RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.4

Indefinite Integrals Ex 19.4 Q1

$$\text{Let } I = \int \frac{x^2 + 5x + 2}{x + 2} dx$$

Using long division method, we have

$$\frac{x^2 + 5x + 2}{x + 2} = x + 3 - \frac{4}{x + 2}$$

$$\therefore \quad I = \int \frac{x^2 + 5x + 2}{x + 2} = \int \left(x + 3 - \frac{4}{x + 2} \right) dx$$

$$\Rightarrow \quad I = \int x dx + 3\int dx - 4\int \frac{1}{x + 2} dx$$

$$= \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{2} + 3x - 4\log|x + 2| + c$$

Indefinite Integrals Ex 19.4 Q2

$$\text{Let } I = \int \frac{x^3}{x - 2} dx$$

Using long division method, we have

$$\frac{x^3}{x - 2} = x^2 + 2x + 4 + \frac{8}{x - 2}$$

$$\therefore \quad I = \int \left(x^2 + 2x + 4 + \frac{8}{x - 2} \right) dx$$

$$= \int x^2 dx + 2\int x dx + 4\int dx + 8\int \frac{1}{x - 2} dx$$

$$= \frac{x^3}{3} + \frac{2x^2}{2} + 4x + 8\log|x - 2| + c$$

$$= \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

$$\therefore \quad I = \frac{x^3}{3} + x^2 + 4x + 8\log|x - 2| + c$$

Indefinite Integrals Ex 19.4 Q3

Let $I = \int \frac{x^2 + x + 5}{3x + 2} dx$

Using long division method, we have

$$\frac{x^2 + x + 5}{3x + 2} = \frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2}$$

$$\therefore \quad I = \int \left(\frac{x}{3} + \frac{1}{9} + \frac{43}{9} \times \frac{1}{3x + 2} \right) dx$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{9 \times 3} |3x + 2| + c$$

$$= \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log |3x + 2| + c$$

$$\therefore \quad I = \frac{x^2}{6} + \frac{1}{9} \times x + \frac{43}{27} \times \log |3x + 2| + c$$

Indefinite Integrals Ex 19.4 Q4

Let $I = \int \frac{2x + 3}{(x - 1)^2} dx$. Then,

$$I = \int \frac{2x + 2 - 2 + 3}{(x - 1)^2} \times dx$$

$$= \int \frac{2x - 2 + 5}{(x - 1)^2} \times dx$$

$$= 2 \int \frac{(x - 1)}{(x - 1)^2} \times dx + 5 \int \frac{1}{(x - 1)^2} dx$$

$$= 2 \int \frac{1}{x - 1} \times dx + 5 \int (x - 1)^{-2} \times dx$$

$$= 2 \log |x - 1| + 5 \times \frac{(x - 1)^{-1}}{-1} + c$$

$$= 2 \log |x - 1| - \frac{5}{x - 1} + c$$

$$\therefore \quad I = 2 \log |x - 1| - \frac{5}{x - 1} + c.$$

Indefinite Integrals Ex 19.4 Q5

Let $I = \int \frac{x^2+3x-1}{(x+1)^2}dx$. Then,

$$I = \int \frac{x^2+x+2x-1}{(x+1)^2}dx$$

$$= \int \frac{x(x+1)+2x-1}{(x+1)^2}dx$$

$$= \int \frac{x(x+1)}{(x+1)^2}dx + \int \frac{2x-1}{(x+1)^2}dx$$

$$= \int \frac{x}{x+1}dx + \int \frac{\sqrt{2x+2-2-1}}{(x+1)^2}dx$$

$$= \int \frac{x+1-1}{x+1}dx + \int \frac{2(x+1)-3}{(x+1)^2}dx$$

$$= \int \frac{x+1}{x+1}dx - \int \frac{1}{x+1}dx + \int \frac{2(x+1)}{(x+1)^2}dx - 3\int \frac{1}{(x+1)^2}dx$$

$$= \int dx - \int \frac{1}{x+1}dx + 2\int \frac{1}{x+1}dx - 3\int (x+1)^{-2}dx$$

$$= x - \log|x+1| + 2\log|x+1| + \frac{3}{x+1} + c$$

$$= x + \log|x+1| + \frac{3}{x+1} + c$$

$\therefore \quad I = x + \log|x+1| + \frac{3}{x+1} + c$

Indefinite Integrals Ex 19.4 Q6

Let $I = \int \frac{2x-1}{(x-1)^2}dx$. Then,

$$I = \int \frac{2x-1+2-2}{(x-1)^2}dx$$

$$= \int \frac{2x-2+1}{(x-1)^2}dx$$

$$= \int \frac{2(x-1)}{(x-1)^2}dx + 1\int \frac{1}{(x-1)^2}dx$$

$$= 2\int \frac{1}{x-1}dx + \int (x-1)^2 dx$$

$$= 2\log|x-1| - (x-1)^{-1} + c$$

$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$\therefore \quad I = \frac{-1}{x-1} + 2\log|x-1| + c.$