RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.27

Indefinite Integrals Ex 19.27 Q1

Let $I = \int e^{ax} \cos bx dx$

Intergrating by parts,

$$I = e^{ax} \frac{\sin bx}{b} - a\int e^{ax} \frac{\sin bx}{b} fx$$

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx$$

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b}\left[-e^{ax} \frac{\cos bx}{b} + \int ae^{ax} \frac{\cos bx}{b} dx\right]$$

$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{b^2}\left[b \sin bx + a \cos bx\right] - \frac{a^2}{b^2} I + c$$

$$\Rightarrow \qquad I.\left\{\frac{a^2 + b^2}{b^2}\right\} = \frac{e^{ax}}{b^2}\left[b \cos bx + a \cos bx\right] + c$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2}\left[a \cos bx + a \cos bx\right] + c$$

Indefinite Integrals Ex 19.27 Q2

Let $I = \int e^{ax} \sin(bx + c)\, dx$

$$\Rightarrow \quad -e^{ax} \frac{\cos(bx + c)}{b} + \int a e^{ax} \frac{\cos(bx + c)}{b} dx$$

$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c)\, dx$$

$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b}\left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int a e^{ax} \frac{\sin(bx + c)}{b} dx\right] + c_1$$

$$= \frac{e^{ax}}{b^2}\{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c)\, dx + c_1$$

$$\Rightarrow \quad I = \frac{e^{ax}}{b^2}\{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} I + c_1$$

$$\Rightarrow \quad I = \left\{\frac{a^2 + b^2}{b^2}\right\} - \frac{e^{ax}}{b^2}\{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

$$\Rightarrow \quad I = \frac{e^{ax}}{a^2 + b^2}\{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad dx = xdt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int \cos(\log x)dx = \int e^t \cos t dt$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos bx + b \sin bx\} + c$$

Here $a = 1$, $b = 1$

So, $I = \frac{e^t}{2}\{\cos t + \sin t\} + c$

Hence,

$$I = \int \cos(\log x)dx = \frac{e^{\log x}}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow \quad I = \frac{x}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

Let $I = \int e^{2x} \cos(3x + 4)dx$

Integrating by parts

$$I = e^{2x}\frac{\sin(3x + 4)}{3} - \int 2e^{2x}\frac{\sin(3x + 4)}{3}dx$$

$$= \frac{1}{3}e^{2x} \sin(3x + 4) - \frac{2}{3}\int e^{2x} \sin(3x + 4)dx$$

$$= \frac{1}{3}e^{2x} \sin(3x + 4) - \frac{2}{3}\left[-e^{2x}\frac{\cos(3x + 4)}{3} + \int 2e^{2x}\frac{\cos(3x + 4)}{3}dx\right] + c$$

$$I = \frac{e^{2x}}{9}\{2\cos(3x + 4) + 3\sin(3x + 4)\} + c$$

Hence,

$$I = \frac{e^{2x}}{13}\{2\cos(3x + 4) + 3\sin(3x + 4)\} + c$$

Indefinite Integrals Ex 19.27 Q5

Let $I = \int e^{2x} \sin x \cos x dx$

$$= \frac{1}{2}\int e^{2x} 2 \sin x \cos x dx$$
$$= \frac{1}{2}\int e^{2x} \sin 2x dx$$

We know that

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2}\left\{a \sin bx - b \cos bx\right\} + c$$

$$\Rightarrow \quad \int e^{2x} \sin 2x dx = \frac{e^{2x}}{8}\left\{2 \sin 2x - 2 \cos 2x\right\} + c$$

$$\therefore \quad I = \frac{1}{2} \cdot \frac{e^{2x}}{8}\left\{2 \sin 2x - 2 \cos 2x\right\} + c$$

$$\therefore \quad I = \frac{e^{2x}}{8}\left\{\sin 2x - \cos 2x\right\} + c$$

Indefinite Integrals Ex 19.27 Q6

Let $I = \int e^{2x} \sin x \, dx$ ...(1)

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \qquad \text{[From (1)]}$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} \left[2 \sin x - \cos x \right] + C$$

Indefinite Integrals Ex 19.27 Q8

Let $I = \int e^x \sin^2 x dx$

$$= \frac{1}{2}\int e^x 2\sin^2 x dx$$
$$= \frac{1}{2}\int e^x (1 - \cos 2x) dx$$
$$= \frac{1}{2}\int e^x dx - \frac{1}{2}\int e^x \cos 2x dx$$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\{a\cos bx - b\sin bx\} + c$

$\therefore \quad I = \frac{1}{2}\left[e^x - \frac{e^x}{5}\{\cos 2x + 2\sin 2x\}\right] + c$

$\therefore \quad I = \frac{e^x}{2} - \frac{e^x}{10}\{\cos 2x + 2\sin 2x\} + c$

Indefinite Integrals Ex 19.27 Q9

Let $I = \int \frac{1}{x^3} \sin(\log x) dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int e^{-2t} \sin t dt$

We know that,

$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2}\{a\sin bx - b\cos bx\} + c$

$\therefore \quad \int e^{-2t} \sin t dt = \frac{e^{-2t}}{5}\{-2\sin t - \cos t\} + c$

$\therefore \quad I = \frac{x^{-2}}{5}\{-2\sin(\log x) - \cos(\log x)\} + c$

Hence,

$\int \frac{1}{x^3} \sin(\log x) dx = \frac{-1}{5x^2}\{2\sin(\log x) + \cos(\log x)\} + c$

Indefinite Integrals Ex 19.27 Q10

Let $I = \int e^{2x} \cos^2 x dx$

$$= \frac{1}{2}\int e^{2x} 2\cos^2 x dx$$
$$= \frac{1}{2}\int e^{2x} \left(1 + \cos 2x\right) dx$$
$$= \frac{1}{2}\int e^{2x} dx + \frac{1}{2}\int e^{2x} \cos 2x dx$$

$$\because \qquad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\left\{a \cos bx - b \sin bx\right\} + c$$

$$\therefore \qquad I = \frac{1}{4}e^{2x} + \frac{1}{2}\frac{e^{2x}}{8}\left\{2\cos 2x + 2\sin 2x\right\} + c$$

Hence,

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{16}\left\{2\cos 2x + 2\sin 2x\right\} + c$$

or

$$I = \frac{e^{2x}}{4} + \frac{e^{2x}}{8}\left\{\cos 2x + \sin 2x\right\} + c$$

Indefinite Integrals Ex 19.27 Q11

Let $I = \int e^{-2x} \sin x dx$

$$\because \qquad \int e^{ax} \sin bx = \frac{e^{ax}}{a^2 + b^2}\left\{a \sin bx - b \cos bx\right\} + c$$

$$\therefore \qquad I = \frac{e^{-2x}}{5}\left\{-2\sin x - \cos x\right\} + c$$

Indefinite Integrals Ex 19.27 Q12

Let $I = \int x^2 e^{x^3} \cos x^3 dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\therefore \quad I = \frac{1}{3}\int e^t \cos t dt$

$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2}\{a \cos bx + b \sin bx\} + c$

$\therefore \quad I = \frac{1}{3}\left\{\frac{e^t}{2}(\cos t + \sin t)\right\} + c$

$\therefore \quad I = \frac{1}{3}\left\{\frac{e^{x^3}}{2}\left(\cos x^3 + \sin x^3\right)\right\} + c$