

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.19

Indefinite Integrals Ex 19.19 Q1

$$\text{Let } I = \int \frac{x}{x^2 + 3x + 2} dx$$

$$\text{Let } x = \lambda \frac{d}{dx} (x^2 + 3x + 2) + \mu$$

$$= \lambda(2x + 3) + \mu$$

$$x = (2\lambda)x + (3\lambda + \mu)$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$3\lambda + \mu = 0 \Rightarrow 3\left(\frac{1}{2}\right) + \mu = 0$$

$$\mu = -\frac{3}{2}$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x + 3) - \frac{3}{2}}{x^2 + 3x + 2} dx$$

$$I = \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{x^2 + 3x + 2} dx$$

$$= \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{x^2 + 2x \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2} dx$$

$$I = \frac{1}{2} \int \frac{2x + 3}{x^2 + 3x + 2} dx - \frac{3}{2} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$= \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{x + \frac{3}{2} - \frac{1}{2}}{x + \frac{3}{2} + \frac{1}{2}} \right| + c \quad \left[\text{since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \frac{1}{2} \log|x^2 + 3x + 2| - \frac{3}{2} \log \left| \frac{x + 1}{x + 2} \right| + c$$

Indefinite Integrals Ex 19.19 Q2

$$\text{Let } I = \int \frac{x+1}{x^2+x+3} dx$$

$$\text{Let } x+1 = \lambda \frac{d}{dx} \{x^2+x+3\} + \mu$$

$$x+1 = \lambda(2x+1) + \mu$$

$$x+1 = (2\lambda)x + (\lambda + \mu)$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\lambda + \mu = 1 \Rightarrow \left(\frac{1}{2}\right) + \mu = 0$$

$$\mu = -\frac{1}{2}$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{x^2+x+3} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{x^2+2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 3} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{11}{4}\right)} dx$$

$$I = \frac{1}{2} \int \frac{2x+1}{x^2+x+3} dx + \frac{1}{2} \int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{11}}{2}\right)^2} dx$$

$$= \frac{1}{2} \log|x^2+x+3| + \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{11}}{2}\right)} \tan^{-1} \left(\frac{x + \frac{1}{2}}{\frac{\sqrt{11}}{2}} \right) + c \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \frac{1}{2} \log|x^2+x+3| + \frac{1}{\sqrt{11}} \tan^{-1} \left(\frac{2x+1}{\sqrt{11}} \right) + c$$

$$\text{Let } I = \int \frac{x-3}{x^2+2x-4} dx$$

$$\begin{aligned} \text{Let } x-3 &= \lambda \frac{d}{dx}(x^2+2x-4) + \mu \\ &= \lambda(2x+2) + \mu \\ x-3 &= (2\lambda)x + (2\lambda + \mu) \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$2\lambda + \mu = -3 \Rightarrow 2\left(\frac{1}{2}\right) + \mu = -3$$

$$\mu = -4$$

$$\text{so, } I = \int \frac{\frac{1}{2}(2x+2) - 4}{x^2+2x-4} dx$$

$$I = \frac{1}{2} \int \frac{2x+2}{x^2+2x-4} dx - 4 \int \frac{1}{x^2+2x+(1)^2 - (1)^2 - 4} dx$$

$$I = \frac{1}{2} \int \frac{2x+2}{x^2+2x-4} dx - 4 \int \frac{1}{(x+1)^2 - (\sqrt{5})^2} dx$$

$$I = \frac{1}{2} \log|x^2+2x-4| - 4 \times \frac{1}{2\sqrt{5}} \log \left| \frac{x+1-\sqrt{5}}{x+1+\sqrt{5}} \right| + c \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{1}{2} \log|x^2+2x-4| - \frac{2}{\sqrt{5}} \log \left| \frac{x+1-\sqrt{5}}{x+1+\sqrt{5}} \right| + c$$

Indefinite Integrals Ex 19.19 Q4

$$\text{Let } I = \int \frac{2x-3}{x^2+6x+13} dx$$

$$\begin{aligned} \text{Let } 2x-3 &= \lambda \frac{d}{dx}(x^2+6x+13) + \mu \\ &= \lambda(2x+6) + \mu \\ 2x-3 &= (2\lambda)x + (6\lambda + \mu) \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 2 \Rightarrow \lambda = 1$$

$$6\lambda + \mu = -3 \Rightarrow 6(1) + \mu = -3$$

$$\mu = -9$$

$$\text{so, } I = \int \frac{1(2x+6) - 9}{x^2+6x+13} dx$$

$$I = \int \frac{2x+6}{x^2+6x+13} dx - 9 \int \frac{1}{x^2+2x(3)+(3)^2 - (3)^2 + 13} dx$$

$$I = \int \frac{2x+6}{x^2+6x+13} dx - 9 \int \frac{1}{(x+3)^2 + (2)^2} dx$$

$$= \log|x^2+6x+13| - 9 \times \frac{1}{2} \tan^{-1} \left(\frac{x+3}{2} \right) + c \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \log|x^2+6x+13| - \frac{9}{2} \tan^{-1} \left(\frac{x+3}{2} \right) + c$$

Indefinite Integrals Ex 19.19 Q5

$$\begin{aligned} \text{Let } I &= \int \frac{x^2}{x^2 + 7x + 10} dx \\ &= \int \left[1 - \frac{7x + 10}{x^2 + 7x + 10} \right] dx \\ I &= x - \int \frac{7x + 10}{x^2 + 7x + 10} dx + c_1 \text{ ---- (i)} \end{aligned}$$

$$\text{Let } I_1 = \int \frac{7x + 10}{x^2 + 7x + 10} dx$$

$$\begin{aligned} \text{Let } 7x + 10 &= \lambda \frac{d}{dx} (x^2 + 7x + 10) + \mu \\ &= \lambda (2x + 7) + \mu \\ 7x + 10 &= (2\lambda)x + 7\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$$

$$7\lambda + \mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right) + \mu = 10$$

$$\mu = -\frac{29}{2}$$

$$\begin{aligned} \text{so, } I &= \int \frac{\frac{1}{6}(6x - 4) - \frac{1}{3}}{3x^2 - 4x + 3} dx \\ I &= \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{x^2 - \frac{4}{3}x + 1} dx \\ I &= \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{x^2 - 2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + (2)^2} dx \\ I &= \frac{1}{6} \int \frac{6x - 4}{3x^2 - 4x + 3} dx - \frac{1}{9} \int \frac{1}{\left(x - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dx \\ &= \frac{1}{6} \log|3x^2 - 4x + 3| - \frac{1}{9} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1} \left(\frac{x - \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right) + c \quad \left[\text{since, } \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right] \\ I &= \frac{1}{6} \log|3x^2 - 4x + 3| - \frac{\sqrt{5}}{15} \tan^{-1} \left(\frac{3x - 2}{\sqrt{5}} \right) + c \end{aligned}$$

We need to evaluate the integral $\int \frac{2x}{2+x-x^2} dx$

write the numerator in the following form

$$2x = \lambda \left\{ \frac{d}{dx} (2+x-x^2) \right\} + \mu$$

i.e. $2x = \lambda(-2x+1) + \mu$

Equating the coefficients will give the values of λ, μ

$$\lambda = -1, \mu = 1$$

$$\begin{aligned} \int \frac{2x}{2+x-x^2} dx &= \int \frac{\lambda(-2x+1) + \mu}{2+x-x^2} dx \\ &= \int \frac{-1(-2x+1) + 1}{2+x-x^2} dx \\ &= \int \frac{-1(-2x+1)}{2+x-x^2} dx + \int \frac{1}{2+x-x^2} dx \\ &= -1 \log |2+x-x^2| + \int \frac{1}{2+x-x^2} dx \\ &= -1 \log |2+x-x^2| - \int \frac{1}{(x^2-x-2)} dx \end{aligned}$$

$$= -1 \log |2+x-x^2| - \int \frac{1}{\left(x^2 - x + \frac{1}{4} - 2 - \frac{1}{4}\right)} dx$$

$$= -1 \log |2+x-x^2| - \int \frac{1}{\left(x^2 - x + \frac{1}{4} - \frac{9}{4}\right)} dx$$

$$= -1 \log |2+x-x^2| - \int \frac{1}{\left(\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right)} dx$$

$$= -1 \log |2+x-x^2| - \frac{1}{3} \log \left| \frac{\left(x - \frac{1}{2}\right) - \left(\frac{3}{2}\right)}{\left(x - \frac{1}{2}\right) + \left(\frac{3}{2}\right)} \right| + C$$

$$= -1 \log |2+x-x^2| - \frac{1}{3} \log \left| \frac{(x-2)}{(x+1)} \right| + C$$

$$\text{Let } I = \int \frac{1-3x}{3x^2+4x+2} dx$$

$$\text{Let } 1-3x = \lambda \frac{d}{dx} (3x^2+4x+2) + \mu$$

$$= \lambda (6x+4) + \mu$$

$$1-3x = (6\lambda)x + (4\lambda + \mu)$$

Comparing the coefficients of like powers of x ,

$$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$$

$$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right) + \mu = 1$$

$$\mu = 3$$

$$\text{so, } I = \int \frac{-\frac{1}{2}(6x+4)+3}{3x^2+4x+2} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + 3 \int \frac{1}{3x^2+4x+2} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \frac{3}{3} \int \frac{1}{x^2 + \frac{4}{3}x + \frac{2}{3}} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{x^2+2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + \frac{2}{3}} dx$$

$$= -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{\left(x + \frac{2}{3}\right)^2 + \frac{2}{9}} dx$$

$$I = -\frac{1}{2} \int \frac{6x+4}{3x^2+4x+2} dx + \int \frac{1}{\left(x + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} dx$$

$$= -\frac{1}{2} \log|3x^2+4x+2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{x + \frac{2}{3}}{\frac{\sqrt{2}}{3}} \right) + c \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = -\frac{1}{2} \log|3x^2+4x+2| + \frac{3}{\sqrt{2}} \tan^{-1} \left(\frac{3x+2}{\sqrt{2}} \right) + c$$

Indefinite Integrals Ex 19.19 Q8

$$\text{Let } I = \int \frac{2x+5}{x^2-x-2} dx$$

$$\text{Let } 2x+5 = \lambda \frac{d}{dx}(x^2-x-2) + \mu$$

$$= \lambda(2x-1) + \mu$$

$$2x+5 = (2\lambda)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$$

$$-\lambda + \mu = 5 \quad \Rightarrow \quad -1 + \mu = 5$$

$$\mu = 6$$

$$\text{so, } I = \int \frac{(2x-1)+6}{x^2-x-2} dx$$

$$I = \int \frac{(2x-1)}{x^2-x-2} dx + 6 \int \frac{1}{x^2-2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2} dx$$

$$I = \int \frac{2x-1}{x^2-x-2} dx + 6 \int \frac{1}{\left(x-\frac{1}{2}\right)^2 - \frac{9}{4}} dx$$

$$I = \int \frac{2x-1}{x^2-x-2} dx + 6 \int \frac{1}{\left(x-\frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx$$

$$I = \log|x^2-x-2| + \frac{6}{2\left(\frac{3}{2}\right)} \log \left| \frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}} \right| + c \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \log|x^2-x-2| + 2 \log \left| \frac{x-2}{x+1} \right| + c$$

Indefinite Integrals Ex 19.19 Q9

$$\text{Let } I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$$

$$\text{Let } ax^3 + bx = \lambda \frac{d}{dx}(x^4 + c^2) + \mu$$

$$ax^3 + bx = \lambda(4x^3) + \mu$$

Comparing the coefficients of like powers of x

$$4\lambda = a \quad \Rightarrow \quad \lambda = \frac{a}{4}$$

$$\mu = 0 \quad \Rightarrow \quad \mu = 0$$

$$\text{so, } I = \int \frac{\frac{a}{4}(4x^3) + bx}{x^4 + c^2} dx$$

$$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + b \int \frac{x}{(x^2)^2 + c^2} dx$$

$$I = \frac{a}{4} \int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2} \int \frac{2x}{(x^2)^2 + c^2} dx$$

$$= \frac{a}{4} \log|x^4 + c^2| + \frac{b}{2} I_1 \text{ ---- (i)}$$

Now,

$$I_1 = \int \frac{2x}{(x^2)^2 + c^2} dx$$

Put $x^2 = t$

$$\Rightarrow 2x dx = dt$$

$$I_1 = \int \frac{1}{(t)^2 + c^2} dt$$

$$= \frac{1}{c} \tan^{-1} \left(\frac{t}{c} \right) + c_1$$

$$I_1 = \frac{1}{c} \tan^{-1} \left(\frac{x^2}{c} \right) + c_1 \text{ ---- (ii)}$$

Using equation (ii) in equation (i),

$$I = \frac{a}{4} \log|x^4 + c^2| + \frac{b}{2c} \tan^{-1} \left(\frac{x^2}{c} \right) + k$$

k = Integration constant

$$\text{Let } I = \int \frac{x+2}{2x^2+6x+5} dx$$

$$\begin{aligned} \text{Let } x+2 &= \lambda \frac{d}{dx}(2x^2+6x+5) + \mu \\ &= \lambda(4x+6) + \mu \\ x+2 &= (4\lambda)x + (6\lambda + \mu) \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$$

$$6\lambda + \mu = 2 \quad \Rightarrow \quad 6\left(\frac{1}{4}\right) + \mu = 2$$

$$\mu = \frac{1}{2}$$

$$\text{so, } I = \int \frac{\frac{1}{4}(4x+6) + \frac{1}{2}}{2x^2+6x+5} dx$$

$$I = \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{1}{2x^2+6x+5} dx$$

$$I = \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{x^2+3x+\frac{5}{2}} dx$$

$$I = \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{x^2+2x\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{2}} dx$$

$$= \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}} dx$$

$$I = \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \int \frac{1}{\left(x+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dx$$

$$I = \frac{1}{4} \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{4} \times \frac{1}{\frac{1}{2}} \tan^{-1} \left(\frac{x+\frac{3}{2}}{\frac{1}{2}} \right) + c \quad \left[\text{since, } \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \frac{1}{4} \log|2x^2+6x+5| + \frac{1}{2} \tan^{-1}(2x+3) + c$$

Indefinite Integrals Ex 19.19 Q11

$$\text{Let } I = \int \frac{(3 \sin x - 2) \cos x}{5 - \cos^2 x - 4 \sin x} dx$$

$$\therefore I = \int \frac{(3 \sin x - 2) \cos x}{5 - (1 - \sin^2 x) - 4 \sin x} dx$$

$$\Rightarrow I = \int \frac{(3 \sin x - 2) \cos x}{5 - 1 + \sin^2 x - 4 \sin x} dx$$

Substitute $\sin x = t$

$$\Rightarrow \cos x \, dx = dt$$

Thus,

$$I = \int \frac{(3t - 2)}{4 + t^2 - 4t} dt$$

$$\Rightarrow I = \int \frac{(3t - 2)}{t^2 - 4t + 4} dt$$

$$\Rightarrow I = \int \frac{(3t - 2)}{(t - 2)^2} dt$$

Now let us separate the integrand into the simplest form using partial fractions.

$$\frac{(3t - 2)}{(t - 2)^2} = \frac{A}{(t - 2)} + \frac{B}{(t - 2)^2}$$

$$= \frac{A(t - 2) + B}{(t - 2)^2}$$

$$= \frac{At - 2A + B}{(t - 2)^2}$$

$$\Rightarrow 3t - 2 = At - 2A + B$$

Comparing the coefficients, we have,

$$A = 3$$

and

$$-2A + B = -2$$

Substituting the value of $A = 3$ in the above equation, we have,

$$\Rightarrow -2 \times 3 + B = -2$$

$$\Rightarrow -6 + B = -2$$

$$\Rightarrow B = 6 - 2$$

$$\Rightarrow B = 4$$

Thus, $I = \int \frac{(3t - 2)}{(t - 2)^2} dt$ becomes,

$$\begin{aligned} I &= \int \frac{3}{(t - 2)} dt + \int \frac{4}{(t - 2)^2} dt \\ &= 3 \log|t - 2| - 4 \left(\frac{1}{t - 2} \right) + C \\ &= 3 \log|2 - t| + 4 \left(\frac{1}{2 - t} \right) + C \end{aligned}$$

Now substituting $t = \sin x$, we have,

$$I = 3 \log|2 - \sin x| + 4 \left(\frac{1}{2 - \sin x} \right) + C$$

Indefinite Integrals Ex 19.19 Q12

$$\text{Let } I = \int \frac{5x - 2}{1 + 2x + 3x^2} dx$$

Rewriting the numerator we have,

$$5x - 2 = A \frac{d}{dx} (1 + 2x + 3x^2) + B$$

$$\Rightarrow 5x - 2 = A(2 + 6x) + B$$

$$\Rightarrow 5x - 2 = 6xA + 2A + B$$

Comparing the coefficients, we have,

$$6A = 5 \text{ and } 2A + B = -2$$

$$\Rightarrow A = \frac{5}{6}$$

Substituting the value of A in $2A + B = -2$, we have,

$$2 \times \frac{5}{6} + B = -2$$

$$\Rightarrow \frac{10}{6} + B = -2$$

$$\Rightarrow B = -2 - \frac{10}{6}$$

$$\Rightarrow B = \frac{-12 - 10}{6}$$

$$\Rightarrow B = \frac{-22}{6}$$

$$\Rightarrow B = \frac{-11}{3}$$

$$5x - 2 = \frac{5}{6}(2 + 6x) - \frac{11}{3}$$

Thus, $I = \int \frac{5x - 2}{1 + 2x + 3x^2} dx$ becomes,

$$I = \int \frac{\left[\frac{5}{6}(2 + 6x) - \frac{11}{3} \right]}{3x^2 + 2x + 1} dx$$

$$= \frac{5}{6} \int \frac{(2 + 6x)}{3x^2 + 2x + 1} dx - \frac{11}{3} \int \frac{dx}{3x^2 + 2x + 1}$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{3 \times 3} \int \frac{dx}{x^2 + \frac{2}{3}x + \frac{1}{3}} + C$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \int \frac{dx}{x^2 + \frac{2}{3}x + \left(\frac{4}{3}\right)^2 + \frac{1}{3} - \left(\frac{4}{3}\right)^2} + C$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} + C$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \times \frac{1}{\frac{\sqrt{2}}{3}} \tan^{-1} \left[\frac{\left(x + \frac{1}{3}\right)}{\frac{\sqrt{2}}{3}} \right] + C$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{9} \times \frac{3}{\sqrt{2}} \tan^{-1} \left[\frac{\left(\frac{3x + 1}{3}\right)}{\frac{\sqrt{2}}{3}} \right] + C$$

$$= \frac{5}{6} \log(3x^2 + 2x + 1) - \frac{11}{3\sqrt{2}} \tan^{-1} \left[\frac{3x + 1}{\sqrt{2}} \right] + C$$