# RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.8

## Indefinite Integrals Ex 19.8 Q1

We have,

$$\int \frac{1}{\sqrt{1-\cos 2x}}\,dx = \int \frac{1}{\sqrt{2\sin^2 x}}\,dx$$

$$= \int \frac{1}{\sqrt{2}\sin x}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec} x\,dx$$

$$= \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1-\cos 2x}}\,dx = \frac{1}{\sqrt{2}}\log\left|\tan\frac{x}{2}\right| + c$$

## Indefinite Integrals Ex 19.8 Q2

We have,

$$\int \frac{1}{\sqrt{1+\cos x}}\,dx = \int \frac{1}{\sqrt{2\cos^2\frac{x}{2}}}\,dx$$

$$= \int \frac{1}{\sqrt{2}\cos\frac{x}{2}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \sec\frac{x}{2}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(\frac{\pi}{2}+\frac{x}{2}\right)dx$$

$$= \frac{2}{\sqrt{2}}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

$$\because \quad \int \frac{1}{\sqrt{1+\cos x}}\,dx = \sqrt{2}\log\left|\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right| + c$$

## Indefinite Integrals Ex 19.8 Q3

Let $I = \int \sqrt{\frac{1+\cos 2x}{1-\cos 2x}}dx$ then,

$$I = \int \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}dx$$

$$= \int \sqrt{\cot^2 x}\,dx$$

$$= \int \cot x\,dx$$

$$= \log\left|\sin x\right| + c \qquad \left[\because \quad \int \cot x = \log\left|\sin x\right| + c\right]$$

$$I = \log\left|\sin x\right| + c$$

## Indefinite Integrals Ex 19.8 Q4

Let $I = \int \sqrt{\frac{1-\cos x}{1+\cos x}}dx$ then,

$$I = \int \sqrt{\frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}}dx$$

$$= \int \sqrt{\tan^2 \frac{x}{2}}dx$$

$$= \int \tan\frac{x}{2}dx$$

$$= -2\log\left|\cos\frac{x}{2}\right| + c \qquad \left[\because \quad \int \tan x\,dx = \log\left|\cos x\right| + c\right]$$

$$\because \qquad I = -2\log\left|\cos\frac{x}{2}\right| + c$$

## Indefinite Integrals Ex 19.8 Q5

Let $I = \int \frac{\sec x}{\sec 2x}dx$, then,

$$I = \int \frac{\frac{1}{\cos x}}{\frac{1}{\cos 2x}}dx$$

$$= \int \frac{\cos 2x}{\cos x}dx$$

$$= \int \frac{2\cos^2 x - 1}{\cos x}dx$$

$$= \int 2\cos x\,dx - \int \frac{1}{\cos x}dx$$

$$= 2\int \cos dx - \int \sec x\,dx$$

$$= 2\sin x - \log\left|\sec x + \tan x\right| + c$$

$$\therefore \qquad I = 2\sin x - \log\left|\sec x + \tan x\right| + c$$

## Indefinite Integrals Ex 19.8 Q6

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\int\frac{\cos 2x}{(1+\sin 2x)}\,dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\ dx=dt$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log|\sin x+\cos x|+C$$

## Indefinite Integrals Ex 19.8 Q7

Let $I=\int\frac{\sin(x-a)}{\sin(x-b)}dx$ then

$$I=\int\frac{\sin(x-a+b-b)}{\sin(x-b)}dx$$

$$=\int\frac{\sin(x-b+b-a)}{\sin(x-b)}dx$$

$$=\int\frac{\sin(x-b)\cos(b-a)+\cos(x-b)\sin(b-a)}{\sin(x-b)}dx$$

$$=\int(\cos(b-a)+\cot(x-b)\sin(b-a))\,dx$$

$$=\cos(b-a)\int dx+\sin(b-a)\int\cot(x-b)\,dx$$

$$=x\cos(b-a)+\sin(b-a)\log|\sin(x-b)|+c$$

$$\therefore \quad I=x\cos(b-a)+\sin(b-a)\log|\sin(x-b)|+c$$

## Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$ then,

$$I = \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \frac{\sin(x+\alpha)\cos 2\alpha - \cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)} dx$$

$$= \int \left[\frac{\sin(x+\alpha)\cos 2\alpha}{\sin(x+\alpha)} - \frac{\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}\right] dx$$

$$= \int \left(\cos 2\alpha - \cot(x+\alpha)\sin 2\alpha\right) dx$$

$$= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha) dx$$

$$= x\cos 2\alpha - \sin 2\alpha \log\left|\sin(x+\alpha)\right| + c$$

$\therefore$ $I = x\cos 2\alpha - \sin 2\alpha \log\left|\sin(x+\alpha)\right| + c$

## Indefinite Integrals Ex 19.8 Q9

Let $\quad I = \int \frac{1+\tan x}{1-\tan x}\,dx$

$$I = \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}\,dx$$

$$= \int \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}\,dx$$

$\Rightarrow \quad I = \int \frac{\cos x + \sin x}{\cos x - \sin x}\,dx \text{ - - - - - (i)}$

Let $\cos x - \sin x = t$ then,

$d(\cos x - \sin x) = dt$

$\Rightarrow \quad (-\sin x - \cos x)\,dx = dt$

$\Rightarrow \quad -(\sin x + \cos x)\,dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x + \cos x}$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x + \cos x}$ in equation (i), we get

$$I = \int \frac{\cos x + \sin x}{t} \times \frac{-dt}{\sin x + \cos x}$$

$$= -\int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log|\cos x - \sin x| + c$$

$\therefore \quad I = -\log|\cos x - \sin x| + c$

## Indefinite Integrals Ex 19.8 Q10

$$\text{Let } I = \int \frac{\cos x}{\cos(x-a)}dx \quad \text{then,}$$

$$I = \int \frac{\cos(x+a-a)}{\cos(x-a)}dx$$

$$= \int \frac{\cos(x-a+a)}{\cos(x-a)}dx$$

$$= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)}dx$$

$$= \int \frac{\cos(x-a)\cos a}{\cos(x-a)}dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)}dx$$

$$= \cos a \int dx - \sin a \int \tan(x-a)dx$$

$$= x\cos a - \sin a \log\left|\sec(x-a)\right| + c$$

## Indefinite Integrals Ex 19.8 Q11

$$\text{Let } I = \int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}dx \quad \text{then,}$$

$$I = \int \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{1+\cos\left(\frac{\pi}{2}-2x\right)}}dx$$

$$= \int \sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\cos^2\left(\frac{\pi}{4}-x\right)}}dx$$

$$= \int \sqrt{\tan^2\left(\frac{\pi}{4}-x\right)}dx$$

$$= \int \tan\left(\frac{\pi}{4}-x\right)dx$$

$$= \log\left|\cos\left(\frac{\pi}{4}-x\right)\right| + c$$

## Indefinite Integrals Ex 19.8 Q12

Let $I = \int \frac{e^{3x}}{e^{3x}+1}\,dx$ ------(i)

Let $e^{3x}+1 = t$, then,

$d\left(e^{3x}+1\right) = dt$

$\Rightarrow \quad 3e^{3x}dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3e^{3x}}$

Putting $e^{3x}+1 = t$ and $dx = \frac{dt}{3e^{3x}}$ in equation (i), we get

$$I = \int \frac{e^{3x}}{t} \times \frac{dt}{3e^{3x}}$$

$$= \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|3e^{3x}+1\right| + c$$

$$\therefore \quad = \frac{1}{3}\log\left|3e^{3x}+1\right| + c$$

## Indefinite Integrals Ex 19.8 Q13

Let $I = \int \frac{\sec x \tan x}{3\sec x + 5} dx$ ----- $(i)$

Let $3\sec x + 5 = t$, then,

$\Rightarrow \quad d(3\sec x + 5) = dt$

$\Rightarrow \quad 3\sec x \tan x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\sec x \tan x}$

Putting $3\sec x \tan x dx = t$ and $dx = \frac{dt}{3\sec x \tan x}$ in equation (i), we get

$$I = \int \frac{\sec x \tan x}{t} \times \frac{dt}{3\sec x \tan x}$$

$$= \frac{1}{3}\int \frac{1}{t} dt$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|3\sec x + 5| + c$$

## Indefinite Integrals Ex 19.8 Q14

Let $I = \int \frac{1-\cot x}{1+\cot x}dx$ then,

$$I = \int \frac{1-\frac{\cos x}{\sin x}}{1+\frac{\cos x}{\sin x}}dx$$

$$= \int \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x + \cos x}{\sin x}}dx$$

$\Rightarrow$ $I = \int \frac{\sin x - \cos x}{\sin x + \cos x}dx$ ----(i)

Let $\sin x + \cos x = t$. then,

$d(\sin x + \cos x) = dt$

$\Rightarrow$ $(\cos x - \sin x)dx = dt$

$\Rightarrow$ $-(\sin x - \cos x)dx = dt$

$\Rightarrow$ $dx = -\frac{dt}{\sin x - \cos x}$

Putting $\sin x + \cos x = t$ and $dx = -\frac{dt}{\sin x - \cos x}$ in equation (i), we get,

$$I = \int \frac{\sin x - \cos x}{t} \times \frac{-dt}{\sin x - \cos x}$$

$$= \int \frac{-dt}{t}$$

$$= -\log|t| + c$$

$$= -\log|\sin x + \cos x| + c$$

## Indefinite Integrals Ex 19.8 Q15

Let $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ then,

Let $\log(\tan x) = t$ then,

$d[\log(\tan x)] = dt$

$\Rightarrow \quad \sec x \operatorname{cosec} x \, dx = dt \quad \left[\because \quad \frac{d}{dx}(\log \tan x) = \sec x \operatorname{cosec} x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x \operatorname{cosec} x}$

Putting $\log(\tan x) = t$ and $dx = \frac{dt}{\sec x \operatorname{cosec} x}$ in equation (i), we get,

$$I = \int \frac{\sec x \operatorname{cosec} x}{t} \times \frac{dt}{\sec x \operatorname{cosec} x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log \tan x| + c$$

## Indefinite Integrals Ex 19.8 Q16

Let $I = \int \frac{1}{x(3 + \log x)} dx$ - - - - - (i)

Let $3 + \log x = t$ then,

$d(3 + \log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x \, dt$

Putting $3 + \log x = t$ and $dx = x\,dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x dt$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|(3 + \log x)| + c$$

$\therefore \quad I = \log|(3 + \log x)| + c$

## Indefinite Integrals Ex 19.8 Q17

Let $I = \int \frac{e^x + 1}{e^x + x} dx$ ------ (i)

Let $e^x + x = t$ then,

$d\left(e^x + x\right) = dt$

$\Rightarrow \left(e^x + x\right) dx = dt$

$\Rightarrow dx = \frac{dt}{e^x + 1}$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x + 1}$ in equation (i), we get,

$I = \int \frac{e^x + 1}{t} \times \frac{dt}{e^x + 1}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|e^x + x\right| + c$

$\therefore I = \log\left|e^x + x\right| + c$

## Indefinite Integrals Ex 19.8 Q18

Let $I = \int \frac{1}{x \log x} dx$ ------ (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow dx = x\,dt$

Putting $\log x = t$ and $dx = x\,dt$ in equation (i), we get,

$I = \int \frac{1}{x \times t} \times x\,dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|(\log x)| + c$

$\therefore I = \log|(\log x)| + c$

## Indefinite Integrals Ex 19.8 Q19

Let $I = \int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x}dx$ ------ (i)

Let $a\cos^2 x + b\sin^2 x = t$ then,

$d\left(a\cos^2 x + b\sin^2 x\right) = dt$

$\left[a\left(2\cos x\left(-\sin x\right)\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \quad \left[-a\left(2\sin x\cos x\right) + b\left(2\sin x\cos x\right)\right]dx = dt$

$\Rightarrow \quad \left[-a\sin 2x + b\sin 2x\right]dx = dt$

$\Rightarrow \quad \sin 2x\left(b-a\right)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\left(b-a\right)\sin 2x}$

Putting $a\cos^2 x + b\sin^2 x = t$ and $dx = \frac{dt}{\left(b-a\right)\sin 2x}$ in equation (i), we get,

$$I = \int \frac{\sin 2x}{t} \times \frac{dt}{\left(b-a\right)\sin 2x}$$

$$= \frac{1}{b-a}\int \frac{dt}{t}$$

$$= \frac{1}{b-a}\log|t| + c$$

$$= \frac{1}{b-a}\log\left|a\cos^2 x + b\sin^2 x\right| + c$$

## Indefinite Integrals Ex 19.8 Q20

Let $I = \int \frac{\cos x}{2+3\sin x} dx$ ------ (i)

Let $2+3\sin x = t$ then,

$d(2+3\sin x) = dt$

$\Rightarrow \quad 3\cos x dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\cos x}$

Putting $2+3\sin x = t$ and $dx = \frac{dt}{3\cos x}$ in equation (i), we get,

$I = \int \frac{\cos x}{t} \times \frac{dt}{3\cos x}$

$= \frac{1}{3}\int \frac{dt}{t}$

$= \frac{1}{3}\log|t| + c$

$= \frac{1}{3}\log|2+3\sin x| + c$

## Indefinite Integrals Ex 19.8 Q21

Let $I = \int \frac{1-\sin x}{x+\cos x} dx$ ------ (i)

Let $x+\cos x = t$ then,

$d(x+\cos x) = dt$

$\Rightarrow \quad (1-\sin x)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-\sin x}$

Putting $x+\cos x = t$ and $dx = \frac{dt}{1-\sin x}$ in equation (i), we get,

$I = \int \frac{1-\sin x}{t} \times \frac{dt}{1-\sin x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|x+\cos x| + c$

$\therefore \quad I = \log|x+\cos x| + c$

## Indefinite Integrals Ex 19.8 Q22

Let $I = \int \frac{a}{b + ce^x} dx$ then,

$$I = \int \frac{a}{e^x \left[\frac{b}{e^x} + c\right]} dx$$

$$\Rightarrow \quad I = \int \frac{a}{e^x \left[be^{-x} + c\right]} dx \text{ --------- } (i)$$

Let $be^{-x} + c = t$ then,

$$d\left(be^{-x} + c\right) = dt$$

$$\Rightarrow \quad -be^{-x} dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{be^{-x}}$$

$$= -\frac{e^x dt}{b}$$

Putting $be^{-x} + c = t$ and $dx = \frac{-e^x\, dt}{b}$ in equation $(i)$, we get,

$$I = \int \frac{a}{e^x \times t} \times \frac{-e^x\, dt}{b}$$

$$= -\frac{a}{b} \int \frac{dt}{t}$$

$$= -\frac{a}{b} \log|t| + c$$

$$= -\frac{a}{b} \log\left|be^{-x} + c\right| + c$$

**Indefinite Integrals Ex 19.8 Q23**

Let $I = \int \frac{1}{e^x + 1} dx$ then,

$$I = \int \frac{1}{e^x \left[1 + \frac{1}{e^x}\right]} dx$$

$\Rightarrow$ $$I = \int \frac{1}{e^x \left[1 + e^{-x}\right]} dx \text{ - - - - - - - - } (i)$$

Let $1 + e^{-x} = t$ then,

$d\left(1 + e^{-x}\right) = dt$

$\Rightarrow$ $-e^{-x} dx = dt$

$\Rightarrow$ $dx = \frac{-dt}{e^{-x}}$

$dx = -dt \times e^x$

Putting $1 + e^{-x} = t$ and $dx = -e^x dt$ in equation $(i)$, we get,

$$I = \int \frac{1}{e^x \times t} \times -e^x \, dt$$

$$= -\int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log\left|1 + e^{-x}\right| + c$$

$\therefore$ $$= -\log\left|1 + e^{-x}\right| + c$$

**Indefinite Integrals Ex 19.8 Q24**

Let $I = \int \frac{\cot x}{\log \sin x} dx$ -------- (i)

Let $\log \sin x = t$ then,

$d(\log \sin x) = dt$

$\Rightarrow \quad \frac{\cos x}{\sin x} dx = dt$

$\Rightarrow \quad \cot x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\cot x}$

Putting $\log \sin x = t$ and $dx = \frac{dt}{\cot x}$ in equation (i), we get,

$$I = \int \frac{\cot x}{t} \times \frac{dt}{\cot x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log \sin x| + c$$

## Indefinite Integrals Ex 19.8 Q25

Let $I$ $\quad I = \int \frac{e^{2x}}{e^{2x} - 2} dx$ - - - - - - - (i)

Let $e^{2x} - 2 = t$ then,

$d\left(e^{2x} - 2\right) = dt$

$\Rightarrow \quad 2e^{2x} dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2e^{2x}}$

Putting $e^{2x} - 2 = t$ and $dx = \frac{dt}{2e^{2x}}$ in equation (i), we get,

$$I = \int \frac{2e^{2x}}{t} \times \frac{dt}{2e^{2x}}$$

$$= \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \log|t| + c$$

$$= \frac{1}{2} \log\left|e^{2x} - 2\right| + c$$

$$\therefore \quad = \frac{1}{2} \log\left|e^{2x} - 2\right| + c$$

## Indefinite Integrals Ex 19.8 Q26

$$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$$

Let $3\cos x + 2\sin x = t$

$$(-3\sin x + 2\cos x)dx = dt$$

$$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log|2\sin x + 3\cos x| + C$$

## Indefinite Integrals Ex 19.8 Q27

Let $I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$ - - - - - - - - (i)

Let $x^2 + \sin 2x + 2x = t$ then,

$d\left(x^2 + \sin 2x + 2x\right) = dt$

$\Rightarrow \quad \left(2x + 2\cos 2x + 2\right) dx = dt$

$\Rightarrow \quad 2\left(\cos 2x + x + 1\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$

Putting $x^2 + \sin 2x + 2x = t$ and $dx = \frac{dt}{2\left(\cos 2x + x + 1\right)}$ in equation (i), we get,

$$I = \int \frac{\cos 2x + x + 1}{t} \times \frac{dt}{2\left(\cos 2x + x + 1\right)}$$

$$= \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \log|t| + c$$

$$= \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$$

$\therefore \quad I = \frac{1}{2} \log\left|x^2 + \sin 2x + 2x\right| + c$

## Indefinite Integrals Ex 19.8 Q29

Let $\quad I = \int \frac{-\sin x + 2\cos x}{2\sin x + \cos x} dx$ - - - - - - - (i)

Let $2\sin x + \cos x = t$ then,

$d(2\sin x + \cos x) = dt$

$\Rightarrow \quad (2\cos x - \sin x)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{-\sin x + 2\cos x}$

Putting $2\sin x + \cos x = t$ and $dx = \frac{dt}{-\sin x + 2\cos x}$ in equation (i), we get,

$I = \int \frac{-\sin x + 2\cos x}{t} \times \frac{dt}{-\sin x + 2\cos x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|2\sin x + \cos x| + c$

$\therefore \quad I = \log|2\sin x + \cos x| + c$

## Indefinite Integrals Ex 19.8 Q30

$\int \frac{\cos 4x - \cos 2x}{\sin 4x - \sin 2x} dx$

$= -\int \frac{2\sin 3x \sin x}{2\cos 3x \sin x} dx$

$= -\int \frac{\sin 3x}{\cos 3x} dx$

Putting $\cos 3x = t$, and $-3\sin 3x dx = dt$

$= \frac{1}{3}\int \frac{dt}{t}$

$= \frac{1}{3}\log|t| + c$

$= \frac{1}{3}\log|\cos 3x| + C$

## Indefinite Integrals Ex 19.8 Q31

Let $I = \int \frac{\sec x}{\log(\sec x + \tan x)} dx$ -------- (i)

Let $\log(\sec x + \tan x) = t$ then,

$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \sec x \, dx = dt$ $\left[\because \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

$\Rightarrow dx = \frac{dt}{\sec x}$

Putting $\log(\sec x + \tan x) = t$ and $dx = \frac{dt}{\sec x}$ in equation (i), we get,

$I = \int \frac{\sec x}{t} \times \frac{dt}{\sec x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\log(\sec x + \tan x)| + c$

$\therefore I = \log|\log(\sec x + \tan x)| + c$

## Indefinite Integrals Ex 19.8 Q32

Let $$I = \int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx \text{ - - - - - - - - (i)}$$

Let $\log \tan \frac{x}{2} = t$ then,

$$d\left[\log \tan \frac{x}{2}\right] = dt$$

$$\Rightarrow \quad \operatorname{cosec} x \, dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\operatorname{cosec} x}$$

Putting $\log \tan \frac{x}{2} = t$ and $dx = \frac{dt}{\operatorname{cosec} x}$ in equation (i),we get,

$$I = \int \frac{\operatorname{cosec} x}{t} \times \frac{dt}{\operatorname{cosec} x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|\log \tan \frac{x}{2}\right| + c$$

$$\therefore \quad I = \log\left|\log \tan \frac{x}{2}\right| + c$$

**Indefinite Integrals Ex 19.8 Q33**

Let $\quad I = \int \frac{1}{x \log x \log(\log x)} dx$ - - - - - - - - (i)

Let $\log(\log x) = t$ then,

$d[\log(\log x)] = dt$

$\Rightarrow \quad \frac{1}{x} \times \frac{1}{\log x} dx = dt$

$\Rightarrow \quad dx = x \log x \, dt$

Putting $\log(\log x) = t$ and $dx = x \log x \, dt$ in equation (i), we get,

$$I = \int \frac{1}{x \log x \, t} \times x \log x \, dt$$

$$= \int \frac{1}{t} dt$$

$$= \log|t| + c$$

$$= \log|\log(\log x)| + c$$

$\therefore \quad I = \log|\log(\log x)| + c$

**Indefinite Integrals Ex 19.8 Q34**

Let $\quad I = \int \frac{\operatorname{cosec}^2 x}{1+\cot x} dx$ - - - - - - - - (i)

Let $1+\cot x = t$ then,

$d\left[1+\cot x\right] = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x\, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $1+\cot x = t$ and $dx = \frac{-dt}{\operatorname{cosec}^2 x}$ in equation (i), we get,

$$I = \int \frac{\operatorname{cosec}^2 x}{t} \times -\frac{dt}{\operatorname{cosec}^2 x}$$

$$= -\int \frac{1}{t} dt$$

$$= -\log\left|t\right| + c$$

$$= -\log\left|1+\cot x\right| + c$$

$\therefore \quad I = -\log\left|1+\cot x\right| + c$

## Indefinite Integrals Ex 19.8 Q35

Let $I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$ -------- (i)

Let $10^x + x^{10} = t$ then,

$d\left(10^x + x^{10}\right) = dt$

$\Rightarrow \left(10^x \log_e 10 + 10x^9\right) dx = dt$

$\Rightarrow dx = \frac{dt}{10x^9 + 10^x \log_e 10}$

Putting $10^x + x^{10} = t$ and $dx = \frac{dt}{10x^9 + 10^x \log_e 10}$ in equation (i), we get,

$$I = \int \frac{10x^9 + 10^x \log_e 10}{t} \times \frac{dt}{10x^9 + 10^x \log_e 10}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|10^x + x^{10}\right| + c$$

$\therefore I = \log\left|10^x + x^{10}\right| + c$

## Indefinite Integrals Ex 19.8 Q36

Let $I = \int \frac{1-\sin 2x}{x+\cos^2 x} dx$ ----- (i)

Let $x + \cos^2 x = t$ then,

$d\left(x+\cos^2 x\right) = dt$

$\Rightarrow$ $\left(1 - 2\cos x \sin x\right) dx = dt$

$\Rightarrow$ $dx = \frac{dt}{1 - 2\cos x \sin x}$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1-2\cos x \sin x}$ in equation (i), we get

$$I = \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-2\cos x \sin x}$$
$$= \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-\sin 2x}$$
$$= \int \frac{dt}{t}$$
$$= \log\left|t\right| + c$$
$$= \log\left|x+\cos^2 x\right| + c$$

$\therefore$ $I = \log\left|x + \cos^2 x\right| + c$

## Indefinite Integrals Ex 19.8 Q37

Let $I = \int \frac{1+\tan x}{x+\log\sec x}dx$ -------- (i)

Let $x+\log\sec x = t$ then,

$d\left(x+\log\sec x\right) = dt$

$\Rightarrow$ $\left(1+\tan x\right)dx = dt$ $\left[\because \frac{d}{dx}\left(\log\sec x\right) = \tan x\right]$

$\Rightarrow$ $dx = \frac{dt}{1+\tan x}$

Putting $x+\log\sec x = t$ and $dx = \frac{dt}{1+\tan x}$ in equation (i), we get,

$$I = \int \frac{1+\tan x}{t} \times \frac{dt}{1+\tan x}$$

$$= \int \frac{dt}{t}$$

$$= \log\left|t\right| + c$$

$\Rightarrow$ $I = \log\left|x+\log\sec x\right| + c$

## Indefinite Integrals Ex 19.8 Q38

Let $I = \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$ -------- (i)

Let $a^2 + b^2 \sin^2 x = t$ then,

$d\left(a^2 + b^2 \sin^2 x\right) = dt$

$\Rightarrow$ $b^2 (2 \sin x \cos x) dx = dt$

$\Rightarrow$ $dx = \frac{dt}{b^2 (2 \sin x \cos x)}$

$= \frac{dt}{b^2 \sin 2x}$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i), we get,

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x}$

$= \frac{1}{b^2} \int \frac{dt}{t}$

$= \frac{1}{b^2} \log |t| + c$

$= \frac{1}{b^2} \log \left| a^2 + b^2 \sin^2 x \right| + c$

$\Rightarrow$ $I = \frac{1}{b^2} \log \left|a^2 + b^2 \sin^2 x\right| + c$

## Indefinite Integrals Ex 19.8 Q39

Let $I = \int \frac{x+1}{x(x+\log x)} dx$ - - - - - - - (i)

Let $(x+\log x) = t$ then,

$d(x+\log x) = dt$

$\Rightarrow \left(1+\frac{1}{x}\right) dx = dt$

$\Rightarrow \left(\frac{x+1}{x}\right) dx = dt$

$\Rightarrow dx = \frac{x}{x+1} dt$

Putting $(x+\log x) = t$ and $dx = \frac{x}{x+1}$ in equation (i), we get,

$$I = \int \frac{x+1}{x \times t} \times \frac{x}{x+1} dt$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|x+\log x| + c$$

$\Rightarrow I = \log|x+\log x| + c$

## Indefinite Integrals Ex 19.8 Q40

Let $I = \int \frac{1}{\sqrt{1-x^2}\left(2+3\sin^{-1}x\right)}dx$ ---------(i)

Let $2+3\sin^{-1}x = t$ then,

$d\left(2+3\sin^{-1}x\right) = dt$

$\Rightarrow \quad 3\times\frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \quad dx = \frac{\sqrt{1-x^2}}{3}dt$

Putting $2+3\sin^{-1}x = t$ and $dx = \frac{\sqrt{1-x^2}}{3}$ in equation (i), we get,

$$I = \int \frac{\sqrt{1-x^2}}{3}\times\frac{1}{\sqrt{1-x^2}\,t}dt$$

$$= \frac{1}{3}\int\frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$$

$\Rightarrow \quad I = \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$

**Indefinite Integrals Ex 19.8 Q41**

Let $\quad I = \int \frac{\sec^2 x}{\tan x + 2} dx$ - - - - - - - (i)

Let $\tan x + 2 = t$ then,

$d(\tan x + 2) = dt$

$\Rightarrow \quad \sec^2 x dx = dt$

$\Rightarrow \quad dx = \frac{1}{\sec^2 x} dt$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i),we get,

$I = \int \frac{\sec^2 x}{t} \times \frac{1}{\sec^2 x} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\tan x + 2| + c$

$\Rightarrow \quad I = \log|\tan x + 2| + c$

## Indefinite Integrals Ex 19.8 Q42

Let $\quad I = \int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx$ - - - - - - - (i)

Let $\sin 2x + \tan x - 5 = t$ then,

$d(\sin 2x + \tan x - 5) = dt$

$\Rightarrow \quad (2\cos 2x + \sec^2 x) dx = dt$

$\Rightarrow \quad dx = \frac{1}{2\cos 2x + \sec^2 x} dt$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2\cos 2x + \sec^2 x}$ in equation (i),we get,

$I = \int \frac{2\cos 2x + \sec^2 x}{t} \times \frac{1}{2\cos 2x + \sec^2 x} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\sin 2x + \tan x - 5| + c$

$\therefore \quad I = \log|\sin 2x + \tan x - 5| + c$

## Indefinite Integrals Ex 19.8 Q43

Let $I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx$ then,

$$I = \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$$
$$= \int \cot 3x \, dx - \int \cot 5x \, dx$$
$$= \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$$

$$\therefore \quad I = \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$$

## Indefinite Integrals Ex 19.8 Q44

Let $I = \int \frac{1 + \cot x}{x + \log \sin x} dx$ ------ (i)

Let $x + \log \sin x = t$ then,

$d(x + \log \sin x) = dt$

$\Rightarrow \quad (1 + \cot x) dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log \sin x) = \cot x\right]$

$\Rightarrow \quad dx = \frac{dt}{1 + \cot x}$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$$I = \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|x + \log \sin x| + c$$

$$\therefore \quad I = \log|x + \log \sin x| + c$$

## Indefinite Integrals Ex 19.8 Q45

Let $I = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)} dx$ - - - - - (i)

Let $\sqrt{x}+1 = t$ then,

$d\left(\sqrt{x}+1\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

Putting $\sqrt{x}+1 = t$ and $dx = 2\sqrt{x}\, dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{x}\, t} \times 2\sqrt{x}\, dt$$
$$= 2\int \frac{dt}{t}$$
$$= 2\log|t| + c$$
$$= 2\log\left|\sqrt{x}+1\right| + c$$
$$\therefore \quad I = 2\log\left|\sqrt{x}+1\right| + c$$

## Indefinite Integrals Ex 19.8 Q46

Let $I = \int \tan 2x \tan 3x \tan 5x dx$ - - - - - - - - - (i)

Now,

$$\tan(5x) = \tan(2x+3x)$$
$$= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$$\Rightarrow \quad \tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$
$$\Rightarrow \quad \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$$
$$\Rightarrow \quad \tan 5x - \tan 2x - \tan 3x = \tan 2x \tan 3x \tan 5x \text{ - - - - - (ii)}$$

Using equation (i) and equation (ii), we get

$$I = \int\left[\tan 5x - \tan 2x - \tan 3x\right] dx$$
$$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$$

$$\therefore \quad I = \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$$

## Indefinite Integrals Ex 19.8 Q47

Since,

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$\therefore \quad \tan(x+\theta-x) = \frac{\tan(x+\theta)-\tan x}{1+\tan(x+\theta)\tan x}$$

$$\Rightarrow \quad 1+\tan(x+\theta)\tan x = \frac{\tan(x+\theta)-\tan x}{\tan\theta}$$

$$\Rightarrow \quad \int 1+\tan(x+\theta)\tan x dx$$

$$= \frac{1}{\tan\theta}\left[\int \tan(x+\theta)dx - \int \tan x dx\right]$$

$$= \frac{1}{\tan\theta}\left[-\log\left|\cos(x+\theta)\right| + \log\left|\cos x\right|\right] + C$$

$$= \frac{1}{\tan\theta}\left[\log\left|\cos x\right| - \log\left|\cos(x+\theta)\right|\right] + C$$

$$= \frac{1}{\tan\theta}\log\left|\frac{\cos x}{\cos(x+\theta)}\right| + C$$

## Indefinite Integrals Ex 19.8 Q48

$$\text{Consider } I = \int\left(\frac{\sin 2x}{\sin\left(x-\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}\sin^2 x - \frac{1}{4}\cos^2 x\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}(1-\cos^2 x) - \frac{1}{4}\cos^2 x\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}-\cos^2 x\right)}\right)dx$$

$$let \ \cos^2 x = t \rightarrow \sin 2x dx = -dt$$

$$I = \int\left(\frac{-dt}{\left(\frac{3}{4}-t\right)}\right)$$

$$I = \log\left|\sin^2 x - \frac{1}{4}\right| + C$$

## Indefinite Integrals Ex 19.8 Q49

$$\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$$

$$= \frac{1}{e} \int \frac{e^x + ex^{e-1}}{e^x + x^e} dx$$

Let $e^x + x^e = u$

$$\Rightarrow \quad \left(e^x + ex^{e-1}\right) dx = du$$

$$= \frac{1}{e} \int \frac{1}{4} du = \frac{1}{e} \log |u| + C$$

$$= \frac{1}{e} \log \left|e^x + x^e\right| + C$$

## Indefinite Integrals Ex 19.8 Q50

Let $I = \int \frac{1}{\sin x \cos^2 x} dx$, then,

$$I = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \cos^2 x} dx$$

$$= \int \sec x \tan x \, dx + \int \operatorname{cosec} x \, dx$$

$$= \sec x + \log \left|\tan \frac{x}{2}\right| + c$$

$$\therefore \quad I = \sec x + \log \left|\tan \frac{x}{2}\right| + c$$

## Indefinite Integrals Ex 19.8 Q51

Let $I = \int \frac{1}{\cos 3x - \cos x} dx,$ then,

$$I = \int \frac{\sin^2 x + \cos^2 x}{-2\sin 2x \sin x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{-4\sin^2 x \cos x} dx$$

$$= -\frac{1}{4}\int \left[\frac{\sin^2 x}{\sin^2 x \cos x} + \frac{\cos^2 x}{\sin^2 x \cos x}\right] dx$$

$$= -\frac{1}{4}\int [\sec x + \operatorname{cosec} x \cot x] dx$$

$$= -\frac{1}{4}\left[\log|\sec x + \tan x| - \operatorname{cosec} x\right] + c$$

$$\therefore \quad I = \frac{1}{4}\left[\operatorname{cosec} x - \log|\sec x + \tan x|\right] + c$$