RD Sharma Solutions

Class 12 Maths

Chapter 19

Ex 19.3

Indefinite Integrals Ex 19.3 Q1

Let $I = \int \left[(2x - 3)^5 + \sqrt{3x + 2} \right] dx$. Then,

$$I = \int (2x - 3)^5 dx + \int (3x + 2)^{\frac{1}{2}} dx$$

$$= \frac{(2x - 3)^6}{2 \times 6} + \frac{(3x + 2)^{\frac{3}{2}}}{3 \times \frac{3}{2}} + c$$

$$= \frac{(2x - 3)^6}{12} + \frac{2}{9}(3x + 2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{(2x - 3)^6}{12} + \frac{2}{9}(3x + 2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.3 Q2

Let $I = \int \left[\frac{1}{(7x - 5)^3} + \frac{1}{\sqrt{5x - 4}} \right] dx$. Then,

$$I = \int (7x - 5)^{-3} dx + \int (5x - 4)^{\frac{-1}{2}} dx$$

$$= \frac{(7x - 5)^{-2}}{7 \times (-2)} + \frac{(5x - 4)^{\frac{1}{2}}}{5 \times \frac{1}{2}} + c$$

$$= -\frac{(7x - 5)^{6-2}}{14} + \frac{2}{5}\sqrt{(5x - 4)} + c$$

$$\therefore \quad I = \frac{-1}{14}(7x - 5)^{-2} + \frac{2}{5} \times \sqrt{5x - 4} + c.$$

Indefinite Integrals Ex 19.3 Q3

Let $I = \int \dfrac{1}{2-3x} + \dfrac{1}{\sqrt{3x-2}}dx$. Then,

$$I = \int \dfrac{1}{2-3x}dx + \int \dfrac{1}{\sqrt{3x-2}}dx$$

$$= \dfrac{\log|2-3x|}{-3} + \dfrac{2}{3}(3x-2)^{\frac{1}{2}}c$$

$$= \dfrac{-1}{3} \times \log|2x-3| + \dfrac{2}{3} \times \sqrt{3x-2} + c$$

Indefinite Integrals Ex 19.3 Q4

Let $I = \int \dfrac{x+3}{(x+1)^4}dx$. Then,

$$I = \int \dfrac{x+1+2}{(x+1)^4}dx$$

$$= \int \dfrac{x+1}{(x+1)^4} \times dx + 2\int \dfrac{1}{(x+1)^4} \times dx$$

$$= \int \dfrac{1}{(x+1)^3} \times dx + 2\int \dfrac{1}{(x+1)^4} \times dx$$

$$= \int (x+1)^{-3} \times dx + 2\int (x-1)^{-4}dx$$

$$= \dfrac{(x+1)^{-2}}{-2} + 2\dfrac{(x+1)^{-3}}{-3} + c$$

$$= -\dfrac{1}{2} \times \dfrac{1}{(x+1)^2} - \dfrac{2}{3} \times \dfrac{1}{(x+1)^3} + c$$

$$\therefore \quad I = \dfrac{-1}{2(x+1)^2} - \dfrac{2}{3(x+1)^3} + c$$

Indefinite Integrals Ex 19.3 Q5

Let $I = \int \dfrac{1}{\sqrt{x+1}+\sqrt{x}}dx$. Then,

$$I = \int \dfrac{1}{\sqrt{x+1}+\sqrt{x}} \times \dfrac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}} \times dx$$

$$= \int \dfrac{\sqrt{x+1}-\sqrt{x}}{\left(\sqrt{x+1}\right)^2 - \left(\sqrt{x}\right)^2} \times dx$$

$$= \int \dfrac{\sqrt{x+1}-\sqrt{x}}{x+1-x} \times dx$$

$$= \int \left(\sqrt{x+1}-\sqrt{x}\right) \times dx$$

$$= \int (x+1)^{\frac{1}{2}}dx - \int x^{\frac{1}{2}}dx$$

$$= \dfrac{2}{3}(x+1)^{\frac{3}{2}} - \dfrac{2}{3}x^{\frac{3}{2}} + c$$

$$\therefore \quad I = \dfrac{2}{3}(x+1)^{\frac{3}{2}} - \dfrac{2}{3}x^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q6

Let $I = \int \dfrac{1}{\sqrt{2x+3}+\sqrt{2x-3}}\,dx$. Then,

$$I = \int \dfrac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \dfrac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}} \times dx$$

$$= \int \dfrac{\sqrt{2x+3}-\sqrt{2x-3}}{\left(\sqrt{2x+3}\right)^2 - \left(\sqrt{2x-3}\right)^2} \times dx$$

$$= \int \dfrac{\sqrt{2x+3}-\sqrt{2x-3}}{2x+3-2x+3} \times dx$$

$$= \dfrac{1}{6}\int (2x+3)^{\frac{1}{2}}dx - \dfrac{1}{6}\int (2x-3)^{\frac{1}{2}}dx$$

$$= \dfrac{1}{6} \times \dfrac{(2x+3)^{\frac{3}{2}}}{\dfrac{3}{2}\times 2} - \dfrac{1}{6}\dfrac{(2x-3)^{\frac{3}{2}}}{\dfrac{3}{2}\times 2} + c$$

$$= \dfrac{1}{18} \times (2x+3)^{\frac{3}{2}} - \dfrac{1}{18}(2x-3)^{\frac{3}{2}} + c$$

$\therefore \qquad I = \dfrac{1}{18}(2x+3)^{\frac{3}{2}} - \dfrac{1}{18}(2x-3)^{\frac{3}{2}} + c.$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \dfrac{2x}{(2x+1)^2}\,dx$. Then,

$$I = \int \dfrac{2x+1-1}{(2x+1)^2} \times dx$$

$$= \int \dfrac{2x+1}{(2x+1)^2} \times dx - \int \dfrac{1}{(2x+1)^2} \times dx$$

$$= \int \dfrac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx$$

$$= \dfrac{1}{2}\log|2x+1| - \dfrac{(2x+1)^{-1}}{-1\times 2} + c$$

$$= \dfrac{1}{2}\log|2x+1| + \dfrac{1}{2} \times \dfrac{1}{2x+1} + c$$

$\therefore \qquad I = \dfrac{1}{2}\log|2x+1| + \dfrac{1}{2(2x+1)} + c.$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \dfrac{1}{\sqrt{x+a} + \sqrt{x+b}}\, dx$. Then,

$$I = \int \dfrac{1}{\sqrt{x+a} + \sqrt{x+b}} \times \dfrac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}} \times dx$$

$$= \int \dfrac{\sqrt{x+a} - \sqrt{x+b}}{x+a-x-b} \times dx$$

$$= \int \dfrac{\sqrt{x+a} - \sqrt{x+b}}{a-b} \times dx$$

$$= \dfrac{1}{a-b}\left[\dfrac{2}{3}(x+a)^{\frac{3}{2}} - \dfrac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$$

$$= \dfrac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \dfrac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c.$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin\sqrt{1 + \cos 2x}\, dx$

$$I = \int \sin x \times \sqrt{2\cos^2 x} \times dx$$

$$= \int \sin x \times \sqrt{2} \times \cos x \times dx$$

$$= \sqrt{2}\int \sin x \times \cos x \times dx$$

$$= \dfrac{\sqrt{2}}{2}\int 2\sin x \times \cos x \times dx$$

$$= \dfrac{\sqrt{2}}{2}\int \sin 2x\, dx$$

$$= \dfrac{\sqrt{2}}{2} \times \dfrac{-\cos 2x}{2} + c$$

$$= \dfrac{-1}{2\sqrt{2}} \times \cos 2x + c$$

$$\therefore \quad I = \dfrac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \dfrac{1 + \cos x}{1 - \cos x} dx$. Then,

$$I = \int \dfrac{2\cos^2 \dfrac{x}{2}}{2\sin^2 \dfrac{x}{2}} \times dx$$

$$= \int \dfrac{\cos^2 \dfrac{x}{2}}{\sin^2 \dfrac{x}{2}} \times dx$$

$$= \int \cot^2 \dfrac{x}{2} \times dx$$

$$= \int \left(\cosec^2 \dfrac{x}{2} - 1 \right) dx$$

$$= \dfrac{-\cot \dfrac{x}{2}}{\dfrac{1}{2}} - x + c$$

$$= -2\cot \dfrac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q11

Let $I = \int \dfrac{1 - \cos x}{1 + \cos x} \times dx$. Then,

$$I = \int \dfrac{2\sin^2 \dfrac{x}{2}}{2\cos^2 \dfrac{x}{2}} \times dx$$

$$= \int \dfrac{\sin^2 \dfrac{x}{2}}{\cos^2 \dfrac{x}{2}} \times dx$$

$$= \int \tan^2 \dfrac{x}{2} dx$$

$$= \int \left(\sec^2 \dfrac{x}{2} - 1 \right) dx$$

$$= \dfrac{\tan \dfrac{x}{2}}{\dfrac{1}{2}} - x + c$$

$$= 2\tan \dfrac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \dfrac{1}{1 - \sin\frac{x}{2}}$. Then,

$$I = \int \dfrac{1}{1 - \sin\frac{x}{2}} \times \dfrac{1 + \sin\frac{x}{2}}{1 + \sin\frac{x}{2}}\,dx$$

$$= \int \dfrac{1 + \sin\frac{x}{2}}{1 - \sin^2\frac{x}{2}} \times dx$$

$$= \int \dfrac{1 + \sin\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \dfrac{1}{\cos^2\frac{x}{2}}\,dx + \int \dfrac{\sin\frac{x}{2}}{\cos^2\frac{x}{2}}\,dx$$

$$= \int \sec^2\frac{x}{2}\,dx + \int \sec\frac{x}{2}\,\tan\frac{x}{2}\,dx$$

$$= \dfrac{\tan\frac{x}{2}}{\frac{1}{2}}\,2 + + \dfrac{\sec\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan\frac{x}{2} + 2\sec\frac{x}{2} + c$$

$$\therefore \quad I = 2\left(\tan\frac{x}{2} + \sec\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \dfrac{1}{1 + \cos 3x} \times dx$. Then,

$$I = \int \dfrac{1}{1 + \cos 3x} \times \dfrac{1 - \cos 3x}{1 - \cos 3x} \times dx$$

$$= \int \dfrac{1 - \cos 3x}{1 - \cos^2 3x} \times dx$$

$$= \int \dfrac{1 - \cos 3x}{\sin^2 3x} \times dx$$

$$= \int \left(\dfrac{1}{\sin^2 3x} - \dfrac{\cos 3x}{\sin^2 3x}\right)dx$$

$$= \int \left(\cosec^2 3x - \cosec 3x\,\cot 3x\right)dx$$

$$= \dfrac{-\cot 3x}{3} + \dfrac{\cosec 3x}{3} + c$$

$$= \dfrac{-1}{3} \times \dfrac{\cos 3x}{\sin 3x} + \dfrac{1}{3} \times \dfrac{1}{\sin 3x} + c$$

$$= \dfrac{1 - \cos 3x}{3\sin 3x} + c$$

$$\therefore \quad I = \dfrac{1 - \cos 3x}{3\sin 3x} + c.$$

Indefinite Integrals Ex 19.3 Q14

Consider $I = \int \left(e^x + 1\right)^2 e^x \, dx$

let $\left(e^x + 1\right) = t \rightarrow e^x dx = dt$

$$I = \int \left(e^x + 1\right)^2 e^x \, dx$$

$$= \int (t)^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{\left(e^x + 1\right)^3}{3} + C$$

Indefinite Integrals Ex 19.3 Q15

Let $I = \int \left(e^x + \dfrac{1}{e^x}\right)^2 dx$. Then,

$$I = \int \left(e^x + \frac{1}{e^x}\right)^2 xdx$$

$$= \int \left(e^{2x} + \frac{1}{e^{2x}} + 2\right) dx$$

$$= \frac{e^{2x}}{2} - \frac{1}{2} e^{-2x} + 2x + c$$

$\therefore \qquad I = \dfrac{1}{2} \times e^{2x} + 2x - \dfrac{1}{2} \times e^{-2x} + c$

Indefinite Integrals Ex 19.3 Q16

Let $I = \int \dfrac{1 + \cos 4x}{\cot x - \tan x} \, x \, dx$. Then,

$$I = \int \dfrac{2\cos^2 2x}{\dfrac{\cos x}{\sin x} - \dfrac{\sin x}{\cos x}} \, dx$$

$$= \int \dfrac{2\cos^2 2x}{\dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x}} \, dx$$

$$= \int \dfrac{2\cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} \, dx$$

$$= \int \dfrac{\cos^2 2x \times \sin 2x}{\cos^2 2x} \, dx$$

$$= \int \cos 2x \times \sin 2x \times dx$$

$$= \dfrac{1}{2} \int 2 \sin 2x \cos 2x \, dx$$

$$= \dfrac{1}{2} \int \left[\sin(2x + 2x) + \sin(2x - 2x) \right] dx$$

$$= \dfrac{1}{2} \int \left(\sin 4x + \sin 0 \right) dx$$

$$= \dfrac{1}{2} \int \left(\sin 4x + 0 \right) dx$$

$$= \dfrac{1}{2} \int \sin 4x$$

$$= -\dfrac{1}{2} \times \dfrac{\cos 4x}{4} + c$$

$$= -\dfrac{1}{8} \times \cos 4x + c$$

Indefinite Integrals Ex 19.3 Q17

Let $I = \int \dfrac{1}{\sqrt{x+3} - \sqrt{x+2}} \, dx$. Then,

$$I = \int \dfrac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \dfrac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} \, dx$$

$$= \int \dfrac{\sqrt{x+3} + \sqrt{x+2}}{x+3 - x - 2} \, dx$$

$$= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}} \right] dx$$

$$= \dfrac{(x+3)^{\frac{3}{2}}}{\dfrac{3}{2}} + \dfrac{(x+2)^{\frac{3}{2}}}{\dfrac{3}{2}} + c$$

$$= \dfrac{2}{3} \times (x+3)^{\frac{3}{2}} + \dfrac{2}{3}(x+2)^{\frac{3}{2}} + c$$

$$= \dfrac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + c$$

$$\therefore \quad I = \dfrac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + c$$

Indefinite Integrals Ex 19.3 Q18

$$\tan^2(2x-3) = \sec^2(2x-3)-1$$

Let $2x - 3 = t$

$$\Rightarrow 2dx = dt$$

$$\Rightarrow \int \tan^2(2x-3)\,dx = \int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$

$$= \frac{1}{2}\int\left(\sec^2 t\right)dt - \int 1\,dx$$

$$= \frac{1}{2}\int \sec^2 t\,dt - \int 1\,dx$$

$$= \frac{1}{2}\tan t - x + C$$

$$= \frac{1}{2}\tan(2x-3) - x + C$$

Indefinite Integrals Ex 19.3 Q19

Consider $I = \displaystyle\int \frac{1}{\cos^2 x\,(1-\tan x)^2}\,dx$

$$= \int \frac{1}{\cos^2 x\left(1-\dfrac{\sin x}{\cos x}\right)^2}\,dx$$

$$= \int \frac{1}{(\cos x - \sin x)^2}\,dx$$

$$= \int \frac{1}{1-\sin 2x}\,dx$$

$$= \int \frac{1}{1+\cos\left(\dfrac{\pi}{2}+2x\right)}\,dx$$

$$= \int \frac{1}{2\cos^2\left(\dfrac{\pi}{4}+x\right)}\,dx$$

$$= \frac{1}{2}\int \sec^2\left(\frac{\pi}{4}+x\right)dx$$