RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.21

Indefinite Integrals Ex 19.21 Q1

Let $I = \int \frac{x}{\sqrt{x^2 + 6x + 10}} dx$

Let $x = \lambda \frac{d}{dx}\left(x^2 + 6x + 10\right) + \mu$

$= \lambda (2x + 6) + \mu$

$x = (2\lambda) x + 6\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$

$6\lambda + \mu = 0 \Rightarrow 6\left(\frac{1}{2}\right) + \mu = 0$

$\mu = -3$

so, $I_1 = \int \frac{\frac{1}{2}(2x + 6) - 3}{\sqrt{x^2 + 6x + 10}} dx$

$= \frac{1}{2} \int \frac{(2x + 6)}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{x^2 + 2x(3) + (3)^2 - (3)^2 + 10}} dx$

$I_1 = \frac{1}{2} \int \frac{2x + 6}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{(x + 3)^2 + (1)^2}} dx$

$I_1 = \frac{1}{2}\left(2\sqrt{x^2 + 6x + 10}\right) - 3 \log\left|x + 3 + \sqrt{(x + 3)^2 + 1}\right| + c$ $\left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = \sqrt{x^2 + 6x + 10} - 3 \log\left|x + 3 + \sqrt{x^2 + 6x + 10}\right| + c$

Indefinite Integrals Ex 19.21 Q2

Let $I = \int \frac{2x + 1}{\sqrt{x^2 + 2x - 1}} dx$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 + 2x - 1\right) + \mu$

$= \lambda (2x + 2) + \mu$

$2x + 1 = (2\lambda) x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \Rightarrow \lambda = 1$

$2\lambda + \mu = 1 \Rightarrow 2(1) + \mu = 1$

$\mu = -1$

so, $I = \int \frac{(2x + 2) - 1}{\sqrt{x^2 + 2x - 1}} dx$

$= \int \frac{(2x + 2)}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{x^2 + 2x + (1)^2 - (1)^2 - 1}} dx$

$I = \int \frac{2x + 2}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{(x + 1)^2 - (\sqrt{2})^2}} dx$

$I = \left(2\sqrt{x^2 + 2x - 1}\right) - \log\left|(x + 1) + \sqrt{(x + 1)^2 - (\sqrt{2})^2}\right| + c$ $\left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$I = 2\sqrt{x^2 + 2x - 1} - \log\left|x + 1 + \sqrt{x^2 + 2x - 1}\right| + c$

Indefinite Integrals Ex 19.21 Q3

Let $I = \int \frac{x+1}{\sqrt{4+5x-x^2}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(4+5x-x^2\right)+\mu$

$= \lambda\left(5-2x\right)+\mu$

$x = \left(-2\lambda\right)x+5\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$5\lambda+\mu = 1 \quad \Rightarrow \quad 5\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = \frac{7}{2}$

so, $\quad I = \int \frac{-\frac{1}{2}\left(5-2x\right)+\frac{7}{2}}{\sqrt{4+5x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(5-2x\right)}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-5x-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{5}{2}\right)^2-\left(\frac{\sqrt{41}}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{5}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\left(2\sqrt{4+5x-x^2}\right)+\frac{7}{2}\sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{\sqrt{41}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\sqrt{4+5x-x^2}+\frac{7}{2}\sin^{-1}\left(\frac{2x-5}{\sqrt{41}}\right)+c$

Indefinite Integrals Ex 19.21 Q4

Let $I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}}dx$

Let $3x^2-5x+1 = t$

$\left(6x-5\right)dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t}+c$

$I = 2\sqrt{3x^2-5x+1}+c$

Indefinite Integrals Ex 19.21 Q5

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right) + \mu$

$= \lambda\left(-2-2x\right) + \mu$

$3x+1 = \left(-2\lambda\right)x - 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right) + \mu = 1$

$\mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right) - 2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left[\left(\sqrt{6}\right)^2-\left(x+1\right)^2\right]}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

Indefinite Integrals Ex 19.21 Q6

Let $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

Let $x = \lambda \frac{d}{dx}\left(8+x-x^2\right) + \mu$

$= \lambda\left(1-2x\right) + \mu$

$x = \left(-2\lambda\right)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(-\frac{1}{2}\right) + \mu = 0$

$\mu = \frac{1}{2}$

so, $I = \int \frac{-\frac{1}{2}\left(1-2x\right)+\frac{1}{2}}{\sqrt{8+x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-x-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{2}\right)^2-\left(\frac{33}{4}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{33}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\times 2\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{33}}{2}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -\sqrt{8+x-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right) + c$

Indefinite Integrals Ex 19.21 Q7

Let $I = \int \frac{x+2}{\sqrt{x^2+2x-1}}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+2x-1\right) + \mu$

$x+2 = \lambda\left(2x+2\right) + \mu$

$x+2 = \left(2\lambda\right)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda + \mu = 2 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right) + \mu = 2$

$\Rightarrow \quad \mu = 1$

so, $I_1 = \int \frac{\frac{1}{2}\left(2x+2\right)+1}{\sqrt{x^2+2x-1}}dx$

$= \frac{1}{2}\int \frac{\left(2x+2\right)}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2-1}}dx$

$I = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \frac{1}{2}2\sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x},\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+2x-1} + \log\left|x+1+\sqrt{x^2+2x-1}\right| + c$

Indefinite Integrals Ex 19.21 Q8

$$\text{Let } x+2=A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$

$$\Rightarrow x+2=A(2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A=1\Rightarrow A=\frac{1}{2}$$

$$B=2$$

From (1), we obtain

$$(x+2)=\frac{1}{2}(2x)+2$$

$$\text{Then, }\int\frac{x+2}{\sqrt{x^2-1}}dx=\int\frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$

$$=\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx+\int\frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$

$$\text{In }\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx,\text{ let } x^2-1=t\ \Rightarrow\ 2xdx=dt$$

$$\text{In }\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx,\text{ let } x^2-1=t\ \Rightarrow\ 2xdx=dt$$

$$\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx=\frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

$$=\frac{1}{2}\left[2\sqrt{t}\right]$$

$$=\sqrt{t}$$

$$=\sqrt{x^2-1}$$

$$\text{Then, }\int\frac{2}{\sqrt{x^2-1}}dx=2\int\frac{1}{\sqrt{x^2-1}}dx=2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int\frac{x+2}{\sqrt{x^2-1}}dx=\sqrt{x^2-1}+2\log\left|x+\sqrt{x^2-1}\right|+C$$

Indefinite Integrals Ex 19.21 Q9

$$\int \frac{x-1}{\sqrt{x^2+1}}dx =$$

$$\int \frac{x-1}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}dx - \int \frac{1}{\sqrt{x^2+1}}dx$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}\left(2\sqrt{t}\right) - \int \frac{1}{\sqrt{x^2+1}}dx = \sqrt{t} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

$$= \sqrt{x^2+1} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

Indefinite Integrals Ex 19.21 Q10

Let $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x+1\right) + \mu$

$= \lambda\left(2x+1\right) + \mu$

$x = \left(2\lambda\right)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(\frac{1}{2}\right) + \mu = 0$

$\Rightarrow \quad \mu = -\frac{1}{2}$

so, $I = \int \frac{\frac{1}{2}\left(2x+1\right) + \frac{1}{2}}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x+1\right)}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}}dx$

$I = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + c$

Indefinite Integrals Ex 19.21 Q11

Let $I = \int \frac{x+1}{\sqrt{x^2+1}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+1\right) + \mu$

$x+1 = \lambda\left(2x\right) + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(2x\right)+1}{\sqrt{x^2+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x\right)}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+1}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c$

Indefinite Integrals Ex 19.21 Q12

Let $I = \int \frac{2x+5}{\sqrt{x^2+2x+5}}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2+2x+5\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$2x+5 = \left(2\lambda\right)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda+\mu = 5 \quad \Rightarrow \quad 2\left(1\right)+\mu = 5$

$\Rightarrow \quad \mu = 3$

so, $I = \int \frac{\left(2x+2\right)+3}{\sqrt{x^2+2x+5}}dx$

$= \int \frac{\left(2x+3\right)}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2+5}}dx$

$I = \int \frac{2x+3}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{\left(x+1\right)^2+\left(2\right)^2}}dx$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{\left(x+1\right)^2+\left(2\right)^2}\right|+c$ $\left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = 2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$

Indefinite Integrals Ex 19.21 Q13

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$= \lambda\left(-2-2x\right)+\mu$

$3x+1 = \left(-2\lambda\right)x-2\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda+\mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu = 1$

$\Rightarrow \quad \mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2+5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2-\left(x+1\right)^2}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$ $\left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

Indefinite Integrals Ex 19.21 Q14

Let $I = \int \sqrt{\frac{1-x}{1+x}}dx$

$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$

$= \int \frac{1-x}{\sqrt{1-x^2}}dx$

Let $1-x = \lambda \frac{d}{dx}\left(1-x^2\right) + \mu$

$= \lambda\left(-2x\right) + \mu$

$1-x = \left(-2\lambda\right)x + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = -1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(-2x\right)+1}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{\left(-2x\right)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$

$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}\left(x\right) + c\right]$

$I = \sqrt{1-x^2} + \sin^{-1}x + c$

Indefinite Integrals Ex 19.21 Q15

Let $I = \int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+4x+3\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+1 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(1\right) + \mu = 1$

$\Rightarrow \quad \mu = -3$

so, $I = \int \frac{\left(2x+4\right)-3}{\sqrt{x^2+4x+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{\left(x+2\right)^2-1}}dx$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{\left(x+2\right)^2-1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{x^2+4x+3}\right| + c$

Indefinite Integrals Ex 19.21 Q16

Let $I = \int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

Let $2x+3 = \lambda \frac{d}{dx}\left(x^2+4x+5\right)+\mu$

$= \lambda\left(2x+4\right)+\mu$

$2x+3 = \left(2\lambda\right)x+4\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda+\mu = 3 \quad \Rightarrow \quad 4\left(1\right)+\mu = 3$

$\Rightarrow \quad \mu = -1$

so, $I = \int \frac{\left(2x+4\right)-1}{\sqrt{x^2+4x+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{\left(x+2\right)^2+\left(1\right)^2}}dx$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{\left(x+2\right)^2+1}\right|+c$ $\quad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{x^2+4x+5}\right|+c$

Indefinite Integrals Ex 19.21 Q17

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$\rightarrow let\ 5x+3 = \lambda\left(2x+4\right)+\mu$

$\lambda = \frac{5}{2}, \mu = -7$

$$\int \frac{\lambda\left(2x+4\right)+\mu}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}\left(2x+4\right)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}\left(2x+4\right)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}dt}{\sqrt{t}} - \int \frac{7}{\sqrt{\left(x+2\right)^2+6}}dx$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|\left(x+2\right)+\sqrt{x^2+4x+10}\right|+C$$

Indefinite Integrals Ex 19.21 Q18

Let $I=\int \frac{x+2}{\sqrt{x^2+2x+3}}$

$x+2=A\frac{d}{dx}\left[x^2+2x+3\right]+B$

$\Rightarrow x+2=2Ax+2A+B$

Comparing the coefficients, we have,

$2A=1$ and $2A+B=2$

$\Rightarrow A=\frac{1}{2}$

Substituting the value of A in $2A+B=2$, we have,

$2\times\frac{1}{2}+B=2$

$\Rightarrow 1+B=2$

$\Rightarrow B=2-1$

$\Rightarrow B=1$

Thus we have,

$x+2=\frac{1}{2}[2x+2]+1$

Hence,

$$I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx$$

$$=\int \frac{\left[\frac{1}{2}[2x+2]+1\right]}{\sqrt{x^2+2x+3}}dx$$

$$=\int \frac{\left[\frac{1}{2}[2x+2]\right]}{\sqrt{x^2+2x+3}}dx+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$=\frac{1}{2}\int \frac{[2x+2]}{\sqrt{x^2+2x+3}}dx+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

Substituting $t=x^2+2x+3$ and $dt=2x+2$

in the first integrand, we have,

$$I=\frac{1}{2}\int \frac{dt}{\sqrt{t}}+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$=\frac{1}{2}\times 2\sqrt{t}+\int \frac{dx}{\sqrt{x^2+2x+1+2}}+C$$

$$=\sqrt{t}+\int \frac{dx}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}+C$$

$$I=\sqrt{x^2+2x+3}+\log\left[|x+1|+\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}\right]+C$$

$$\Rightarrow I=\sqrt{x^2+2x+3}+\log\left[|x+1|+\sqrt{x^2+2x+3}\right]+C$$