x) = 0$$

$$\Rightarrow -(x-1)^2(x+1)(5x+1) = 0$$

$$\Rightarrow x = 1, -1, -\frac{1}{5}$$

Now,

$$f''(1) = 0$$

$\therefore x = 1$ is inflection point

$$f''(-1) = -4 \times -4 = 16 > 0$$

$\therefore x = -1$ is point of minima

$$f''\left(-\frac{1}{5}\right) = -5\left(\frac{36}{25}\right) \times \frac{4}{5} = \frac{-144}{25} < 0$$

$\therefore x = -\frac{1}{5}$ is point of maxima

Hence,

$$\text{local max value} = f\left(-\frac{1}{5}\right) = \frac{3456}{3125}$$

$$\text{local min value} = f(-1) = 0.$$

Maxima and Minima 18.3 Q3

We have,

$$y = a \log x + bx^2 + x$$

$$\therefore \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

$$\text{and } \frac{d^2y}{dx^2} = \frac{-a}{x^2} + 2b$$

For maximum and minimum value,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{a}{x} + 2bx + 1 = 0$$

Given that extreme value exist at $x = 1, 2$

$$\Rightarrow a + 2b = -1 \quad \text{--- (i)}$$

$$\frac{a}{2} + 4b = -1$$

$$\Rightarrow a + 8b = -2 \quad \text{--- (ii)}$$

Solving (i) and (ii), we get

$$a = \frac{-2}{3}, b = \frac{-1}{6}.$$

Maxima and Minima 18.3 Q4

The given function is $f(x) = \frac{\log x}{x}$.

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

$$\text{Now, } f'(x) = 0$$

$$\Rightarrow 1 - \log x = 0$$

$$\Rightarrow \log x = 1$$

$$\Rightarrow \log x = \log e$$

$$\Rightarrow x = e$$

$$\begin{aligned}\text{Now, } f''(x) &= \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4} \\ &= \frac{-x - 2x(1 - \log x)}{x^4} \\ &= \frac{-3 + 2\log x}{x^3}\end{aligned}$$

$$\text{Now, } f''(e) = \frac{-3 + 2\log e}{e^3} = \frac{-3 + 2}{e^3} = \frac{-1}{e^3} < 0$$

Therefore, by second derivative test, f is the maximum at $x = e$.

Maxima and Minima 18.3 Q5

$$f(x) = \frac{4}{x+2} + x$$

$$\therefore f'(x) = \frac{-4}{(x+2)^2} + 1$$

$$f''(x) = \frac{8}{(x+2)^3}$$

For maximum and minimum value,

$$f'(x) = 0$$

$$\Rightarrow \frac{-4}{(x+2)^2} + 1 = 0$$

$$\Rightarrow (x+2)^2 = 4$$

$$\Rightarrow x^2 + 4x = 0$$

$$\Rightarrow x(x+4) = 0$$

$$x = 0, -4$$

Now,

$$f''(0) = 1 > 0$$

$\therefore x = 0$ is point of minima

$$f''(-4) = -1 < 0$$

$\therefore x = -4$ is point of maxima

\therefore local max value = $f(-4) = -6$

local min value = $f(0) = 2$.

Maxima and Minima 18.3 Q6

We have,

$$y = \tan x - 2x$$

$$\therefore y' = \sec^2 x - 2$$

$$y'' = 2 \sec^2 x \tan x$$

For maximum and minimum value,

$$y' = 0$$

$$\Rightarrow \sec^2 x = 2$$

$$\Rightarrow \sec x = \pm\sqrt{2}$$

$$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\therefore y''\left(\frac{\pi}{4}\right) = 4 > 0$$

$\therefore x = \frac{\pi}{4}$ is point of minima

$$y''\left(\frac{3\pi}{4}\right) = -4 < 0$$

$\therefore x = \frac{3\pi}{4}$ is point of maxima

Hence,

$$\text{max value} = f\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$$

$$\text{min value} = f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}.$$

Maxima and Minima 18.3 Q7

Consider the function

$$f(x) = x^3 + ax^2 + bx + c$$

$$\text{Then } f'(x) = 3x^2 + 2ax + b$$

It is given that $f(x)$ is maximum at $x = -1$.

$$\therefore f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$$

$$\Rightarrow f'(-1) = 3 - 2a + b = 0 \dots (1)$$

It is given that $f(x)$ is minimum at $x = 3$.

$$\therefore f'(3) = 3(3)^2 + 2a(3) + b = 0$$

$$\Rightarrow f'(3) = 27 + 6a + b = 0 \dots (2)$$

Solving equations (1) and (2), we have,

$$a = -3 \text{ and } b = -9$$

Since $f'(x)$ is independent of constant c , it can be any real number.