RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.28

Indefinite Integrals Ex 19.28 Q1

$$\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-(x-1)^2}\ dx$$

Let $x - 1 = t$, so that $dx = dt$

Thus, $\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-t^2}\ dt$

$$= \frac{1}{2} t\sqrt{4-t^2} + \frac{4}{2}\sin^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$$

Indefinite Integrals Ex 19.28 Q2

Let $I = \int \sqrt{x^2+x+1}dx$

$$= \int \sqrt{x^2+x+\frac{1}{4}+\frac{3}{4}}dx$$

$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}.\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$$

$$= \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(\frac{2x+1}{2}\right)+\frac{1}{2}\sqrt{x^2+x+1}\right| + c$$

$$\therefore \quad I = \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|2x+1+\sqrt{x^2+x+1}\right| + c$$

Indefinite Integrals Ex 19.28 Q3

Let $I = \int \sqrt{x - x^2}dx$

$$= \int \sqrt{\frac{1}{4} - \frac{1}{4} + x - x^2}dx \qquad \left[\text{Add and subtract } \frac{1}{4}\right]$$

$$= \int \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2} - x\right)^2}dx$$

$$= -\left(\frac{1-2x}{4}\right)\sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2} - x\right)^2} - \frac{\left(\frac{1}{2}\right)^2}{2}\sin^{-1}\left(\frac{\frac{1-2x}{2}}{\frac{1}{2}}\right) + c$$

$$\therefore \quad I = \left(\frac{2x-1}{4}\right)\sqrt{x - x^2} + \frac{1}{8}\sin^{-1}(2x-1) + c$$

Indefinite Integrals Ex 19.28 Q4

Let $I = \int \sqrt{1 + x - 2x^2}dx$

$$= \sqrt{2}\int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2}dx$$

$$= \sqrt{2}\int \sqrt{\frac{9}{16} - \left(\frac{1}{16} - \frac{x}{2} + x^2\right)}dx$$

$$= \sqrt{2}\int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{\left(x - \frac{1}{4}\right)}{2}\sqrt{\frac{1}{2} + \frac{x}{2} - x^2} + \frac{9}{32}\sin^{-1}\left(\frac{x - \frac{1}{4}}{\frac{3}{4}}\right)\right\} + c$$

$$I = \frac{1}{8}(4x-1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x-1}{3}\right) + c$$

Indefinite Integrals Ex 19.28 Q5

Let $I = \int \cos x\sqrt{4 - \sin^2 x}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x dx = dt$

$\Rightarrow \quad I = \int \sqrt{4 - t^2}dt$

$= \int \sqrt{2^2 - t^2}dt$

$= \frac{t}{2}\sqrt{2^2 - t^2} + \frac{4}{2}\sin^{-1}\frac{t}{2} + c$

$\therefore \quad I = \frac{1}{2}\sin x\sqrt{4 - \sin^2 x} + 2\sin^{-1}\left(\frac{\sin x}{2}\right) + c$

Indefinite Integrals Ex 19.28 Q6

Let $I = \int e^x\sqrt{e^{2x} + 1}dx$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$\therefore \quad I = \int \sqrt{t^2 + 1^2}dt$

$= \frac{t}{2}\sqrt{t^2 + 1} + \frac{1}{2}\log\left|t + \sqrt{t^2 + 1}\right| + c$

$\therefore \quad I = \frac{e^x}{2}\sqrt{e^{2x} + 1} + \frac{1}{2}\log\left|e^x + \sqrt{e^{2x} + 1}\right| + c$

Indefinite Integrals Ex 19.28 Q7

Let $I = \int \sqrt{3^2 - x^2}$

We know that,

$\int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

$\therefore \quad I = \frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + c$

Indefinite Integrals Ex 19.28 Q8

Let $I = \int \sqrt{16x^2 + 25}dx$

$$= \int \sqrt{(4x)^2 + 5^2}dx$$

$$= 4\int \sqrt{x^2 + \left(\frac{5}{4}\right)^2}dx$$

$$= 4\left\{\frac{x}{2}\sqrt{x^2 + \left(\frac{5}{4}\right)^2} + \frac{\left(\frac{5}{4}\right)^2}{2}\log\left|x + \sqrt{x^2 + \left(\frac{5}{4}\right)^2}\right| + c\right\}$$

$$\therefore \quad I = 2x\sqrt{x^2 + \frac{25}{16}} + \frac{25}{8}\log\left|x + \sqrt{x^2 + \frac{25}{16}}\right| + c$$

Indefinite Integrals Ex 19.28 Q9

Let $I = \int \sqrt{4x^2 - 5}dx$

$$= 2\int \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}dx$$

$$= 2\left\{\frac{x}{2}\sqrt{x^2 - \frac{5}{4}} - \frac{5}{8}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c\right.$$

$$\therefore \quad I = x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$$

Indefinite Integrals Ex 19.28 Q10

Let $I = \int \sqrt{2x^2 + 3x + 4}dx$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + 2}dx$$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + \frac{9}{16} + \frac{23}{16}}dx$$

$$= \sqrt{2}\int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{\left(x + \frac{3}{4}\right)}{2}\sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c\right\}$$

$$\therefore \quad I = \frac{4x + 3}{8}\sqrt{2x^2 + 3x + 4} + \frac{23\sqrt{2}}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c$$

Indefinite Integrals Ex 19.28 Q11

Let $I = \int \sqrt{3 - 2x - 2x^2}dx$

$$= \sqrt{2}\int \sqrt{\frac{3}{2} - x - x^2}dx$$

$$= \sqrt{2}\int \sqrt{\frac{7}{4} - \left(\frac{1}{4} + x + x^2\right)}dx \qquad \left[\text{Adding and subtracting } \frac{1}{4}\right]$$

$$= \sqrt{2}\int \sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{x + \frac{1}{2}}{2}\sqrt{\frac{3}{2} - x - x^2} + \frac{7}{8}\sin^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{7}}{2}}\right) + c\right.$$

$$\therefore \quad I = \frac{2x + 1}{4}\sqrt{3 - 2x - 2x^2} + \frac{7\sqrt{2}}{8}\sin^{-1}\left(\frac{2x + 1}{\sqrt{7}}\right) + c$$

Indefinite Integrals Ex 19.28 Q12

Let $x^2 = t$

$\Rightarrow \quad 2xdx = dt$

$\therefore \quad I = \frac{1}{2}\int \sqrt{t^2 + 1^2}dt$

$= \frac{1}{2}\left\{\frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\log\left|t + \sqrt{t^2+1}\right\}\right. + c$

$\therefore \quad I = \frac{1}{2}\left\{\frac{x^2}{2}\sqrt{x^4+1} + \frac{1}{2}\log\left|x^2 + \sqrt{x^4+1}\right|\right\} + c$

Indefinite Integrals Ex 19.28 Q13

Let $I = \int x^2\sqrt{a^6 - x^6}dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2dx = dt$

$\therefore \quad I = \frac{1}{3}\int \sqrt{a^6 - t^2}dt$

$= \frac{1}{3}\left\{\frac{t}{2}\sqrt{a^6 - t^2} + \frac{a^6}{2}\sin^{-1}\left(\frac{t}{a^3}\right)\right\} + c$

$\therefore \quad I = \frac{x^3}{6}\sqrt{a^6 - x^6} + \frac{a^6}{6}\sin^{-1}\left(\frac{x^3}{a^3}\right) + c$

Indefinite Integrals Ex 19.28 Q14

Let $I = \int \frac{\sqrt{16 + (\log x)^2}}{x}dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\therefore \quad I = \int \sqrt{16 + t^2}dt$

$= \int \sqrt{4^2 + t^2}dt$

$= \frac{t}{2}\sqrt{16 + t^2} + \frac{16}{2}\log\left|t + \sqrt{16 + t^2}\right| + c$

$\therefore \quad I = \frac{\log x}{2}\sqrt{16 + (\log x)^2} + 8\log\left|\log x + \sqrt{16 + (\log x)^2}\right| + c$

Indefinite Integrals Ex 19.28 Q15

Let $I = \int \sqrt{2ax - x^2}dx$

$= \int \sqrt{a^2 - \left(a^2 - 2ax + x^2\right)}dx$ [Adding and subtracting a^2]

$= \int \sqrt{a^2 - (a - x)^2}dx$

$= \int \sqrt{a^2 - (x - a)^2}dx$

$= \frac{(x - a)}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$

$\therefore \quad I = \frac{1}{2}(x - a)\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x - a}{a}\right) + c$

Indefinite Integrals Ex 19.28 Q16

Let $I = \int \sqrt{3 - x^2}dx$

$= \int \sqrt{\left(\sqrt{3}\right)^2 - x^2}dx$

$I = \frac{x}{2}\sqrt{3 - x^2} + \frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$