RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.32

Let $I = \int \frac{1}{(x-1)\sqrt{2x+3}} dx$

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Let $2x + 3 = t^2$

$\Rightarrow \quad 2dx = 2tdt$

$\therefore \quad I = \int \frac{t\, dt}{\left(\frac{t^2-3}{2} - 1\right)t}$

$$= 2\int \frac{dt}{t^2 - 5}$$

$$= \frac{2}{2\sqrt{5}} log \left|\frac{t - \sqrt{5}}{t + \sqrt{5}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{5}} log \left|\frac{\sqrt{2x+3} - \sqrt{5}}{\sqrt{2x+3} + \sqrt{5}}\right| + c$$

Indefinite Integrals Ex 19.32 Q3

Let $I = \int \frac{1}{(x-1)\sqrt{x+2}} dx$

$$I = \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}} dx$$

$$I = \int \frac{dx}{\sqrt{x+2}} + 2\int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x + 2 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$$

$$= \frac{2\times 1}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_2$$

$$= \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| +_2$$

Thus, form (A),

$$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.32 Q4

Let $I = \int \frac{x^2}{(x-1)\sqrt{x+2}} dx$

$$= \int \frac{\left(x^2 - 1 + 1\right)}{(x-1)\sqrt{x+2}} dx$$

$$= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+1)}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+2)-1}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$I = \int \sqrt{x+2}\,dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \sqrt{x+2}\,dx = \frac{2}{3}(x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x + 2 = t^2$

$$\Rightarrow \quad dx = 2t\,dt$$

$$\therefore \quad 2\int \frac{t\,dt}{\left(t^2 - 3\right)t} = 2\int \frac{dt}{t^2 - 3}$$

$$= \frac{2}{2\sqrt{3}} \log\left|\frac{t - \sqrt{3}}{t + \sqrt{3}}\right| + c_3$$

$$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{x+2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}}\right| + c_3$$

Thus, form (A)

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{x+2} - \sqrt{3}}{\sqrt{x+2} + \sqrt{3}}\right| + c \qquad [\text{when } c = c_1 + c_2 + c_3]$$

Indefinite Integrals Ex 19.32 Q5

Let $I = \int \frac{x}{(x-3)\sqrt{x+1}} dx$

$$= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}} dx$$

$$I = \int \frac{dx}{\sqrt{x+1}} + 3\int \frac{dx}{(x-3)\sqrt{x+1}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-3)\sqrt{x+1}}$$

Let $x+1 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = 2\int \frac{tdt}{(t^2-4)t}$$

$$= 2\left|\frac{dt}{t^2-4}\right|$$

$$= \frac{2}{2 \times 2} log \left|\frac{t-2}{t+2}\right| + c_2$$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2} log \left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c_2$$

Thus, form (A)

$$I = 2\sqrt{x+1} + \frac{3}{2} log \left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c \qquad [\text{when } c = c_1 + c_2]$$

Indefinite Integrals Ex 19.32 Q6

Let $I = \int \frac{x}{\left(x^2+1\right)\sqrt{x}}dx$

Let $x = t^2$

$\Rightarrow \quad dx = 2tdt$

$\therefore \quad 2\int \frac{tdt}{\left(t^2+1\right)t}$

$= 2\left|\frac{dt}{t^4+1}\right|$

Dividing numerator and denominator by t^2

$$I = 2\int \frac{\frac{t}{t^2}}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt - \int \frac{\left(1-\frac{1}{t^2}\right)}{\left(1+\frac{1}{t}\right)^2-2}dt$$

Let $t - \frac{1}{t} = z$

$\Rightarrow \quad \left(1+\frac{1}{t^2}\right)dt = dz \qquad$ [For Ist part]

and,

$t + \frac{1}{t} = y$

$\Rightarrow \quad \left(1-\frac{1}{t^2}\right)dt = dy \qquad$ [For IInd part]

$\therefore \quad I = \int \frac{dz}{z^2+2} - \int \frac{dy}{y^2-2}$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x+1-\sqrt{2x}}{x+1+\sqrt{2x}}\right| + c$$

Indefinite Integrals Ex 19.32 Q7

Let $I = \int \frac{x}{\left(x^2+2x+2\right)\sqrt{x+1}}\,dx$

Let $x+1 = t^2$

$\Rightarrow \quad dx = 2t\,dt$

$$= 2\int \frac{\left(t^2-1\right)t\,dt}{\left(t^4+1\right)t}$$

$$= 2\int \frac{\left(t^2-1\right)dt}{\left(t^4+1\right)}$$

$$= 2\int \frac{\left(1-\frac{1}{t^2}\right)dt}{t^2+\frac{1}{t^2}}$$

$$= 2\int \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2-2}$$

Let $t+\frac{1}{t} = y$

$\Rightarrow \quad \left(1-\frac{1}{t^2}\right)dt = dy$

$\therefore \quad I = 2\int \frac{dy}{y^2-2}$

$$= \frac{2}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{2}}\log\left|\frac{t^2+1-\sqrt{2}t}{t^2+1+\sqrt{2}t}\right| + c$$

Hence,

$$I = \frac{1}{\sqrt{2}}\log\left|\frac{x+2-\sqrt{2(x+1)}}{x+2+\sqrt{2(x+1)}}\right| + c$$

Indefinite Integrals Ex 19.32 Q8

Let $I = \int \frac{1}{(x-1)\sqrt{x^2+1}} dx$

Let $x - 1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2} dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2} dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}+1\right)^2 + 1}}$

$= -\int \frac{dt}{\sqrt{2t^2 + 2t + 1}}$

$= -\frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{t^2 + t + \frac{1}{2}}}$

$= -\frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2 + \frac{1}{4}}}$

$\therefore \quad I = -\frac{1}{\sqrt{2}} \log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2 + \frac{1}{4}}\right| + c \quad \left[\text{When } t = \frac{1}{x-1}\right]$

Indefinite Integrals Ex 19.32 Q9

Let $\quad I = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$

Let $x + 1 = \frac{1}{t}$

$\Rightarrow \qquad dx = -\frac{1}{t^2}\,dt$

$\therefore \qquad I = -\int \frac{\frac{1}{t^2}\,dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t^2}+\frac{1}{t}-1\right)}}$

$= -\int \frac{dt}{\sqrt{1+t-t^2}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(\frac{1}{4}-t+t^2\right)}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{1}{2}\right)^2}}$

$= -\sin^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right)+c$

$\therefore \qquad I = -\sin^{-1}\left(\frac{2t-1}{\sqrt{5}}\right)+c \qquad \left[\text{When } t = \frac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q11

$$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+1}}\,dx$$

Let $x^2+1 = u^2$

$$\Rightarrow \quad 2x\,dx = 2u\,du$$

$$\therefore \quad I = \int \frac{u}{\left(u^2+3\right)u}\,du$$

$$= \int \frac{1}{u^2+3}\,du$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)+C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x^2+1}{3}}\right)+C$$

Indefinite Integrals Ex 19.32 Q12

Let $\quad I = \int \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \qquad dx = -\frac{1}{t^2}dt$

$\therefore \qquad I = \int \frac{-\frac{1}{t^2}dt}{\left(\frac{1}{t^2}+1\right)\sqrt{\left(1-\frac{1}{t^2}\right)}}$

$= -\int \frac{tdt}{\left(t^2+1\right)\sqrt{t^2-1}}$

Let $t^2 - 1 = u^2$

$\Rightarrow \qquad 2tdt = 2udu$

$I = -\int \frac{udu}{\left(u^2+2\right)u}$

$= -\int \frac{du}{u^2+2}$

$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$

$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right)+c$

Thus,

$I = -\frac{1}{\sqrt{2}}\tan^{-1}\left|\sqrt{\frac{1-x^2}{2x^2}}\right|+c$

Indefinite Integrals Ex 19.32 Q13

Let $I = \int \frac{1}{\left(2x^2+3\right)\sqrt{x^2-4}} dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2} dt$

$\therefore \quad I = \int \frac{-\frac{1}{t^2} dt}{\left(\frac{2}{t^2}+3\right)\sqrt{\left(\frac{1}{t^2}-4\right)}}$

$= -\int \frac{t dt}{\left(2+3t^2\right)\sqrt{1-4t^2}}$

Let $1-4t^2 = u^2$

$\Rightarrow \quad -8t dt = 2u du$

$\therefore \quad I = \frac{1}{4}\int \frac{u du}{\frac{\left(11-3u^2\right)}{4} u}$

$= \frac{1}{3}\int \frac{du}{\frac{11}{3}-u^2}$

$= \frac{1}{2\sqrt{33}} log \left|\frac{u-\sqrt{\frac{11}{3}}}{u+\sqrt{\frac{11}{3}}}\right| + c$

$= \frac{1}{2\sqrt{33}} log \left|\frac{\sqrt{1-4t^2}-\sqrt{\frac{11}{3}}}{\sqrt{1-4t^2}+\sqrt{\frac{11}{3}}} + c\right|$

Hence,

$I = \frac{1}{2\sqrt{33}} log \left|\frac{\sqrt{11}x+\sqrt{3x^2-12}}{\sqrt{11}x-\sqrt{3x^2-12}}\right| + c$

Indefinite Integrals Ex 19.32 Q14

$$I = \int \frac{x}{\left(x^2 + 4\right)\sqrt{x^2 + 9}} dx$$

Let $x^2 + 9 = u^2$

$$\Rightarrow \qquad 2x dx = 2u\, du$$

$$\therefore \qquad I = \int \frac{u}{\left(u^2 - 5\right) u} du$$

$$= \int \frac{du}{u^2 - 5}$$

$$= \frac{1}{2\sqrt{5}} \log\left(\frac{u - \sqrt{5}}{u + \sqrt{5}}\right) + C$$

$$= \frac{1}{2\sqrt{5}} \log\left(\frac{\sqrt{x^2 + 9} - \sqrt{5}}{\sqrt{x^2 + 9} + \sqrt{5}}\right) + C$$