RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.12

Indefinite Integrals Ex 19.12 Q1

Let $I = \int \sin^4 x \cos^3 x dx$

Here, power of $\cos x$ is odd, so we substitute

$$\sin x = t$$

$$\Rightarrow \quad \cos x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cos x}$$

$$\therefore \quad I = \int t^4 \cos^3 x \frac{dt}{\cos x}$$

$$= \int t^4 \cos^2 x dt$$

$$= \int t^4 \left(1 - \sin^2 x\right) dt$$

$$= \int t^4 \left(1 - t^2\right) dt$$

$$= \int \left(t^4 - t^6\right) dt$$

$$= \frac{t^5}{5} - \frac{t^7}{7} + c$$

$$\therefore \quad I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q2

Let $I = \int \sin^5 x dx$. Then

$$I = \int \sin^3 x \sin^2 x dx$$
$$= \int \sin^3 x \left(1 - \cos^2 x\right) dx$$
$$= \int \left(\sin^3 x - \sin^3 x \cos^2 x\right) dx$$
$$= \int \left[\sin x \left(1 - \cos^2 x\right) - \sin^3 x \cos^2 x\right] dx$$
$$= \int \left(\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x\right) dx$$

$\Rightarrow \quad I = \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt$$
$$= \int \sin x dx + \int t^2 dt + \int \left(1 - \cos^2 x\right) t^2 dt$$
$$= \int sixdx + \int t^2 dt + \int \left(1 - t^2\right) t^2 dt$$
$$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$$
$$= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + c$$
$$= -\cos x + \frac{2}{3}\left(\cos^3 x\right) - \frac{1}{5}\left(\cos^5 x\right) + c$$

$\therefore \quad I = -\left[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right] + c$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$I = \int \cos^2 x \cos^3 x dx$$
$$= \int \left(1 - \sin^2 x\right) \cos^3 x dx$$
$$= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx$$
$$= \int \cos^2 x \cos x dx - \int \sin^2 x \left(1 - \sin^2 x\right) \cos x dx$$
$$= \int \left(\cos x - \sin^2 x \cos x\right) dx - \int \left(\sin^2 x \cos x - \sin^4 x \cos x\right) dx$$

$\Rightarrow \quad I = \int \cos x dx - 2\int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \cos x dx - 2\int t^2 dt + \int t^4 dt$$
$$= \sin x - \frac{2}{3} t^3 + \frac{t^5}{5} + c$$
$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

$\therefore \quad I = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

Indefinite Integrals Ex 19.12 Q4

Let $I = \int \sin^5 x \cos x \, dx$ ---(i)

Let $\sin x = t$. Then,

$$d(\sin x) = dt$$

$\Rightarrow \quad \cos x dx = dt$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$

$$= \frac{t^6}{6} + c$$

$$= \frac{\sin^6 x}{6} + c$$

$\therefore \quad I = \frac{1}{6}\sin^6 x + c$

Indefinite Integrals Ex 19.12 Q5

Let $I = \int \sin^3 x \cos^6 x dx$

Here, power of $\sin x$ is odd, so we substitute

$$\cos x = t$$

$\Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \sin^2 x t^6 (-dt)$

$$= -\int \left(1 - \cos^2 x\right) t^6 dt$$

$$= -\int \left(t^6 - t^8\right) dt$$

$$= -\frac{t^7}{7} + \frac{t^9}{9} + c$$

$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + c$$

$\therefore \quad I = -\frac{1}{7}\cos^7 x + \frac{1}{9}\cos^9 x + c$

Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$I = \int \cos^6 x \cos x dx$$

$$= \int \left(\cos^2 x\right)^3 \cos x dx$$

$$= \int \left(1 - \sin^2 x\right)^3 \cos x dx$$

$$= \int \left[1 - \sin^6 x - 3\sin^2 x + 3\sin^4 x\right] \cos x dx$$

$$= \int \left[\cos x - \sin^6 x \cos x - 3\sin^2 x \cos x + 3\sin^4 x \cos x\right] dx + c$$

$$\Rightarrow \quad I = \int \cos x dx - \int \sin^6 x \cos x dx - 3\int \sin^2 x \cos x dx + 3\int \sin^4 x \cos x dx$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$I = \int \cos x dx - \int t^6 dt - 3\int t^2 dt + 3\int t^4 dt$$

$$= \sin x - \frac{t^7}{7} - \frac{3}{3}t^3 + \frac{3}{5}t^5 + c$$

$$= \sin x - \frac{1}{7}\sin^7 x - \sin^3 x + \frac{3}{5}\sin^5 x + c$$

$$\therefore \quad I = \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q7

Let $I = \int x \cos^3 x^2 \sin x^2 dx$

Let $\cos x^2 = t$. Then

$$d\left(\cos x^2\right) = dt$$

$$\Rightarrow \quad -2x \sin x^2 x = dt$$

$$\Rightarrow \quad x \sin x^2 dx = -\frac{dt}{2}$$

$$\therefore \quad I = \int t^3 \times \frac{-dt}{2}$$

$$= -\frac{t^4}{8} + c$$

$$= -\frac{1}{8}\cos^4 x^2 + c$$

$$\therefore \quad I = -\frac{1}{8}\cos^4 x^2 + c$$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$I = \int \sin^6 x \sin x dx$$
$$= \int \left(\sin^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x\right) \sin x dx$$

$\Rightarrow \quad I = \int \sin x dx - \int \cos^6 x \sin x dx + 3\int \cos^4 x \sin x dx - 3\int \cos^2 x \sin x dx$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2ne, 3rd and 4th integral, we get

$$I = \int \sin x dx - \int t^6 (-dt) + 3\int t^4 (-dt) - 3\int t^2 (-dt)$$
$$= -\cos x + \frac{t^7}{7} - \frac{3}{5}t^5 + \frac{3}{3}t^3 + c$$
$$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5}\cos^5 x + \cos^3 x + c$$

$\therefore \quad I = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$d(\cos x) = dt$$

$\Rightarrow \quad -\sin x dx = dt$

$\Rightarrow \quad dx = \frac{-dt}{\sin x}$

$\therefore \quad I = \int \sin^3 x t^5 \frac{-dt}{\sin x}$

$$= -\int \sin^2 x t^5 dt$$
$$= -\int \left(1 - \cos^2 x\right) t^5 dt$$
$$= -\int \left(1 - t^2\right) t^5 dt$$
$$= -\int \left(t^5 - t^7\right) dt$$
$$= -\frac{t^6}{6} + \frac{t^8}{8} + c$$
$$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

$\therefore \quad I = \frac{-1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$

Indefinite Integrals Ex 19.12 Q10

Let $I = \int \frac{1}{\sin^4 x \cos^2 x} dx$ ---(i)

Then, $I = \int \sin^{-4} x \cos^{-2} x dx$

Since $-4-2=-6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx$$

$$= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^6 x}{\tan^4 x} dx$$

$$= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(\sec^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^4 x + 2\tan^2 x\right) \times \sec^2 x}{\tan^4 x} dx \qquad \text{---(ii)}$$

Let $\tan x = t$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int \frac{\left(1+t^4+2t^2\right)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int \left(t^{-4}+1+2t^{-2}\right) dt$$

$$= -\frac{t^{-3}}{3} + t - 2t^{-1} + c$$

$$= -\frac{1}{3t^3} + t - \frac{2}{t} + c$$

$$= -\frac{1}{3\tan^3 x} + \tan x - \frac{2}{\tan x} + c$$

$$= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + c$$

$$\therefore \quad I = \frac{-1}{3} \times \cot^3 x - 2\cot x + \tan x + c$$

Indefinite Integrals Ex 19.12 Q11

Let $I = \int \frac{1}{\sin^3 x \cos^5 x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-5} x dx$

Since $-3-5=-8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^3}{\tan^3 x} \times \sec^2 x dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^6 x + 3\tan^4 x + 3\tan^2 x\right) \times \sec^2 x}{\tan^3 x} dx \qquad \text{---(ii)}$$

Let $t = \tan x$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\therefore \quad I = \int \frac{\left(1+t^6+3t^4+3t^2\right)}{t^3} dt$$

$$= \int \left(t^{-3}+t^3+3t+3t^{-1}\right) dt$$

$$= -\frac{t^{-2}}{2}+\frac{t^4}{4}+\frac{3}{2}t^2+3\log t + c$$

$$= -\frac{1}{2t^2}+\frac{t^4}{4}+\frac{3}{2}t^2+3\log t + c$$

$$= -\frac{1}{2} \times \frac{1}{\tan^2 x}+\frac{\tan^4 x}{4}+\frac{3}{2} \times \tan^2 x + 3\log\left|\tan x\right| + c$$

$$\therefore \quad I = \frac{-1}{2\tan^2 x} + 3\log\left|\tan x\right| + \frac{3}{2}\tan^2 x + \frac{1}{4} \times \tan^4 x + c$$

Indefinite Integrals Ex 19.12 Q12

Let $I = \int \frac{1}{\sin^3 x \cos x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-1} x dx$

Since $-3-1=-4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)}{\tan^3 x} \times \sec^2 x dx$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \frac{1+t^2}{t^3} dt$$

$$= \int \left(t^{-3} + \frac{1}{t}\right) dt$$

$$= -\frac{t^{-2}}{2} + \log|t| + c$$

$$= -\frac{1}{2t^2} + \log|t| + c$$

$$= -\frac{1}{2\tan^2 x} + \log|\tan x| + c$$

Indefinite Integrals Ex 19.12 Q13

$$\frac{1}{\sin x\cos^3 x} = \frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$

$$= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x\cos x}$$

$$= \tan x\sec^2 x + \frac{\frac{1\cos^2 x}{\sin x\cos x}}{\cos^2 x}$$

$$= \tan x\sec^2 x + \frac{\sec^2 x}{\tan x}$$

$$\therefore \int\frac{1}{\sin x\cos^3 x}dx = \int\tan x\sec^2 x\ dx + \int\frac{\sec^2 x}{\tan x}\ dx$$

Let $\tan x = t \Rightarrow \sec^2 x\ dx = dt$

$$\Rightarrow \int\frac{1}{\sin x\cos^3 x}dx = \int t\,dt + \int\frac{1}{t}dt$$

$$= \frac{t^2}{2} + \log|t| + C$$

$$= \frac{1}{2}\tan^2 x + \log|\tan x| + C$$