

RD Sharma
Solutions Class
12 Maths
Chapter 19
Ex 19.26

Indefinite Integrals Ex 19.26 Q1

$$\begin{aligned} \text{Let } I &= \int e^x (\cos x - \sin x) dx \\ &= \int e^x \cos x dx - \int e^x \sin x dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \cos x - \int e^x \left(\frac{d}{dx} \cos x \right) dx - \int e^x \sin x dx \\ &= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx \\ &= e^x \cos x + c \end{aligned}$$

$$\therefore \int e^x (\cos x - \sin x) dx = e^x \cos x + c$$

Indefinite Integrals Ex 19.26 Q2

$$\begin{aligned} I &= \int e^x (x^{-2} - 2x^{-3}) dx \\ &= \int e^x x^{-2} dx - 2 \int e^x x^{-3} dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x x^{-2} - \int e^x \left(\frac{d}{dx} (x^{-2}) \right) dx - 2 \int e^x x^{-3} dx \\ &= e^x x^{-2} + 2 \int e^x x^{-3} dx - 2 \int e^x x^{-3} dx \\ &= \frac{e^x}{x^2} + c \end{aligned}$$

$$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3} \right) dx = \frac{e^x}{x^2} + c$$

Indefinite Integrals Ex 19.26 Q3

$$\begin{aligned}
& e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) \\
&= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right) \\
&= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}} \\
&= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2 \\
&= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2 \\
&= \frac{1}{2} e^2 \left(1 + \tan \frac{x}{2} \right)^2 \\
&= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&= \frac{1}{2} e^x \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right] \\
&\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \quad \dots(1)
\end{aligned}$$

$$\text{Let } \tan \frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x (1 + \sin x)}{(1 + \cos x)} dx = e^x \tan \frac{x}{2} + C$$

$$\text{Let } I = \int e^x (\cot x - \operatorname{cosec}^2 x) dx$$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Integrating by parts

$$= e^x \cot x - \int e^x \left(\frac{d}{dx} \cot x \right) dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + c$$

$$\int e^x (\cot x - \operatorname{cosec}^2 x) dx = e^x \cot x + c$$

Indefinite Integrals Ex 19.26 Q5

$$\text{Let } I = \int e^x \frac{1}{2x} dx - \int e^x \frac{1}{2x^2} dx$$

Integrating by parts

$$= \frac{e^x}{2x} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{2x} \right) \right) dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + \int \frac{e^x}{2x^2} dx - \int \frac{e^x}{2x^2} dx$$

$$= \frac{e^x}{2x} + c$$

Indefinite Integrals Ex 19.26 Q7

$$\begin{aligned}\text{Let } I &= \int e^x (\tan x - \log \cos x) dx \\ &= \int e^x \tan x dx - \int e^x \log \cos x dx\end{aligned}$$

Integrating by parts

$$\begin{aligned}&= \int e^x \tan x dx - \left\{ e^x \log \cos x - \int e^x \left(\frac{d}{dx} \log \cos x \right) dx \right\} \\ &= \int e^x \tan x dx - \left\{ e^x \log \cos x + \int e^x \tan x dx \right\} \\ &= \int e^x \tan x dx - e^x \log \cos x - \int e^x \tan x dx + c \\ &= -e^x \log \cos x + c \\ &= e^x \log \sec x + c \quad \left[\because \log \sec x = -\log \cos x \right]\end{aligned}$$

Indefinite Integrals Ex 19.26 Q8

$$\text{Let } I = \int e^x [\sec x + \log(\sec x + \tan x)] dx$$

$$= \int e^x \sec x dx + \int e^x \log(\sec x + \tan x) dx$$

Integrating by parts

$$= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \left\{ \frac{d}{dx} \log(\sec x + \tan x) \right\} dx$$

$$= \int e^x \sec x dx + e^x \log(\sec x + \tan x) - \int e^x \sec x dx$$

$$= e^x \log(\sec x + \tan x) + c$$

Indefinite Integrals Ex 19.26 Q9

$$\text{Let } I = \int e^x (\cot x + \log \sin x) dx$$

$$= \int e^x \cot x dx + \int e^x \log \sin x dx$$

Integrating by parts

$$= \int e^x \log \sin x dx + \int e^x \cot x dx$$

$$= (\log \sin x) e^x - \int e^x \left(\frac{d}{dx} \log \sin x \right) dx + \int e^x \cot x dx$$

$$= e^x \log \sin x - \int e^x \cot x dx + \int e^x \cot x dx$$

$$= e^x \log \sin x + c$$

Indefinite Integrals Ex 19.26 Q10

$$\begin{aligned}
 \text{Let } I &= \int e^x \frac{x+1-2}{(x+1)^3} \\
 &= \int e^x \left\{ \frac{1}{(x+1)^2} + \frac{-2}{(x+1)^3} \right\} dx \\
 &= \int e^x \frac{1}{(x+1)^2} dx + \int e^x \frac{(-2)}{(x+1)^3} dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x \frac{1}{(x+1)^2} - \int e^x \left(\frac{d}{dx} (x+1)^{-2} \right) dx + \int e^x \frac{(-2)}{(x+1)^3} dx \\
 &= e^x \frac{1}{(x+1)^2} - \int e^x \frac{(-2)}{(x+1)^3} dx + \int e^x \frac{(-2)}{(x+1)^3} dx \\
 &= e^x \frac{1}{(x+1)^2} + c
 \end{aligned}$$

Indefinite Integrals Ex 19.26 Q11

$$\begin{aligned}
 \text{Let } I &= \int e^x \left(\frac{\sin 4x - 4}{2 \sin^2 2x} \right) dx \\
 &= \int e^x \left\{ \frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x} \right\} dx \\
 &= \int e^x \left(\cot 2x - 2 \operatorname{cosec}^2 2x \right) dx \\
 &= \int e^x \cot 2x dx - 2 \int e^x \operatorname{cosec}^2 2x dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x \cot 2x - \int e^x \frac{d}{dx} (\cot 2x) dx - 2 \int e^x \operatorname{cosec}^2 2x dx \\
 &= e^x \cot 2x + 2 \int e^x \operatorname{cosec}^2 2x - 2 \int e^x \operatorname{cosec}^2 2x dx \\
 &= e^x \cot 2x + c
 \end{aligned}$$

Indefinite Integrals Ex 19.26 Q12

$$\begin{aligned} \text{Let } I &= \int \frac{2-x}{(1-x)^2} e^x dx \\ &= \int e^x \left\{ \frac{(1-x)+1}{(1-x)^2} \right\} dx \\ &= \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx \end{aligned}$$

$$\text{Here, } f(x) = \frac{1}{1-x} \text{ and } f'(x) = \frac{1}{(1-x)^2}$$

And we know that,

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\therefore \int e^x \left\{ \frac{1}{1-x} + \frac{1}{(1-x)^2} \right\} dx = e^x \cdot \frac{1}{1-x} + c$$

Hence,

$$I = \frac{e^x}{1-x} + c$$

$$\begin{aligned}
 \text{Let } I &= \int e^x \frac{1+x}{(2+x)^2} dx \\
 &= \int e^x \left(\frac{x+2-1}{(2+x)^2} \right) dx \\
 &= \int e^x \left\{ \frac{1}{x+2} - \frac{1}{(x+2)^2} \right\} dx \\
 &= \int e^x \frac{1}{x+2} dx - \int e^x \frac{1}{(x+2)^2} dx
 \end{aligned}$$

Integrating by parts

$$\begin{aligned}
 &= e^x \frac{1}{x+2} - \int e^x \left(\frac{d}{dx} \left(\frac{1}{x+2} \right) \right) dx - \int e^x \frac{1}{(x+2)^2} dx \\
 &= e^x \frac{1}{x+2} + \int e^x \frac{1}{(x+2)^2} dx - \int e^x \frac{1}{(x+2)^2} dx \\
 &= \frac{e^x}{x+2} + c
 \end{aligned}$$

$$\text{Let } I = \int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} dx$$

$$\text{Put } \frac{x}{2} = t$$

$$\Rightarrow x = 2t$$

$$dx = 2dt$$

$$\therefore \int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} dx$$

$$= 2 \int \frac{\sqrt{1 - \sin 2t}}{1 + \cos 2t} e^{-t} dt \quad [\because \sin^2 t + \cos^2 t = 1]$$

$$= 2 \int \frac{\sqrt{\sin^2 t + \cos^2 t - 2 \sin t \cos t}}{1 + \cos 2t} e^{-t} dt$$

$$= 2 \int \frac{\sqrt{(\cos t - \sin t)^2}}{2 \cos^2 t} e^{-t} dt$$

$$= 2 \int \frac{(\cos t - \sin t)}{2 \cos^2 t} e^{-t} dt$$

$$= \int (\sec t - \tan t \sec t) e^{-t} dt$$

$$= \int \sec t e^{-t} dt - \int \tan t \sec t e^{-t} dt$$

Integrating by parts

$$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt} (\sec t) dt - \int \tan t \sec t e^{-t} dt$$

$$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t e^{-t} dt$$

$$= -e^{-t} \sec t + c$$

Putting the value of t

$$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$$

We have,

$$I = \int e^x \left(\log x + \frac{1}{x} \right) dx$$

We know that

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\text{Here } f(x) = \log x \text{ and } f'(x) = \frac{1}{x}$$

$$\therefore \int e^x \left(\log x + \frac{1}{x} \right) dx = e^x \log x + c$$

Indefinite Integrals Ex 19.26 Q16

We have,

$$\begin{aligned} I &= \int e^x \left(\log x + \frac{1}{x^2} \right) dx \\ &= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= \int e^x \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \frac{d}{dx} \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= e^x \left(\log x - \frac{1}{x} \right) + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q18

$$\text{Let } I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$I = \int e^x \sin^{-1} x + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

Integrating by parts

$$= e^x \sin^{-1} x - \int e^x \left(\frac{d}{dx} (\sin^{-1} x) \right) dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

$$= e^x \sin^{-1} x - \int e^x \frac{1}{\sqrt{1-x^2}} dx + \int e^x \frac{1}{\sqrt{1-x^2}} dx$$

$$= e^x \sin^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q19

$$\begin{aligned}\text{Let } I &= \int e^{2x} (-\sin x + 2 \cos x) dx \\ &= -\int e^{2x} \sin x dx + 2\int e^{2x} \cos x dx\end{aligned}$$

Applying by parts in the 2nd integrand

$$\begin{aligned}\therefore I &= -\int e^{2x} \sin x dx + 2 \left\{ \frac{1}{2} e^{2x} \cos x + \int \frac{1}{2} e^{2x} \sin x dx \right\} \\ &= -\int e^{2x} \sin x dx + e^{2x} \cos x + \int e^{2x} \sin x dx + c \\ &= e^{2x} \cos x + c\end{aligned}$$

Thus,

$$I = e^{2x} \cos x + c$$

Indefinite Integrals Ex 19.26 Q20

$$\text{Let } I = \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$$

$$\text{Here, } f(x) = \tan^{-1} x \text{ and } f'(x) = \frac{1}{1+x^2}$$

And we know that,

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\therefore \int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx = e^x \tan^{-1} x + c$$

Thus,

$$I = e^x \tan^{-1} x + c$$

Indefinite Integrals Ex 19.26 Q21

$$\begin{aligned}
 \text{Let } I &= \int e^x \left(\frac{\sin x \cos x - 1}{\sin^2 x} \right) dx \\
 &= \int e^x (\cot x - \operatorname{cosec}^2 x) dx \\
 &= \int e^x (\cot x + (-\operatorname{cosec}^2 x)) dx
 \end{aligned}$$

$$\therefore \int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\text{Here } f(x) = \cot x$$

$$\Rightarrow f'(x) = -\operatorname{cosec}^2 x$$

$$\therefore \int e^x (\cot x - \operatorname{cosec}^2 x) dx = e^x \cot x + c$$

Thus,

$$I = e^x \cot x + c$$

Indefinite Integrals Ex 19.26 Q23

$$\text{Let } I = \int \frac{e^x (x - 4)}{(x - 2)^3} dx$$

$$= \int e^x \left\{ \frac{(x - 2) - 2}{(x - 2)^3} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{(x - 2)^2} - \frac{2}{(x - 2)^3} \right\} dx$$

$$\text{Here, } f(x) = \frac{1}{(x - 2)^2} \text{ and } f'(x) = \frac{-2}{(x - 2)^3}$$

And we know that,

$$\int e^{ax} (af(x) + f'(x)) dx = e^{ax} f(x) + c$$

$$\therefore \int e^x \left\{ \frac{1}{(x - 2)^2} - \frac{2}{(x - 2)^3} \right\} dx = \frac{e^x}{(x - 2)^2} + c$$

$$\therefore I = \frac{e^x}{(x - 2)^2} + c$$

$$\text{Let } I = \int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

We have, $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned} I &= \int e^{2x} \left(\frac{1 - \sin 2x}{1 - (1 - 2\sin^2 x)} \right) dx \\ &= \int e^{2x} \left(\frac{1 - \sin 2x}{2\sin^2 x} \right) dx \\ &= \int e^{2x} \left(\frac{\operatorname{cosec}^2 x}{2} - \frac{2\sin x \cos x}{2\sin^2 x} \right) dx \\ &= \int e^{2x} \left(\frac{\operatorname{cosec}^2 x}{2} - \frac{\cos x}{\sin x} \right) dx \\ &= \int e^{2x} \left(\frac{\operatorname{cosec}^2 x}{2} - \cot x \right) dx \\ &= \frac{1}{2} \int e^{2x} \operatorname{cosec}^2 x dx - \int e^{2x} \cot x dx \end{aligned}$$

That is

$$I = I_1 + I_2, \text{ where, } I_1 = \frac{1}{2} \int e^{2x} \operatorname{cosec}^2 x dx \text{ and } I_2 = - \int e^{2x} \cot x dx$$

$$\text{Consider } I_1 = \frac{1}{2} \int e^{2x} \operatorname{cosec}^2 x dx$$

Take e^{2x} as the first function and $\operatorname{cosec}^2 x$ as the second function.

$$\text{So, } u = e^{2x}; \quad du = 2e^{2x} dx$$

and

$$\int \operatorname{cosec}^2 x dx = \int dv$$

$$\Rightarrow v = -\cot x$$

$$I_1 = \frac{1}{2} \left[e^{2x} (-\cot x) - \int (-\cot x) 2e^{2x} dx \right]$$

$$\Rightarrow I_1 = \frac{1}{2} \left[e^{2x} (-\cot x) + 2 \int \cot x e^{2x} dx \right]$$

$$\Rightarrow I_1 = \frac{1}{2} \left[e^{2x} (-\cot x) \right] + \int \cot x e^{2x} dx$$

Thus,

$$I = \frac{1}{2} \left[e^{2x} (-\cot x) \right] + \int \cot x e^{2x} dx - \int e^{2x} \cot x dx$$

$$= I = \frac{1}{2} \left[e^{2x} (-\cot x) \right] + C$$