RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.18

Indefinite Integrals Ex 19.18 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{\left(x^2\right)^2 + \left(a^2\right)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + \left(a^2\right)^2}}$

$= \frac{1}{2} \log \left| t + \sqrt{t^2 + \left(a^2\right)^2} \right| + c$ $\quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c\right]$

$I = \frac{1}{2} \log \left| x^2 + \sqrt{\left(x^2\right)^2 + \left(a^2\right)^2} \right| + c$

$I = \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + a^4} \right| + c$

Indefinite Integrals Ex 19.18 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\,dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log \left| t + \sqrt{t^2 + 4} \right| + C$$

$$= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C$$

Indefinite Integrals Ex 19.18 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

$$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$$

Indefinite Integrals Ex 19.18 Q4

Let $I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$$

$$= \log\left|t + \sqrt{(2)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$$

$$I = \log\left|\sin x + \sqrt{4 + \sin^2 x}\right| + c$$

Indefinite Integrals Ex 19.18 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x\, dx = dt$

$\Rightarrow \quad \sin x\, dx = -\frac{dt}{2}$

$$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$$

Indefinite Integrals Ex 19.18 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{2} \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2} \sin^{-1}\left(\frac{x^2}{2}\right) + c$

Indefinite Integrals Ex 19.18 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}} dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x} dx = dt$

$\Rightarrow \quad \frac{1}{x} dx = \frac{dt}{3}$

$I = \frac{1}{3} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{3} \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{3} \sin^{-1}\left(\frac{3\log x}{2}\right) + c$

Indefinite Integrals Ex 19.18 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9+(\sin 4x)^4}}dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\,dx = dt$

$\Rightarrow \quad \sin 8x\,dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \frac{1}{4}\log\left|t+\sqrt{(3)^2+t^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \frac{1}{4}\log\left|\sin^2 4x+\sqrt{9+\sin^4 4x}\right|+c$

Indefinite Integrals Ex 19.18 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x+8}}dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2+(2\sqrt{2})^2}}$

$= \frac{1}{2}\log\left|t+\sqrt{t^2+(2\sqrt{2})^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = \frac{1}{2}\log\left|\sin 2x+\sqrt{\sin^2 2x+8}\right|+c$

Indefinite Integrals Ex 19.18 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4\sin^2 x - 2}}dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x \cos x\, dx = dt$

$\Rightarrow \quad \sin 2x\, dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2 + 4t - 2}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}}dx$$

$$= \int \frac{dt}{\sqrt{(t+2)^2 - 6}}$$

Let $t + 2 = u$

$dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\sqrt{6}\right)^2}}$$

$$= \log\left|u + \sqrt{u^2 - 6}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2 - a^2}}dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|t + 2 + \sqrt{(t+2)^2 - 6}\right| + c$$

$$I = \log\left|\sin^2 x + 2 + \sqrt{\sin^4 x + 4\sin^2 x - 2}\right| + c$$

Indefinite Integrals Ex 19.18 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}}dx =$$

$let\ t = \cos^2 x \rightarrow -dt = 2\cos x \sin x dx$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}}dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}}dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}}dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}}dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}}dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.18 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Indefinite Integrals Ex 19.18 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3}-1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$

$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$

Indefinite Integrals Ex 19.18 Q14

Let $I = \int \frac{1}{\sqrt{\left(1-x^2\right)\left[9+\left(\sin^{-1}x\right)^2\right]}}dx$

Let $\sin^{-1}x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$I = \int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \log\left|t+\sqrt{9+t^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \log\left|\sin^{-1}x+\sqrt{9+\left(\sin^{-1}x\right)^2}\right|+c$

Indefinite Integrals Ex 19.18 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2-2t-3}}$

$= \int \frac{dt}{\sqrt{t^2-2t+(1)^2-(1)^2-3}}$

$= \int \frac{dt}{\sqrt{(t-1)^2-(2)^2}}$

Let $t-1 = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{du}{\sqrt{u^2-(2)^2}}$

$= \log\left|u+\sqrt{u^2-4}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$= \log\left|t-1+\sqrt{(t-1)^2+4}\right|+c$

$I = \log\left|\sin x - 1 + \sqrt{\sin^2 x - 2\sin x - 3}\right|+c$

Indefinite Integrals Ex 19.18 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1}\,dx$

$$= \int \sqrt{\frac{1 - \sin x}{\sin x}}\,dx$$

$$= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x\,(1 + \sin x)}}\,dx$$

$$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}}\,dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}}\,dx$$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$$= \int \frac{dt}{\sqrt{t^2 + t}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

Let, $t + \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}$$

$$== \log\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|\left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c$$

$$I = \log\left|\sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x}\right| + c$$

Indefinite Integrals Ex 19.18 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}}\,dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\,dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x)\,dx = dt$

Now

$$\int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}}\,dx = -\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\ln\left|t + \sqrt{t^2 - 1}\right| + c$$

$$= -\ln\left|\sin x + \cos x + \sqrt{\sin 2x}\right| + c$$