RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.24

Indefinite Integrals Ex 19.24 Q1

Let $I = \int \frac{1}{1 - \cot x} dx$

$= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx$

$= \int \frac{\sin x}{\sin x - \cos x} dx$

Let $\sin x = \lambda \frac{d}{dx}(\sin x - \cos x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \lambda \frac{d}{dx}(\cos x + \sin x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \cos(\lambda - \mu) + \sin x (\lambda + \mu) + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$\lambda + \mu = 1 \quad \ldots(1)$

$\lambda - \mu = 1 \quad \ldots(2)$

$\nu = 0 \quad \ldots(3)$

Equation (1),(2),(3) gives

$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx$

$= \frac{1}{2}\int \frac{(\cos x + \sin x)}{(\sin x - \cos x)} dx + \frac{1}{2}\int dx$

$I = \frac{1}{2}\log|\sin x - \cos x| + \frac{1}{2}x + c$

Indefinite Integrals Ex 19.24 Q2

Let $I = \int \frac{1}{1 - \tan x} dx$

$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx$

$= \int \frac{\cos x}{\cos x - \sin x} dx$

Let $\cos x = \lambda \frac{d}{dx}(\cos x - \sin x) + \mu(\cos x - \sin x) + \nu$

$= \lambda \frac{d}{dx}(-\sin x - \cos x) + \mu(\cos x - \sin x) + \nu$

$\cos x = \sin x(-\lambda - \mu) + \cos x(-\lambda + \mu) + \nu$

Camparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$-\lambda - \mu = 0 ------(1)$

$-\lambda + \mu = 1 ------(2)$

$\nu = 0 ----------(3)$

Equation $(1),(2),(3)$ gives

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)} dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\cos x - \sin x)} dx + \frac{1}{2}\int dx$

$= -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$

$I = \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.24 Q3

Let $\quad I = \int \frac{3 + 2\cos x + 4\sin x}{2\sin x + \cos x + 3} dx$

Let $\quad 3 + 2\cos x + 4\sin x = \lambda \frac{d}{dx}\left(2\sin x + \cos x + 3\right) + \mu\left(2\sin x + \cos x + 3\right) + \nu$

$3 + 2\cos x + 4\sin x = \lambda\left(2\cos x - \sin x\right) + \mu\left(2\sin x + \cos x + 3\right) + \nu$

$3 + 2\cos x + 4\sin x = \left(-\lambda + 2\mu\right)\sin x + \left(2\lambda + \mu\right)\cos x + 3\mu + \nu$

Camparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$-\lambda + 2\mu = 4 - - - - - - - (1)$

$2\lambda + \mu = 2 - - - - - - (2)$

$2\mu + \nu = 3 - - - - - - - - - - (3)$

Solving equation $(1), (2)$ and (3), we get

$\lambda = 0, \mu = 2, \nu = -3$

$I = \int \frac{2\left(2\sin x + \cos x + 3\right) - 3}{\left(2\sin x + \cos x + 3\right)} dx$

$= 2\int dx - 3\int \frac{1}{2\sin x + \cos x + 3} dx$

$I = 2x - 3I_1 + C_1 - - - - - - - (4)$

Let $\quad I_1 = \int \frac{1}{2\sin x + \cos x + 3} dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{\left(1 + \tan^2\frac{x}{2}\right)}$

$$I_1 = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 3} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3\left(1 + \tan^2\frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} = dt$$

$$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$$

$$= \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$= \int \frac{dt}{(t + 1)^2 + 1}$$

$$= \tan^{-1}(t + 1) + C_2$$

$$= \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C_2$$

Now, using equation (1),

$$I = 2x - 3\tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C$$

Indefinite Integrals Ex 19.24 Q4

Let $I = \int \frac{1}{p + q\tan x} dx$

$$= \int \frac{1}{p + q\left(\frac{\sin x}{\cos x}\right)} dx$$

$$= \int \frac{\cos x}{p\cos x + q\sin x} dx$$

Let $\cos x = \lambda \frac{d}{dx}(p\cos x + q\sin x) + \mu(p\cos x + q\sin x) + \nu$

$$\cos x = \lambda(-p\sin x + q\cos x) + \mu(p\cos x + q\sin x) + \nu$$

$$\cos x = (-p\lambda + q\mu)\sin x + (q\lambda + p\mu)\cos x + \nu$$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$-p\lambda + q\mu = 0$ ------ (1)

$q\lambda + p\mu = 1$ ------- (2)

$\nu = 0$ ---------- (3)

Solving equation (1), (2) and (3),

$$\lambda = \frac{q}{(p^2 + q^2)}$$

$$\mu = \frac{p}{(p^2 + q^2)}$$

$$\nu = 0$$

Now,

$$I = \int \frac{q}{(p^2 + q^2)} \frac{(-p\sin x + q\cos x)}{(p\cos x + q\sin x)} dx + \int \frac{p}{(p^2 + q^2)} \frac{(p\cos x + q\sin x)}{(p\cos x + q\sin x)} dx$$

$$I = \frac{q}{(p^2 + q^2)}\left(\log|p\cos x + q\sin x|\right) + \frac{p}{(p^2 + q^2)} x + c$$

Indefinite Integrals Ex 19.24 Q5

Let $I = \int \frac{5\cos x + 6}{2\cos x + \sin x + 3} dx$

Let $(5\cos x + 6) = \lambda \frac{d}{dx}(2\cos x + \sin x + 3) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = \lambda(-2\sin x + \cos x) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = (-2\lambda + \mu)\sin x + (\lambda + 2\mu)\cos x + (3\mu + \nu)$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-2\lambda + \mu = 0 - - - - - - - (1)$

$\lambda + 2\mu = 5 - - - - - - - - (2)$

$3\mu + \nu = 6 - - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

Now,

$$I = \int \frac{(-2\sin x + \cos x)}{(2\cos x + \sin x + 3)} dx + 2\int dx$$

$$I = \log|2\cos x + \sin x + 3| + 2x + c$$

Indefinite Integrals Ex 19.24 Q6

Let $I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $(2\sin x + 3\cos x) = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$$(2\sin x + 3\cos x) = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$$

$$(2\sin x + 3\cos x) = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 ------- (1)$$

$$-4\lambda + 3\mu = 2 ------ (2)$$

$$\nu = 0 -------- (3)$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{1}{25}$$

$$\mu = \frac{18}{25}$$

$$\nu = 0$$

$$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)}dx + \frac{18}{25}\int dx$$

$$I = \frac{1}{25}\log|3\sin x + 4\cos x| + \frac{18}{25}x + c$$

Indefinite Integrals Ex 19.24 Q7

Let $I = \int \frac{1}{3 + 4\cot x} dx$

$= \int \frac{\sin x}{3\sin x + 4\cos x} dx$

Let $\sin x = \lambda \frac{d}{dx}\left(3\sin x + 4\cos x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\sin x = \lambda\left(3\cos x - 4\sin x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$\sin x = \left(3\lambda + 4\mu\right)\cos x + \left(-4\lambda + 3\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 0 - - - - - - - (1)$

$-4\lambda + 3\mu = 1 - - - - - - (2)$

$\nu = 0 - - - - - - - - (3)$

Solving the equation $(1), (2)$ and (3), we get

$\lambda = -\frac{4}{25}$

$\mu = \frac{3}{25}$

$\nu = 0$

$I = -\frac{4}{25}\int \frac{\left(3\cos x - 4\sin x\right)}{\left(3\sin x + 4\cos x\right)} dx + \frac{3}{25}\int dx$

$I = -\frac{4}{25}\log\left|3\sin x + 4\cos x\right| + \frac{3}{25}x + c$

Indefinite Integrals Ex 19.24 Q8

Let $I = \int \frac{2\tan x + 3}{3\tan x + 4} dx$

$= \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $2\sin x + 3\cos x = \lambda \frac{d}{dx}\left(3\sin x + 4\cos x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$2\sin x + 3\cos x = \lambda\left(3\cos x - 4\sin x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$2\sin x + 3\cos x = \left(3\lambda + 4\mu\right)\cos x + \left(-4\lambda + 3\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 3 - - - - - - - (1)$

$-4\lambda + 3\mu = 2 - - - - - (2)$

$\nu = 0$

Solving the equation $(1), (2)$ and (3),

$\mu = \frac{18}{25}$

$\lambda = \frac{1}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{\left(3\cos x - 4\sin x\right)}{\left(3\sin x + 4\cos x\right)} dx + \frac{18}{25}\int dx$

$I = \frac{18}{25}x + \frac{1}{25}\log\left|3\sin x + 4\cos x\right| + c$

Indefinite Integrals Ex 19.24 Q9

Let $I = \int \frac{1}{4+3\tan x} dx$

$I = \int \frac{\cos x}{4\cos x + 3\sin x} dx$

Let $\cos x = \lambda \frac{d}{dx}(4\cos x + 3\sin x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = \lambda(-4\sin x + 3\cos x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = (-4\lambda + 3\mu)\sin x + (3\lambda + 4\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 3\mu = 0 ------(1)$

$3\lambda + 4\mu = 1 -------(2)$

$\nu = 0 --------(3)$

Solving the equation (1), (2) and (3),

$\lambda = \frac{3}{25}$

$\mu = \frac{4}{25}$

$\nu = 0$

$I = \int \frac{3}{25} \frac{(-4\sin x + 3\cos x)}{(4\cos x + 3\sin x)} dx + \frac{4}{25}\int dx$

$I = \frac{3}{25}\log|4\cos x + 3\sin x| + \frac{4}{25}x + c$

Indefinite Integrals Ex 19.24 Q10

Let $I = \int \frac{8\cot x + 1}{3\cot x + 2} dx$

$I = \int \frac{8\cos x + \sin x}{3\cos x + 2\sin x} dx$

Let $8\cos x + \sin x = \lambda \frac{d}{dx}\left(3\cos x + 2\sin x\right) + \mu\left(3\cos x + 2\sin x\right) + \nu$

$8\cos x + \sin x = \lambda\left(-3\sin x + 2\cos x\right) + \mu\left(3\cos x + 2\sin x\right) + \nu$

$8\cos x + \sin x = \left(-3\lambda + 2\mu\right)\sin x + \left(2\lambda + 3\mu\right)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$2\lambda + 3\mu = 8 - - - - - (1)$

$-3\lambda + 2\mu = 1 - - - - - - (2)$

$\nu = 0 - - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

$I = \int \frac{\left(-3\sin x + 2\cos x\right)}{\left(3\cos x + 2\sin x\right)} dx + 2\int dx$

$I = \log\left|3\cos x + 2\sin x\right| + 2x + c$

Indefinite Integrals Ex 19.24 Q11

Let $I = \int \frac{4\sin x + 5\cos x}{5\sin x + 4\cos x} dx$

Let $4\sin x + 5\cos x = \lambda \frac{d}{dx}\left(5\sin x + 4\cos x\right) + \mu\left(5\sin x + 4\cos x\right) + \nu$

$4\sin x + 5\cos x = \lambda\left(5\cos x - 4\sin x\right) + \mu\left(5\sin x + 4\cos x\right) + \nu$

$4\sin x + 5\cos x = \left(5\lambda + 4\mu\right)\cos x + \left(-4\lambda + 5\mu\right)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 5\mu = 4 ------ (1)$

$5\lambda + 4\mu = 5 ------- (2)$

$\nu = 0 -------- (3)$

Solving equation $(1),(2)$ and (3),

$\lambda = \frac{9}{41}$

$\mu = \frac{40}{41}$

$\nu = 0$

Now,

$I = \frac{40}{41}\int dx + \frac{9}{41}\int \frac{\left(5\cos x - 4\sin x\right)}{\left(5\sin x + 4\cos x\right)} dx$

$I = \frac{40}{41}x + \frac{9}{41}\log\left|5\sin x + 4\cos x\right| + c$