RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.31

Indefinite Integrals Ex 19.31 Q1

$$I = \int \frac{x^2+1}{x^4+x^2+1}dx$$

$$= \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $x - \frac{1}{x} = t \Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

$\therefore \quad I = \int \frac{dt}{t^2+3}$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+c$$

$\therefore \quad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+c$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot\theta}\,d\theta$$

Let $\cot\theta = x^2$

$$\Rightarrow \quad -\operatorname{cosec}^2\theta\, d\theta = 2x\,dx$$

$$\Rightarrow \quad d\theta = \frac{-2x}{\operatorname{cosec}^2\theta}dx$$

$$= \frac{-2x}{1+\cot^2\theta}dx$$

$$= \frac{-2x}{1+x^4}dx$$

$$\therefore \quad I = -\int \frac{2x^2}{1+x^4}dx$$

$$= -\int \frac{2}{\frac{1}{x^2}+x^2}dx$$

Dividing numerator and denominator by x^2

$$= -\int \frac{1+\frac{1}{x^2}+1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= -\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2} - \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}$$

Let $x-\frac{1}{x} = t \Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$

and $\quad x+\frac{1}{x} = z \Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dz$

$$\Rightarrow \quad I = -\int \frac{dt}{t^2+2} - \int \frac{dz}{z^2-2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

$$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\cot\theta-1}{\sqrt{2\cot\theta}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{\cot\theta+1-\sqrt{2\cot\theta}}{\cot\theta+1-\sqrt{2\cot\theta}}\right| + c$$

Indefinite Integrals Ex 19.31 Q3

Let $I = \int \frac{x^2+9}{x^4+81} dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{1+\frac{9}{x^2}}{x^2+\frac{81}{x^2}} dx$$

$$= \int \frac{1+\frac{9}{x^2}}{\left(x-\frac{9}{x}\right)^2+18} dx$$

Let $\left(x-\frac{9}{x}\right) = t \Rightarrow \quad \left(1+\frac{9}{x^2}\right)dx = dt$

$\therefore \quad I = \int \frac{dt}{t^2+18}$

$\Rightarrow \quad I = \frac{1}{3\sqrt{2}} tan^{-1}\left(\frac{t}{3\sqrt{2}}\right) + c$

Thus,

$$I = \frac{1}{3\sqrt{2}} tan^{-1}\left(\frac{x^2-9}{3\sqrt{2}x}\right) + c$$

Indefinite Integrals Ex 19.31 Q4

Let $I = \int \frac{1}{x^4 + x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{1 + \frac{1}{x^2} - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \left\{ \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 1} dx \right\}$$

Let $\left(x - \frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt$

and $\quad \left(x + \frac{1}{x}\right) = z$

$\Rightarrow \quad \left(1 - \frac{1}{x^2}\right) dx = dz$

$\therefore \quad I = \frac{1}{2} \int \frac{dt}{t^2 + 3} - \frac{1}{2} \int \frac{dz}{z^2 - 1}$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4} \log\left|\frac{z - 1}{z + 1}\right| + c$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \frac{1}{4} \log\left|\frac{x^2 + 1 - x}{x^2 + 1 + x}\right| + c$

Indefinite Integrals Ex 19.31 Q5

Let $I = \int \frac{x^2 - 3x + 1}{x^4 + x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{1 - \frac{3}{x} + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{3x}{x^4 + x^2 + 1} dx$$

Let $\left(x - \frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt \qquad$ [For Ist part]

Let $\quad x^2 = z$

$\Rightarrow \quad 2x dx = dz \qquad$ [For IInd part]

$$\therefore \quad I = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{z^2 + z + 1}$$

$$\Rightarrow \quad = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$\Rightarrow \quad = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{3}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{z + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2z + 1}{\sqrt{3}}\right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q6

Let $I = \int \frac{x^2+1}{x^4-x^2+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2-1+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right) dx}{\left(x-\frac{1}{x}\right)^2+1}$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right) dx = dt$$

$$\Rightarrow \quad I = \int \frac{dt}{t^2+1}$$

$$= tan^{-1} t + c$$

$$\therefore \quad I = tan^{-1}\left(\frac{x^2-1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q7

Let $I = \int \frac{x^2-1}{x^4+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1-\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2} dx$$

Let $\left(x+\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1-\frac{1}{x^2}\right) dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2-2}$$

$$= \frac{1}{2\sqrt{2}} log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

So,

$$I = \frac{1}{2\sqrt{2}} log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

Indefinite Integrals Ex 19.31 Q8

Let $I = \int \frac{x^2+1}{x^4+7x^2+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+7+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+9} dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right) dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2+9}$$

$$= \frac{1}{3} tan^{-1} \left|\frac{t}{3}\right| + c$$

Hence,

$$I = \frac{1}{3} tan^{-1} \left(\frac{x^2-1}{3x}\right) + c$$

Indefinite Integrals Ex 19.31 Q9

Let $I = \int \frac{(x-1)^2}{x^4+x^2+1}dx$

$= \int \frac{x^2-2x+1}{x^4+x^2+1}dx$

Dividing numerator and denominator by x^2

$\therefore \quad I = \int \frac{1-\frac{2}{x}+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$

$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx - \int \frac{2x}{x^4+x^2+1}dx$

Let $\left(x-\frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$ [For Ist part]

Let $x^2 = z$

$\Rightarrow \quad 2xdx = dz$ [For IInd part]

$\therefore \quad I = \int \frac{dt}{t^2+3} - \int \frac{dz}{z^2+z+1}$

$= \int \frac{dt}{t^3+3} - \int \frac{dz}{\left(z+\frac{1}{2}\right)^2+\frac{3}{4}}$

$\Rightarrow \quad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c$

$\Rightarrow \quad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right)+c$

Hence,

$I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+c$

Indefinite Integrals Ex 19.31 Q10

Let $\quad I = \int \frac{1}{x^4 + 3x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$\therefore \qquad I = \int \frac{\frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right) - \left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \frac{1}{2} \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx$$

Let $\left(x - \frac{1}{x}\right) = t$

$$\Rightarrow \qquad \left(1 + \frac{1}{x^2}\right) dx = dt$$

and $\qquad x + \frac{1}{x} = z$

$$\Rightarrow \qquad \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore \qquad I = \frac{1}{2} \int \frac{dt}{t^2 + 5} - \frac{1}{2} \int \frac{dz}{z^2 + 1}$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) - \frac{1}{2} \tan^{-1}(z) + c$$

Hence,

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{5}x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q11

Consider the integral

$$I = \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

Divide both the numerator and the denominator by $\cos^4 x$, we have,

$$I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{\sec^2 x \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{\left(\tan^2 x + 1\right) \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

Substituting $\tan x = t$; $\sec^2 x dx = dt$

Thus,

$$I = \int \frac{\left(1+t^2\right) dt}{t^4 + t^2 + 1}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} + 1\right)}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} - 2 + 2 + 1\right)}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

Substituting $z = t - \frac{1}{t}$; $dz = \left(1+\frac{1}{t^2}\right) dt$

$$I = \int \frac{dz}{z^2 + 3}$$

$$\Rightarrow I = \int \frac{dz}{z^2 + \left(\sqrt{3}\right)^2}$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{z}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{3}}\right) + C$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan x - \cot x}{\sqrt{3}}\right) + C$$