RD Sharma Solutions Class 12 Maths Chapter 20 Ex 20.1

Definite Integrals Ex 20.1 Q1

We know that $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Now,

$\int_4^9 \frac{1}{\sqrt{x}} dx$

$= \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^9$

$= \left[\frac{\sqrt{x}}{\frac{1}{2}}\right]_4^9$

$= 2[\sqrt{9} - \sqrt{4}]$

$= 2[3 - 2]$

$= 2$

$\therefore \int_4^9 \frac{1}{\sqrt{x}} dx = 2$

Definite Integrals Ex 20.1 Q2

We know that $\int \frac{dx}{x} = \log x + C$

Now,

$\int_{-2}^3 \frac{1}{x+7} dx$

$= [\log(x+7)]_{-2}^3$

$= [\log 10 - \log 5]_{-2}^3$

$= \log \frac{10}{5}$ $\qquad \left[\because \log a - \log b = \log \frac{a}{b}\right]$

$= \log 2$

$\therefore \int_{-2}^3 \frac{1}{x+7} dx = \log 2$

Definite Integrals Ex 20.1 Q3

Let $x = \sin\theta$

$\Rightarrow dx = \cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{6}} d\theta$$

$$= [\theta]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{\pi}{6} - 0\right]$$

$$= \frac{\pi}{6}$$

$$\therefore \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{6}$$

Definite Integrals Ex 20.1 Q4

We have,

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \left[\frac{\pi}{4} - 0\right] \qquad \left[\because \tan^{-1} 1 = \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{1}{1+x^2} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q5

Let $x^2+1=t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \frac{dt}{2}$

Now,

$x = 2 \Rightarrow t = 5$

$x = 3 \Rightarrow t = 10$

$$\therefore \int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_5^{10}\frac{dt}{t} = \frac{1}{2}\left[\log t\right]_5^{10}$$

$$= \frac{1}{2}\left[\log 10 - \log 5\right]$$

$$= \frac{1}{2}\left[\log\frac{10}{5}\right]$$

$$= \frac{1}{2}\left[\log 2\right]$$

$$= \log\sqrt{2}$$

$$\therefore \int_2^3 \frac{x}{x^2+1} = \log\sqrt{2}$$

Definite Integrals Ex 20.1 Q6

We have,

$$\int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx$$

We know that $\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$$\therefore \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx = \frac{1}{b^2}\left[\frac{b}{a}\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{2ab}$$

$$\Rightarrow \int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.1 Q7

We have,

$$\int_{-1}^{1} \frac{1}{1+x^2} dx$$

We know that $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

Now,

$$\int_{-1}^{1} \frac{1}{1+x^2} dx$$

$$= \left[\tan^{-1} x\right]_{-1}^{1}$$

$$= \left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= \left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] \qquad \left[\because \tan^{-1}(-1) = \frac{-\pi}{4}\right]$$

$$= \left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{2\pi}{4}$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.1 Q8

We have,

$$\int_{0}^{\infty} e^{-x} dx$$

We know that $\int e^{-x} dx = -e^{-x}$

Now,

$$\int_{0}^{\infty} e^{-x} dx$$

$$= \left[-e^{-x}\right]_{0}^{\infty}$$

$$= \left[-e^{-\infty} + e^{-0}\right] \qquad \left[\because e^{\infty} = 0, \quad e^{0} = 1\right]$$

$$= \left[-0 + 1\right]$$

$$\therefore \int_{0}^{\infty} e^{-x} dx = 1$$

Definite Integrals Ex 20.1 Q9

We have,

$$\int_0^1 \frac{x}{x+1}\,dx \qquad \left[\text{Add and subtract 1 in numerator}\right]$$

$$= \int_0^1 \frac{(x+1)-1}{x+1}\,dx$$

$$= \int_0^1 1\,dx - \int_0^1 \frac{1}{x+1}\,dx$$

$$= \left[x\right]_0^1 - \left[\log(x+1)\right]_0^1$$

$$= 1 - \left[\log 2 - \log 1\right]$$

$$= 1 - \log\frac{2}{1}$$

$$= 1 - \log 2$$

$$= \log e - \log 2 \qquad \left[\because \log e = 1\right]$$

$$= \log\frac{e}{2}$$

$$\therefore \int_0^1 \frac{x}{x+1}\,dx = \log\frac{e}{2}$$

Definite Integrals Ex 20.1 Q10

We have,

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\,dx + \int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\cos\frac{\pi}{2} + \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$= \left[-0+1\right] + 1$$

$$= 1 + 1$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} (\sin x + \cos x)\,dx = 2$$

Definite Integrals Ex 20.1 Q11

We have,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

We know that $\int \cot x dx = \log(\sin x)$

Now,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

$$= \left[\log(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\log\left(\sin\frac{\pi}{2}\right) - \log\left(\sin\frac{\pi}{4}\right)\right]$$

$$= \left[\log 1 - \log\frac{1}{\sqrt{2}}\right]$$

$$= \left[0 - (\log 1 - \log\sqrt{2})\right]$$

$$= \log\sqrt{2} \qquad \left[\because \log a^n = n\log a\right]$$

$$= \frac{1}{2}\log 2$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2}\log 2$$

Definite Integrals Ex 20.1 Q12

We have,

$$\int_{0}^{\frac{\pi}{4}} \sec x dx$$

We know that $\int \sec x dx = \log(\sec x + \tan x)$

$$\therefore \int_{0}^{\frac{\pi}{4}} \sec x dx$$

$$= \left[\log(\sec x + \tan x)\right]_{0}^{\frac{\pi}{4}}$$

$$= \left[\log(\sqrt{2} + 1) - \log(1 + 0)\right]$$

$$= \log(\sqrt{2} + 1) \qquad \left[\because \log 1 = 0\right]$$

$$\therefore \int_{0}^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$$

Definite Integrals Ex 20.1 Q13

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$\int \operatorname{cosec} x \; dx = \log\left|\operatorname{cosec} x - \cot x\right| = F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$

$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$

$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$

$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1-x}{1+x}dx$$

Let $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

Now,

$$\int_0^1 \frac{1-x}{1+x}dx$$

$$= \int_{\frac{\pi}{4}}^{0} \frac{1-\cos 2\theta}{1+\cos 2\theta} \times (-2\sin 2\theta)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{2\sin^2\theta}{2\cos^2\theta} \times 2\sin 2\theta d\theta \qquad \left[\because -\int_a^b f(x)dx = \int_b^a f(x)dx\right]$$

$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta}d\theta$$

Let $\cos\theta = t$

$\Rightarrow -\sin\theta d\theta = dt$

Now,

$\theta = 0 \Rightarrow t = 1$

$\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{\sqrt{2}}$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta}d\theta$$

$$= -4\int_1^{\frac{1}{\sqrt{2}}} \frac{(1-t^2)}{t}dt$$

$$= -4\left[\log t - \frac{t^2}{2}\right]_1^{\frac{1}{\sqrt{2}}}$$

$$= -4\left[\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - 0 + \frac{1}{2}\right]$$

$$= -4\left[-\log\sqrt{2} + \frac{1}{4}\right]$$

$$\therefore \int_0^1 \frac{1-x}{1+x}dx = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q15

$$I = \int_0^{\pi} \frac{1}{1+\sin x} dx$$

Multiplying Numerator and Denominator by $(1-\sin x)$

$$I = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$= \int_0^{\pi} \frac{(1-\sin x)}{(1^2-\sin^2 x)} dx$$

$$= \int_0^{\pi} \frac{1-\sin x}{(\cos^2 x)} dx$$

$$= \int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi} \sec^2 x dx - \int_0^{\pi} \tan x . \sec x dx$$

$$= [\tan x]_0^{\pi} - [\sec x]_0^{\pi}$$

$$= [\tan \pi - \tan 0] - [\sec \pi - \sec 0]$$

$$= [0 - 0] - [-1 - 1]$$

$$= 2$$

$$I = 2$$

$$\therefore \int_0^{\pi} \frac{1}{1+\sin x} dx = 2$$

Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$$

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{1+\sin x} = \frac{1}{1+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} = \frac{1+\tan^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2} = \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}$$

$$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

If $f(x)$ is an even function $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx = 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

let $1+\tan\frac{x}{2}=t$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Now,

$x=-\frac{\pi}{4} \Rightarrow t = 1-\tan\frac{\pi}{8}$

$x=\frac{\pi}{4} \Rightarrow t = 1+\tan\frac{\pi}{8}$

$$\therefore\ 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx = 2\int_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}} \frac{8dt}{t^2}$$

$$= 2\left[\frac{-1}{t}\right]_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}}$$

$$= 2\left[\frac{1}{1-\tan\frac{\pi}{8}} - \frac{1}{1+\tan\frac{\pi}{8}}\right]$$

$$= 2\left[\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}\right]$$

$$= 2\tan\frac{\pi}{4} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}dx = 2$$

Definite Integrals Ex 20.1 Q17

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx$

$$\int \cos^2 x\,dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2}+\frac{\sin 2x}{4} = \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[F\left(\frac{\pi}{2}\right) - F(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin \pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$= \frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}} \cos^3 x dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos 3x + 3\cos x}{4} dx \qquad \left[\because \cos 3x = 4\cos^3 x - 3\cos x\right]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (\cos 3x + 3\cos x) dx$$

$$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{\sin 3\frac{\pi}{2}}{3} + 3\sin\frac{\pi}{2}\right) - \left(\frac{\sin 0}{3} + 3\sin 0\right)\right]$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3}+3\right) - (0+0)\right] = \frac{2}{3}$$

$$= \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^3 x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \qquad [\because 2\cos C \cos D = \cos(C+D) - \cos(C-D)]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\cos x \cos 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (\cos 3x + \cos x) dx$$

$$= \frac{1}{2}\int \left[\frac{\sin 3x}{3} + \sin x\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\left(\frac{\sin 3\frac{\pi}{6}}{3} + \sin\frac{\pi}{6}\right) - (\sin 0 - \sin 0)\right]$$

$$= \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}}{3} + \sin\frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(\frac{5}{6}\right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin x \sin 2x dx \qquad [\because 2\sin C \times \sin D = \cos(D-C) - \cos(D+C)]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\frac{\sin 3\frac{\pi}{2}}{3} - \frac{\sin 0}{3}\right)\right]$$

$$= \frac{1}{2}\left[(1-0) - \left(\frac{-1}{3} - 0\right)\right] \qquad \left[\because \sin 3\frac{\pi}{2} = -1\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\sin^2 x + \cot^2 x}{\sin x \cos x} \right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{1}{\sin x \cos x} \right)^2 dx$$

Multiplying numerator and denominator by 2

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{2 \sin x \cos x} \right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{\sin 2x} \right)^2 dx \qquad [\because 2 \sin x \cos x = \sin 2x]$$

$$= 4 \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} cosec^2 x dx$$

$$= 4 \left[-\frac{\cot 2x}{2} \right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= 2 \left[-\cot \frac{\pi}{2} + \cot 2 \frac{\pi}{3} \right]$$

$$= 2 \left[\frac{-1}{\sqrt{3}} - 0 \right]$$

$$= \frac{-2}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (1 + \cos 2x)^2 dx \qquad [\because 2\cos^2 x = 1 + \cos 2x]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (1 + \cos^2 2x + 2\cos 2x) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1 + \frac{1 + \cos 4x}{2} + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\left[x + \frac{1}{2}x + \frac{\sin 4x}{8} + \sin 2x\right]_0^{\frac{\pi}{2}} \qquad \left[\because \int \cos 4x dx = \frac{\sin 4x}{4}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2} + \frac{\pi}{4} + 0 + 0 - 0 - 0 - 0 - 0\right]$$

$$= \frac{1}{4} \times \frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

Definite Integrals Ex 20.1 Q23

We have,

$$\int_0^{\frac{\pi}{2}} \left\{a^2 \cos^2 x + b^2 \left(1 - \cos^2 x\right)\right\} dx$$

$$= \int_0^{\frac{\pi}{2}} \left\{\left(a^2 - b^2\right)\cos^2 x + b^2\right\} dx$$

$$= \frac{a^2 - b^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2x\right) dx + b^2 \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{a^2 - b^2}{2} \left[x + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} + b^2 [x]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2 - b^2}{2} \left[\frac{\pi}{2} + 0 - 0 - 0\right] + b^2 \left[\frac{\pi}{2} - 0\right]$$

$$= \frac{a^2 - b^2}{2} \left[\frac{\pi}{2}\right] + b^2 \left[\frac{\pi}{2}\right]$$

$$= a^2 \frac{\pi}{4} + b^2 \left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$$

$$= \frac{\pi}{4}\left(a^2 + b^2\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} (a^2 \cos^2 x + b^2 \sin^2 x) dx = \frac{\pi}{4}\left(a^2 + b^2\right)$$

Definite Integrals Ex 20.1 Q24

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\, dx \qquad \text{We use } \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{1+\tan^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{\sec^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\tan\frac{x}{2}}{\sec\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) dx$$

$$= \left[2\sin\frac{x}{2} - 2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 0 + 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = 2$$

Definite Integrals Ex 20.1 Q25

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}$$

We use $1+\cos x = 2\cos^2\frac{x}{2}$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\frac{x}{2}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\frac{x}{2}dx$$

$$= \sqrt{2}\left[2\sin\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x} = 2$$

Definite Integrals Ex 20.1 Q26

We have,

$$\int x\sin x\,dx = x\int \sin x dx - \int\left(\int \sin x dx\right)\left(\frac{dx}{dx}\right)dx$$

$$= -x\cos x + \int \cos x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x\,dx = \left[-x\cos x+\sin x\right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2}\times 0\right)+1+0-0 = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x dx = 1$$

Definite Integrals Ex 20.1 Q27

We have,

$$\int x\cos x\,dx = x\int \cos x dx - \int\left(\int \cos x dx\right)\frac{dx}{dx}dx = x\sin x - \int \sin x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x\,dx = \left[x\sin x+\cos x\right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2}+0-0-1\right] = \frac{\pi}{2}-1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\cos x dx = \frac{\pi}{2}-1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\int x^2 \cos x\, dx = x^2 \int \cos x dx - \int (2x)\left(\int \cos x dx\right) dx = x^2 \sin x - \int \sin x . 2x dx$$

$$= x^2 \sin x - 2\left[x \int \sin x - \int\left(\int \sin x dx\right) dx\right]$$

$$= x^2 \sin x - 2\left[-x \cos x + \int \cos x dx\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos x\, dx = \left[x^2 \sin x + 2x \cos x - 2 \sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0\right]$$

$$= \frac{\pi^2}{4} - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos x dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\int x^2 \sin x\, dx = x^2 \int \sin x dx - \int 2x \left(\int \sin x dx\right) dx = x^2 \cos x + \int 2x \cos x dx$$

$$= x^2 \cos x + 2\left[x \int \cos x dx - \int\left(\int \cos x dx\right) dx\right]$$

$$= -x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{4}} x^2 \sin x\, dx = \left[-x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{-\pi^2}{16} . \frac{1}{\sqrt{2}} + \frac{\pi}{2} . \frac{1}{\sqrt{2}} + 2 . \frac{1}{\sqrt{2}} + 0 - 0 - 2$$

$$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30

We have,

$$\int x^2 \cos 2x\,dx = x^2\int \cos 2x dx - \int 2x\left(\int \cos 2x dx\right)dx$$

$$= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2} dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x\int \sin 2x dx - \int\left(\int \sin 2x dx\right)dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \left[\frac{x \cos 2x}{2} - \int \frac{x \cos 2x}{2}\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{8} \times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0\right]$$

$$= \frac{-\pi}{4}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x dx = \frac{-\pi}{4}$$

Definite Integrals Ex 20.1 Q31

We have,

$$\int x^2 \cos^2 x dx = \int x^2\left(\frac{1 + \cos 2x}{2}\right)dx = \frac{1}{2}\int\left(x^2 + x^2 \cos 2x\right)dx = \frac{1}{2}\left[\int x^2 dx + \int x^2 \cos 2x dx\right] \quad ..(A)$$

Now,

$$\int_0^{\frac{\pi}{2}} x^2 dx = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24} \quad ...(B)$$

$$\int x^2 \cos 2x\,dx = x^2\int \cos 2x dx - \int 2x\left(\int \cos 2x dx\right)dx = \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{2}.2x dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x\int \sin 2x - \int\left(\int \sin 2x dx\right)dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \int \frac{\cos 2x}{2} dx$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} = \frac{-\pi}{4} \quad -(C)$$

Now, Put (B)&(C)in (A), we get,

$$\int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx = \int_0^{\frac{\pi}{2}} x^2 dx + \int_0^{\frac{\pi}{2}} x^2 \cos 2x\,dx = \frac{1}{2}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \frac{\pi^3}{48} - \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q32

We have,

$$\int \log x dx = \int 1.\log x dx = \log x\int dx - \int\left(\int dx\right).\frac{1}{x}dx = x \log x - \int x.\frac{1}{x} dx = x \log x - \int dx$$

$$\therefore \quad \int_1^2 \log x dx = \left[x \log x - x\right]_1^2 = 2\log 2 - 2 - 0 + 1 = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q33

We have,

$$\int \frac{\log x}{(x+1)^2}dx = \int \frac{1}{(x+1)^2}\log x dx = \log x \int \frac{1}{(x+1)^2}dx - \int \left(\int \frac{1}{(x+1)^2}dx\right)\frac{1}{x}dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)}dx$$

$$= \frac{-\log x}{(x+1)} + \int \left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$$\therefore \quad \int_1^3 \frac{\log x}{(x+1)^2}dx = \left[\frac{-\log x}{x+1} + \log x - \log(x+1)\right]_1^3 = \frac{3}{4}\log 3 - \log 2$$

Definite Integrals Ex 20.1 Q34

Let $I = \int_1^e \frac{e^x}{x}(1 + x\log x)dx$

$$I = \int_1^e \frac{e^x}{x}dx + \int_1^e e^x \log x dx$$

$$I = \left[e^x \log x\right]_1^e - \int_1^e e^x.\log x + \int_1^e e^x \log x$$

$$I = \left[e^x \log x\right]_1^e$$

$$I = \left[e^x \log e - e^1.\log 1\right]$$

$$I = \left[e^e.1 - 0\right]$$

$$I = e^e$$

$$\therefore \int_1^e \frac{e^x}{x}(1 + x\log x)dx = e^e$$

Definite Integrals Ex 20.1 Q35

We have,

$$\int_1^e \frac{\log x}{x} dx$$

Let $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

Now,

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 1$

$$\therefore \int_1^e \frac{\log x}{x} dx = \int_0^1 t\, dt$$

$$= \left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{1}{2}$$

$$\therefore \int_1^e \frac{\log x}{x} dx = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q36

We have,

$$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2} dx\right\}$$

$$I = \int \frac{1}{\log x} .1 dx = \frac{1}{\log x} \int dx - \int \left(\int dx\right). \frac{d}{dx}\left(\frac{1}{\log x}\right) dx = \frac{x}{\log x} + \int \frac{1}{(\log x)^2} .x. \frac{1}{x} dx$$

$$= \frac{x}{\log x} + \int \frac{dx}{(\log x)^2}$$

$$\int_e^{e^2} \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2} dx\right\} = \left[\frac{x}{\log x}\right]_e^{e^2} + \int_e^{e^2} \frac{dx}{(\log x)^2} - \int_e^{e^2} \frac{dx}{(\log x)^2}$$

$$= \left[\frac{x}{\log x}\right]_e^{e^2}$$

$$= \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q37

We have,

$$\int_1^2 \frac{x+3}{x(x+2)}dx$$

$$= \int_1^2 \frac{x}{x(x+2)}dx + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \int_1^2 \frac{dx}{(x+2)} + \int_1^2 \frac{3}{x(x+2)}dx$$

$$= \left[\log(x+2)\right]_1^2 + \frac{3}{2}\int_1^2 \frac{1}{x} - \frac{1}{x+2}dx \qquad \text{[using partial fraction]}$$

$$= \left[\log(x+2)\right]_1^2 + \left[\frac{3}{2}\log x - \frac{3}{2}\log(x+2)\right]_1^2$$

$$= \left[\frac{3}{2}\log x - \frac{1}{2}\log(x+2)\right]_1^2$$

$$= \frac{1}{2}\left[3\log 2 - \log 4 + \log 3\right]$$

$$= \frac{1}{2}\left[3\log 2 - 2\log 2 + \log 3\right] \qquad [\because \log 4 = 2\log 2]$$

$$= \frac{1}{2}\left[\log 2 + \log 3\right]$$

$$= \frac{1}{2}\left[\log 6\right]$$

$$= \frac{1}{2}\log 6$$

$$\therefore \int_1^2 \frac{x+3}{x(x+2)}dx = \frac{1}{2}\log 6$$

Definite Integrals Ex 20.1 Q38

Let $I = \int_0^1 \frac{2x+3}{5x^2+1} dx$

$$\int \frac{2x+3}{5x^2+1} dx = \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx$$
$$= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx$$
$$= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx$$
$$= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx$$
$$= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{\frac{1}{\sqrt{5}}} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}}$$
$$= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}x)$$
$$= F(x)$$

Definite Integrals Ex 20.1 Q39

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-(x^2-x-4)}$$
$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$
$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$
$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{\ldots} \log \frac{\frac{\sqrt{17}}{2}+t}{\ldots} \right]^{\frac{3}{2}}$$

$$\left[2\left(\frac{\sqrt{17}}{2}\right) \quad \frac{\sqrt{17}}{2}-t\right]_{-\frac{1}{2}}$$

$$=\frac{1}{\sqrt{17}}\left[\log\frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}}-\frac{\log\frac{\sqrt{17}}{2}-\frac{1}{2}}{\log\frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$=\frac{1}{\sqrt{17}}\left[\log\frac{\sqrt{17}+3}{\sqrt{17}-3}-\log\frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$=\frac{1}{\sqrt{17}}\log\frac{\sqrt{17}+3}{\sqrt{17}-3}\times\frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{42+10\sqrt{17}}{8}\right)$$

$$=\frac{1}{\sqrt{17}}\log\left(\frac{21+5\sqrt{17}}{4}\right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\int_0^1 \frac{1}{2x^2+x+1}dx$$

$$= \frac{1}{2}\int_0^1 \frac{1dx}{\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{1}{2}-\frac{1}{16}} \qquad \left[\text{Adding } \frac{1}{16} \text{ \& substracting} \frac{1}{16} \text{ in numerator}\right]$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{7}{16}}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{4}{\sqrt{7}}\left[\tan^{-1}\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

Definite Integrals Ex 20.1 Q41

Let $I = \int_0^1 \sqrt{x\left(1-x\right)}\,dx$

let $x = \sin^2\theta$
$\Rightarrow dx = 2\sin\theta.\cos\theta.d\theta$

Now,
$x = 0 \Rightarrow \theta = 0$
$x = 1 \Rightarrow \theta = \frac{\pi}{2}$

$\therefore\ I = \int_0^{\frac{\pi}{2}} \sqrt{\sin^2\theta\left(1-\sin^2\theta\right)}.2\sin\theta.\cos\theta d\theta$

$= \int_0^{\frac{\pi}{2}} 2\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 4\sin^2\theta.\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin^2 2\theta\right) d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 4\theta}{2}\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1-\cos 4\theta\right) d\theta$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}} \cos 4\theta d\theta$

$= \frac{1}{4}\left[\theta\right]_0^{\frac{\pi}{2}} - \frac{1}{4}\left[\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}\left[\frac{\pi}{2} - 0\right] - \frac{1}{16}\left[\sin\pi - \sin 0\right]$

$= \frac{\pi}{8} - \frac{1}{16}\left[0-0\right]$

$= \frac{\pi}{8}$

$I = \frac{\pi}{8}$

$\therefore \int_0^1 \sqrt{x\left(1-x\right)}\,dx = \frac{\pi}{8}$

Definite Integrals Ex 20.1 Q42

We have,

$$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$$

$$\int_0^2 \frac{dx}{\sqrt{3+1-\left(x^2-2x+1\right)}}$$ [Add and subtract 1 in denominator]

$$= \int_0^2 \frac{dx}{\sqrt{(2)^2 (x-1)^2}}$$ $$\left[\because \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^2$$

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}\left(\frac{-1}{2}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) - \sin^{-1}\left[\sin\left(\frac{-\pi}{6}\right)\right]$$

$$= \frac{\pi}{6} + \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\therefore \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$$

Definite Integrals Ex 20.1 Q43

We have,

$$\int_0^4 \frac{dx}{\sqrt{4x-x^2}}$$

$$= \int_0^4 \frac{dx}{\sqrt{4-4+4x-x^2}}$$ [Add and subtract 4 in denominator]

$$= \int_0^4 \frac{dx}{\sqrt{4-\left(x^2-4x+4\right)}}$$

$$= \int_0^4 \frac{dx}{\sqrt{(2)^2-(x-2)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^4$$ $$\left[\because \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4x-x^2}} = \pi$$

Definite Integrals Ex 20.1 Q44

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{(x^2+2x+1)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_{0}^{2} \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q45

We have,

$$\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx$$

Let $2x+1 = t^2$

$\Rightarrow 2dx = 2t\,dt$

Now,

$x = 1 \Rightarrow t = \sqrt{3}$

$x = 4 \Rightarrow t = 3$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \int_{\sqrt{3}}^3 \frac{\left(\frac{t^2-1}{2}\right)^2 + \left(\frac{t^2-1}{2}\right)}{t}\,t\,dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^3 \left(t^4 - 2t^2 + 1 + 2t^2 - 2\right)dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^3 t^4 - 1$$

$$= \frac{1}{4}\left[\frac{t^5}{5} - t\right]_{\sqrt{3}}^3$$

$$= \frac{1}{4}\left[\frac{243 - 9\sqrt{3}}{5} - 3 + \sqrt{3}\right]$$

$$= \frac{1}{4}\left[\frac{228}{5} - \sqrt{3}\,(4)\right]$$

$$= \frac{57 - \sqrt{3}}{5}$$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \frac{57-\sqrt{3}}{5}$$

Definite Integrals Ex 20.1 Q46

We have,

$$\int_0^1 x(1-x)^5 dx$$

Expanding $(1-x)^5$ by Binomial theorem

$$\therefore (1-x)^5 = 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5$$
$$= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$= \int_0^1 x\left(1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5\right) dx$$

$$= \left[\frac{x^2}{2} - \frac{5x^3}{3} + \frac{10x^4}{4} - \frac{10x^5}{5} + \frac{5x^6}{6} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{2} - \frac{5}{3} + \frac{10}{4} - \frac{10}{5} + \frac{5}{6} - \frac{1}{7}$$

$$= \frac{1}{42}$$

$$\therefore \int_0^1 x(1-x)^5 dx = \frac{1}{42}$$

Definite Integrals Ex 20.1 Q47

We have,

$$\int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx = \int_1^2 \frac{xe^x}{x^2} - \int_1^2 \frac{e^x}{x^2} dx = \int_1^2 \frac{e^x dx}{x} - \int_1^2 \frac{e^x}{x^2} dx$$

Expanding 1^{st} integral by by parts we get

$$= \frac{1}{x}\int_1^2 e^x dx - \int_1^2 \left(\int e^x . \frac{d\left(1/x\right)}{dx} dx\right) - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2 + \int_1^2 \frac{e^x}{x^2} dx - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2$$

$$= \frac{e^2}{2} - e$$

$$\therefore \int_1^2 \left(\frac{x-1}{x^2}\right) e^x dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2} \right) dx = \int_0^1 \underset{\text{I}}{x}\, \underset{\text{II}}{e^{2x}}\, dx + \int_0^1 \sin\frac{\pi x}{2}\, dx$$

Applying by parts in first integral

$$= x\int_0^1 e^{2x} dx - \int_0^1 \left(\int e^{2x} dx. \right) \frac{dx}{dx} dx + \left[\frac{-\cos\frac{\pi x}{2}}{\frac{\pi}{2}} \right]_0^1$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x} dx + \frac{2}{\pi}[1-0]$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x} dx + \frac{2}{\pi}[1-0]$$

$$= \left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x} \right]_0^1 + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4}$$

$$= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

$$\therefore \int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2} \right) dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\int_0^1 \left(xe^x + \cos\frac{\pi x}{4}\right)dx$$

$$= \int_0^1 \underset{\text{I}}{x}\,\underset{\text{II}}{e^x}\,dx + \int_0^1 \cos\frac{\pi x}{4}\,dx$$

Applying by by parts in 1st integral we get,

$$= x\int_0^1 e^x dx - \int_0^1 \left(\int e^x dx\right)\frac{dx}{dx}.dx + \int_0^1 \cos\frac{\pi x}{4}dx$$

$$= \left[xe^x\right]_0^1 - \int_0^1 e^x dx + \left[\frac{\sin\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1$$

$$= \left[xe^x - e^x\right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} - 0\right]$$

$$= \left[e^x(x-1)\right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 0 + 1 + \frac{4}{\pi\sqrt{2}}$$

$$= 1 + \frac{2\sqrt{2}}{\pi}$$

$$\therefore \int_0^1 \left(xe^x + \cos\frac{\pi x}{4}\right)dx = 1 + \frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-\sin x}{1-\cos x}dx = \int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}dx \qquad \left[1-\cos x = 2\sin^2\frac{x}{2}\right]$$

$$= -\int_{\frac{\pi}{2}}^{\pi} e^x\left(-\frac{1}{2}\csc^2\frac{x}{2} + \cot\frac{x}{2}\right)dx$$

$$= -e^x\cot\frac{x}{2}\Big|_{\frac{\pi}{2}}^{\pi} \qquad \left[\int e^x\left(F(x)+F'(x)\right)dx = e^x F(x)\right]$$

$$= e^{\frac{\pi}{2}}$$

Definite Integrals Ex 20.1 Q51

We have,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \int_0^{2\pi} e^{x/2}\left(\sin\frac{x}{2}\cos\frac{\pi}{4}+\cos\frac{x}{2}\sin\frac{\pi}{4}\right)dx$$

$$= \int_0^{2\pi} e^{x/2}\sin\frac{x}{2}.\frac{1}{\sqrt{2}}dx + \int_0^{2\pi} e^{x/2}\cos\frac{x}{2}.\frac{1}{\sqrt{2}}dx$$

Expanding 1st part by parts, we get,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}\int_0^{2\pi} e^{x/2}dx - \int_0^{2\pi}\left(\int_0^{2\pi} e^{x/2}dx\right).\frac{d\left(\sin\frac{x}{2}\right)}{dx}dx\right\}+\frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}.\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} - \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}2.\frac{1}{2}\cos\frac{x}{2}dx + \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} = \frac{1}{\sqrt{2}}\{0-0\} = 0$$

$$\therefore \int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = 0$$

Definite Integrals Ex 20.1 Q52

$$\text{Let } I = \int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi} e^x\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$\Rightarrow I = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)e^x\right]_0^{2\pi} + \frac{1}{2}\left[\left\{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right\}_0^{2\pi} - \frac{1}{2}\int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx$$

$$I = \left[\cos\left(\pi+\frac{\pi}{4}\right)e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{4}\right).e^{2\pi} - \sin\frac{\pi}{4} - \frac{1}{2}I\right]$$

$$I = \left[-\cos\frac{\pi}{4}.e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[-\sin\frac{\pi}{4}.e^{2\pi} - \sin\frac{\pi}{4}\right] - \frac{I}{4}$$

$$\frac{5I}{4} = -\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) - \frac{1}{2}.\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) = \frac{-3}{2\sqrt{2}}\left(e^{2\pi}+1\right)$$

$$I = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

$$\therefore \int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

Definite Integrals Ex 20.1 Q53

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$I = \int_0^1 \frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)} \times \frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)} dx$$

$$= \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x} dx$$

$$= \int_0^1 \sqrt{1+x}\, dx + \int_0^1 \sqrt{x}\, dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[(2)^{\frac{3}{2}} - 1\right] + \frac{2}{3}[1]$$

$$= \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{2 \cdot 2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.1 Q54

$$\int_1^2 \frac{x}{(x+1)(x+2)} dx = -\int_1^2 \frac{1}{x+1} dx + \int_1^2 \frac{2}{x+2} dx \qquad \text{[Using Partial Fraction]}$$

$$= -\log(x+1)\Big|_1^2 + 2\log(x+2)\Big|_1^2$$

$$= -(\log 3 - \log 2) + 2(\log 4 - \log 3)$$

$$= -3\log 3 + 5\log 2$$

$$= \log \frac{32}{27}$$

Definite Integrals Ex 20.1 Q55

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sin^3 x\, dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(1 - \cos^2 x\right) \sin x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\, dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x\, dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Definite Integrals Ex 20.1 Q56

$$\text{Let } I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \cos x\ dx$$

$$\int \cos x\ dx = \sin x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(\pi) - \mathrm{F}(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

Definite Integrals Ex 20.1 Q57

$$\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$$

$$= \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt = \int_2^4 e^t\left[f(t)+f'(t)\right]dt$$

$$=\left[e^t f(t)\right]_2^4$$

$$=\left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$=\left[\frac{e^t}{t}\right]_2^4$$

$$=\frac{e^4}{4}-\frac{e^2}{2}$$

$$=\frac{e^4-2e^2}{4}$$

Definite Integrals Ex 20.1 Q58

$$\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}}dx$$

$$= \int_1^2 \frac{1}{\sqrt{-\left(x-\frac{3}{2}\right)^2+\left(\frac{1}{4}\right)}}dx$$

$$= \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}dx$$

$$= \left[\sin^{-1}(2x-3)\right]_1^2$$

$$= \sin^{-1}(1)-\sin^{-1}(-1)$$

$$= \pi$$

Definite Integrals Ex 20.1 Q59

We have,

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\int_0^k \frac{dx}{\left(\frac{1}{2}\right)^2 + x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\left[2\tan^{-1}2x\right]_0^k = \frac{\pi}{16} \qquad \left[\because \int \frac{dx}{a^2-x^2} = 2\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{1}{4}\left[\tan^{-1}2k - \tan^{-1}0\right] = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1}2k - 0 = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}2k = \frac{\pi}{4}$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q60

We have,

$$\int_0^a 3x^2 dx = 8$$

$$\Rightarrow \left[x^3\right]_0^a = 8$$

$$\Rightarrow a^3 = 8$$

$$\Rightarrow a = 2$$

Definite Integrals Ex 20.1 Q61

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)}dx$$

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{2\sin^2 x}dx$$

$$\sqrt{2}\int_{\pi}^{\frac{3\pi}{2}} \sin x dx$$

$$\sqrt{2}(-\cos x)_{\pi}^{\frac{3\pi}{2}}$$

$$= \sqrt{2}$$

Definite Integrals Ex 20.1 Q62

$$I = \int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)^2}\, dx$$

$$\Rightarrow I = \int_0^{2\pi} \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right) dx$$

$$\Rightarrow I = \left[\frac{-\cos\frac{x}{4}}{\frac{1}{4}} + \frac{\sin\frac{x}{4}}{\frac{1}{4}}\right]_0^{2\pi}$$

$$\Rightarrow I = 4(0 + 1 + 1 - 0)$$

$$\Rightarrow I = 8$$

Definite Integrals Ex 20.1 Q63

$$I=\int_0^{\pi/4}(\tan x+\cot x)^{-2}\,dx$$

$$I=\int_0^{\pi/4}\frac{1}{(\tan x+\cot x)^2}\,dx$$

$$I=\int_0^{\pi/4}\frac{1}{\left(\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)^2}\,dx$$

$$I=\int_0^{\pi/4}(\sin x\cos x)^2\,dx$$

$$I=\int_0^{\pi/4}\sin^2 x\left(1-\sin^2 x\right)dx$$

$$I=\int_0^{\pi/4}\sin^2 x\,dx-\int_0^{\pi/4}\sin^4 x\,dx$$

We know that by reduction formula,

$$\int \sin^n x\ dx=\frac{n-1}{n}\int \sin^{n-2} x\ dx-\frac{\cos x\sin^{n-1} x}{n}$$

For n = 2

$$\int \sin^2 x\ dx=\frac{2-1}{2}\int 1\ dx-\frac{\cos x\sin x}{2}$$

$$\int \sin^2 x\ dx=\frac{1}{2}x-\frac{\cos x\sin x}{2}$$

For n = 4

$$\int \sin^4 x\ dx=\frac{4-1}{4}\int \sin^2 x\ dx-\frac{\cos x\sin^3 x}{4}$$

$$\int \sin^4 x\ dx=\frac{3}{4}\left\{\frac{1}{2}x-\frac{\cos x\sin x}{2}\right\}-\frac{\cos x\sin^3 x}{4}$$

Hence,

$$I=\left\{\frac{1}{2}x-\frac{\cos x\sin x}{2}\right\}_0^{\pi/4}-\left\{\frac{3}{4}\left\{\frac{1}{2}x-\frac{\cos x\sin x}{2}\right\}-\frac{\cos x\sin^3 x}{4}\right\}_0^{\pi/4}$$

$$=\left\{\frac{\pi}{8}-\frac{1}{4}\right\}-\left\{\frac{3}{4}\left(\frac{\pi}{8}-\frac{1}{4}\right)-\frac{1}{16}\right\}$$

$$=\frac{\pi}{32}$$

$$\int_0^{\frac{\pi}{2}} (\sin x \cos x)^2 dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x(1-\sin^2 x) dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x - \sin^4 x dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx - \int_0^{\frac{\pi}{2}} \sin^4 x dx$$

We know , By reduction formula

$$\int \sin^n x dx = \frac{n-1}{n} \int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

For n=2

$$\int \sin^2 x dx = \frac{2-1}{2} \int 1 dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 x dx = \frac{1}{2} x - \frac{\cos x \sin x}{2}$$

For n=4

$$\int \sin^4 x dx = \frac{4-1}{4} \int \sin^2 x dx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 x dx = \frac{3}{4}\left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence

$$\left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\}_0^{\frac{\pi}{2}} - \left\{\frac{3}{4}\left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\frac{\pi}{2}}$$

$$\frac{\Pi}{4} - \frac{3}{4}\left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{16}$$

Definite Integrals Ex 20.1 Q64

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\int_0^1 x\log(1+2x)\,dx$$

$$= \left[\frac{x^2\log(1+2x)}{2}\right]_0^1 - \int_0^1 \frac{2x^2}{2(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \int_0^1 \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)}dx$$

$$= \frac{\log(3)}{2} - \left[\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \frac{\log(3)}{2} - \frac{1}{8}\log(3)$$

$$= \frac{3}{8}\log_e(3)$$

Definite Integrals Ex 20.1 Q65

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\tan^2 x + 2\tan x \cot x + \cot^2 x)dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\left(\sec^2 x - 1\right) + 2 + \left(\operatorname{cosec}^2 x - 1\right)\right\}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left\{\sec^2 x + \operatorname{cosec}^2 x\right\}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{cosec}^2 x dx$$

$$\{\tan x\}_{\frac{\pi}{6}}^{\frac{\pi}{3}} + \{-\cot x\}_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\left\{\sqrt{3} - \frac{1}{\sqrt{3}}\right\} - \left\{\frac{1}{\sqrt{3}} - \sqrt{3}\right\}$$

$$2\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\frac{4}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q66

$$I = \int_0^{\pi/4} \left(a^2\cos^2 x + b^2\sin^2 x\right)dx$$

$$I = \int_0^{\pi/4} \left(a^2\left(1-\sin^2 x\right) + b^2\sin^2 x\right)dx$$

$$I = \int_0^{\pi/4} \left(a^2 - a^2\sin^2 x + b^2\sin^2 x\right)dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right)\sin^2 x\, dx$$

$$I = \int_0^{\pi/4} a^2 + \left(b^2 - a^2\right)\frac{(1+\cos 2x)}{2}dx$$

$$I = \left[a^2x + \frac{\left(b^2-a^2\right)}{2}\left(x + \frac{\sin 2x}{2}\right)\right]_0^{\pi/4}$$

$$I = \left[\frac{a^2\pi}{4} + \frac{\left(b^2-a^2\right)}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)\right]$$

$$I = \frac{\left(b^2+a^2\right)\pi}{8} + \frac{\left(b^2-a^2\right)}{4}$$

Definite Integrals Ex 20.1 Q67

$$\int_0^1 \frac{1}{x^4+2x^3+2x^2+2x+1}dx$$

$$\int_0^1 \frac{1}{(x+1)^2(x^2+1)}dx$$

$$\int_0^1 \left\{-\frac{x}{2(x^2+1)} + \frac{1}{2(x+1)} + \frac{1}{2(x+1)^2}\right\}dx$$

$$-\int_0^1 \frac{x}{2(x^2+1)}dx + \int_0^1 \frac{1}{2(x+1)}dx + \int_0^1 \frac{1}{2(x+1)^2}dx$$

$$-\left\{\frac{\log(x^2+1)}{4}\right\}_0^1 + \left\{\frac{\log(x+1)}{2}\right\}_0^1 - \left\{\frac{1}{2(x+1)}\right\}_0^1$$

$$-\frac{\log 2}{4} + \frac{\log 2}{2} - \frac{1}{4} + \frac{1}{2}$$

$$\frac{\log 2}{4} + \frac{1}{4}$$

=(1/4)log(2e)