RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.5

Indefinite Integrals Ex 19.5 Q1

Let $I = \int \frac{x+1}{\sqrt{2x+3}} dx$

Let $x + 1 = \lambda(2x+3) + \mu$

On equating the coefficients of like powers of x on bothy sides, we get

$$I = 2\lambda,\ 3\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } \mu = \frac{-1}{2}$$

Replacing $x + 1$ by $\lambda(2x+3) + \mu$ in the given integral, we get

$$I = \int \frac{\lambda(2x+3) + \mu}{\sqrt{2x+3}} dx$$

$$= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}} dx + \mu \int \frac{1}{\sqrt{2x+3}} dx$$

$$= \lambda \int (2x+3)^{\frac{1}{2}} dx + \mu \int (2x+3)^{\frac{-1}{2}} dx$$

$$= \lambda \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + \mu \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c$$

$$= \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right) \times (2x+3)^{\frac{1}{2}} + c \qquad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2}\right]$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

$$\therefore \quad I = \frac{1}{6} \times (2x+3)^{\frac{3}{2}} - \frac{1}{2}(2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2}dx$. Then,

$$I = \int \{(x+2) - 2\}x + 2dx \qquad [\because x = (x+2) - 2]$$

$$\Rightarrow \quad I = f\left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\}dx$$

$$\Rightarrow \quad I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}} dx$. Then,

$$I = \int \frac{x+4-4-1}{\sqrt{x+4}}$$

$$= \int \frac{x+4-5}{\sqrt{x+4}} dx$$

$$= \int \frac{x+4}{\sqrt{x+4}} dx - 5\int \frac{1}{\sqrt{x+4}} dx$$

$$= \int (x+4)^{\frac{1}{2}} dx - 5\int (x+4)^{\frac{-1}{2}} dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{3} \times (x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.5 Q4

Let $I = \int (x+2)\sqrt{3x+5}dx$

Let $x + 2 = \lambda(3x+5) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \quad \text{and} \quad 5\lambda + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad 5 \times \frac{1}{3} + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3}$$

$$\therefore \quad I = \int \{\lambda(3x+5) + \mu\}\sqrt{3x+5}dx$$

$$= \lambda\int (3x+5)\sqrt{3x+5}dx + \mu\int \sqrt{3x+5}$$

$$= \lambda\int (3x+5)^{\frac{3}{2}} dx + \mu\int (3x+5)^{\frac{1}{2}}dx$$

$$= \lambda \times \frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$

$$= \frac{1}{3}\times\frac{2}{15}\times(3x+5)^{\frac{5}{2}} + \frac{1}{3}\times\frac{2}{9}(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{45}\times(3x+5)^{\frac{5}{2}} + \frac{2}{27}\times(3x+5)^{\frac{3}{2}} + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{1}{5}\times(3x+5)^{1} + \frac{1}{3}\right] + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{3(3x+5)+5}{15}\right] + c$$

$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\frac{(9x+15+5)}{15} + c$$

$$= \frac{2}{135}\times(3x+5)^{\frac{3}{2}}(9x+20) + c$$

$$\therefore \quad I = \frac{2}{135}\times(9x+20)(3x+5)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q5

Let $I = \int \frac{2x+1}{\sqrt{3x+2}}dx$

Let $2x+1 = \lambda(3x+2)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \quad \text{and} \quad 2\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad 2 \times \frac{2}{3} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad \mu = \frac{-1}{3}$$

$$\therefore \quad I = \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$$

$$= \lambda\int \frac{3x+2}{\sqrt{3x+2}}dx + \mu\int \frac{1}{\sqrt{3x+2}}dx$$

$$= \lambda\int (3x+2)^{\frac{1}{2}}dx + \mu\int (3x+2)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times 3} + \mu\frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times 3} + c$$

$$= \frac{2}{3}\times\frac{2}{9}\times(3x+2)^{\frac{3}{2}} - \frac{1}{3}\times\frac{2}{3}(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{4}{27}\times(3x+2)^{\frac{3}{2}} - \frac{2}{9}\times(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{2}{3}\times(3x+2)-1\right]+c$$

$$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{6x+4-3}{3}\right]+c$$

$$= \frac{2}{27}\times\sqrt{3x+2}\,(6x+1)+c$$

$$\therefore \quad I = \frac{2}{27}\times(6x+1)\sqrt{3x+2}+c.$$

Indefinite Integrals Ex 19.5 Q6

Let $I = \int \frac{3x+5}{\sqrt{7x+9}}dx$

Let $3x+5 = \lambda(7x+9)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$7\lambda = 3 \quad \text{and} \quad 9\lambda + \mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad 9\times\frac{3}{7}+\mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad \mu = \frac{8}{7}$$

$$\therefore \quad I = \int \frac{\lambda(7x+9)+\mu}{\sqrt{7x+9}}dx$$

$$= \lambda\int \frac{7x+9}{\sqrt{7x+9}}dx + \mu\int \frac{1}{\sqrt{7x+9}}dx$$

$$= \lambda\int (7x+9)^{\frac{1}{2}}dx + \mu\int (7x+9)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2}\times 7} + \mu\frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2}\times 7} + c$$

$$= \frac{3}{7}\times\frac{2}{21}\times(7x+9)^{\frac{3}{2}} + \frac{8}{7}\times\frac{2}{7}(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{3}{2}} + \frac{16}{49}\times(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+9+8]+c$$

$$= \frac{2}{49}\times(7x+9)^{\frac{1}{2}}[7x+17]+c$$

$$= \frac{2}{49}\times(7x+17)\sqrt{7x+9}+c$$

Indefinite Integrals Ex 19.5 Q7

Let $I = \int \frac{x}{\sqrt{x+4}}\,dx$. Then,

$$I = \int \frac{x+4-4}{\sqrt{x+4}}dx$$

$$= \int \frac{x+4}{\sqrt{x+4}}dx - 4\int \frac{1}{\sqrt{x+4}}dx$$

$$= \int (x+4)^{\frac{1}{2}}dx - 4\int (x+4)^{\frac{-1}{2}}dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + c$$

$$= 2(x+4)^{\frac{1}{2}}\left[\frac{1}{3}(x+4) - 4\right] + c$$

$$= 2(x+4)^{\frac{1}{2}}\left[\frac{(x+4)-12}{3}\right] + c$$

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}\left[x-8\right] + c$$

$$\therefore \quad I = \frac{2}{3} \times (x-8)\sqrt{x+4} + c.$$

Indefinite Integrals Ex 19.5 Q8

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$I = \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx$$

$$= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx$$

$$= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$

$$= -1\int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$

$$= -1\int (1+3x)^{\frac{1}{2}} dx + 3\int (1+3x)^{\frac{-1}{2}} dx$$

$$= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c$$

$$= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[-\frac{1}{9}(1+3x)^{1} + 1\right] + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[\frac{-1-3x+9}{9}\right] + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[\frac{8-3x}{9}\right] + c$$

$$= \frac{2}{9}\sqrt{1+3x}\,(8-3x) + c$$

$$\therefore \quad I = \frac{2}{9}(8-9x)\sqrt{1+3x} + c$$

Indefinite Integrals Ex 19.5 Q9

Let $I = \int 5x + 3\sqrt{2x-1}\,dx$

Let $5x+3 = \lambda(2x-1)+\mu$ comparing both sides, we get

$$2\lambda = 5 \quad \text{and} \quad -\lambda+\mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \frac{-5}{2}+\mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}$$

$$\therefore \quad I = \int \{\lambda(2x-1)+\mu\}\sqrt{2x-1}\,dx$$

$$= \lambda\int (2x-1)\sqrt{2x-1}\,dx + \mu\int \sqrt{2x-1}\,dx$$

$$= \lambda\int (2x-1)^{\frac{3}{2}}dx + \mu\int (2x-1)^{\frac{1}{2}}dx$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{\frac{5}{2}\times 2} + \mu\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}\times 2}$$

$$= \lambda\frac{(2x-1)^{\frac{5}{2}}}{5} + \mu\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{5}{2}\times\frac{(2x-1)^{\frac{5}{2}}}{5} + \frac{11}{2}\times\frac{(2x-1)^{\frac{3}{2}}}{3} + c$$

$$= \frac{(2x-1)^{\frac{5}{2}}}{2} + \frac{11}{6}\times(2x-1)^{\frac{3}{2}} + c$$

$$= \frac{1}{2}(2x-1)^{\frac{3}{2}}\left[(2x-1)+\frac{11}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\left[\frac{6x+8}{3}\right] + c$$

$$= \frac{1}{2}\times(2x-1)^{\frac{3}{2}}\times 2\frac{(3x+4)}{3} + c$$

$$= (2x-1)^{\frac{3}{2}}\times\frac{(3x+4)}{3} + c$$

$$= \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{3}\times(3x+4)(2x-1)^{\frac{3}{2}} + c.$$