RD Sharma Solutions Class 12 Maths Chapter 20 Ex 20.2

Definite Integrals Ex 20.2 Q1

Let $I = \int_2^4 \frac{x}{x^2+1} dx$

$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \frac{1}{2}\log\left(1+x^2\right) = F(x)$

By the second funamental theorem of calculus, we obtain

$I = F(4) - F(2)$

$= \frac{1}{2}\left[\log\left(1+4^2\right) - \log\left(1+2^2\right)\right]$

$= \frac{1}{2}\left[\log 17 - \log 5\right]$

$= \frac{1}{2}\log\left(\frac{17}{5}\right)$

Definite Integrals Ex 20.2 Q2

Let $1 + \log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x} dx = dt$$

Now, $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1 + \log 2$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2} dx = \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[\frac{-1}{t}\right]_1^{1+\log 2}$$

$$= \left[\frac{-1}{1+\log 2} + 1\right]$$

$$= \left[\frac{-1+1+\log 2}{1+\log 2}\right]$$

$$= \left[\frac{\log 2}{1+\log 2}\right] \qquad [\because \log e = 1]$$

$$= \frac{\log 2}{\log e + \log 2} \qquad [\log a + \log b = \log ab]$$

$$= \frac{\log 2}{\log 2e}$$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2} dx = \frac{\log 2}{\log 2e}$$

Definite Integrals Ex 20.2 Q3

Let $9x^2 - 1 = t$

Differentiating w.r.t. x, we get

$$18x\,dx = dt$$

$$3x\,dx = \frac{dt}{6}$$

Now, $x = 1 \Rightarrow t = 8$

$x = 2 \Rightarrow t = 35$

$$\therefore \int_1^2 \frac{3x}{9x^2 - 1}dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6}\left[\log t\right]_8^{35}$$

$$= \frac{1}{6}\left(\log 35 - \log 8\right)$$

$$\therefore \int_1^2 \frac{3x}{9x^2 - 1}dx = \frac{1}{6}\left(\log 35 - \log 8\right)$$

Definite Integrals Ex 20.2 Q4

Put $\sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5\left(1 - \tan^2\frac{x}{2}\right) + 6\tan\frac{x}{2}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5 - 5\tan^2\frac{x}{2} + 6\tan\frac{x}{2}}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5 - 5\tan^2\frac{x}{2} + 6\tan\frac{x}{2}} = \int_0^1 \frac{2dt}{5 - 5t^2 + 6t} = \frac{2}{5}\int \frac{dt}{1 - t^2 + \frac{6}{5}t}$$

Forming perfect square by adding and subtracting $\frac{9}{25}$

$$\frac{2}{5}\int_0^1 \frac{dt}{1-t^2+\frac{6}{5}t}$$

$$=\frac{2}{5}\int_0^1 \frac{dt}{\frac{34}{25}-\left(t-\frac{3}{5}\right)^2}$$

$$=\frac{2}{5}\cdot\frac{1}{2}\sqrt{\frac{25}{34}}\log\left(\frac{\sqrt{\frac{34}{25}}+t-\frac{3}{5}}{\sqrt{\frac{34}{25}}-t+\frac{3}{5}}\right)_0^1 \qquad \left[\because \int \frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left(\frac{x+a}{x-a}\right)\right]$$

$$=\frac{1}{\sqrt{34}}\left\{\log\left(\frac{\sqrt{34}+2}{\sqrt{34}-2}\right)-\log\left(\frac{\sqrt{34}-3}{\sqrt{34}+3}\right)\right\}$$

$$=\frac{1}{\sqrt{34}}\log\left(\frac{\left(\sqrt{34}+2\right)\left(\sqrt{34}-3\right)}{\left(\sqrt{34}-2\right)\left(\sqrt{34}-3\right)}\right)$$

$$=\frac{1}{\sqrt{34}}\log\left(\frac{40+5\sqrt{34}}{40-5\sqrt{34}}\right)$$

$$=\frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x+3\sin x}=\frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

Definite Integrals Ex 20.2 Q5

Let $a^2 + x^2 = t^2$

Differentiating w.r.t. x, we get

$$2x\,dx = 2t\,dt$$
$$x\,dx = t\,dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = a \Rightarrow t = \sqrt{2}a$

$$\therefore \int_0^a \frac{x\,dx}{\sqrt{a^2 + x^2}}dx = \int_a^{\sqrt{2}a} \frac{t\,dt}{t}$$
$$= \int_a^{\sqrt{2}a} dt$$
$$= [t]_a^{\sqrt{2}a}$$
$$= \left[\sqrt{2}a - a\right]$$
$$= a\left(\sqrt{2} - 1\right)$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}}dx = a\left(\sqrt{2} - 1\right)$$

Definite Integrals Ex 20.2 Q6

Let $e^x = t$

Differentiating w.r.t. x, we get

$$e^x\,dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = e$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \int_1^e \frac{dt}{1 + t^2}$$
$$= \left[\tan^{-1} t\right]_1^e \qquad \left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t\right]$$
$$= \left[\tan^{-1} e - \tan^{-1} 1\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$
$$= \tan^{-1} e - \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$
Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,
$x = 0 \Rightarrow t = 0$
$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 xe^{x^2}dx = \int_0^1 \frac{e^t dt}{2}$$
$$= \frac{1}{2}\int_0^1 e^t dt$$

$= \frac{1}{2}\left[e^t\right]_0^1$
$= \frac{1}{2}\left[e^1 - e^0\right] \qquad \left[\because e^0 = 1\right]$
$= \frac{1}{2}(e-1)$

$\therefore \int_0^1 xe^{x^2}dx = \frac{1}{2}(e-1)$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$
Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now,
$x = 0 \Rightarrow t = 0$
$x = 3 \Rightarrow t = \log 3$

$\int_1^3 \frac{\cos(\log x)}{x}dx$
$= \int_0^{\log 3} \cos t\,dt \qquad \left[\because \int \cos t = \sin t\right]$
$= \left[\sin t\right]_0^{\log 3}$
$= \sin(\log 3) - \sin 0$
$= \sin(\log 3)$

$\int_1^3 \frac{\cos(\log x)}{x}dx = \sin(\log 3)$

Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 \frac{2x}{1+x^4}\,dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{2x}{1+x^4}\,dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q10

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$$dx = a\cos\theta d\theta$$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\, a\cos\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \qquad \left[\because \left(1 - \sin^2\theta\right) = \cos^2\theta \text{ and } \frac{1 + \cos 2\theta}{2} = \cos 2\theta\right]$$

$$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2\theta\right) d\theta$$

$$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi a^2}{4}$$

$$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$$

Definite Integrals Ex 20.2 Q11

Let $I=\int_0^{\frac{\pi}{2}}\sqrt{\sin\phi}\cos^5\phi\,d\phi=\int_0^{\frac{\pi}{2}}\sqrt{\sin\phi}\,\cos^4\phi\cos\phi\,d\phi$

Also, let $\sin\phi=t\Rightarrow\cos\phi\,d\phi=dt$

When $\phi=0,\ t=0$ and when $\phi=\frac{\pi}{2},\ t=1$

$$\therefore I=\int_0^1\sqrt{t}\left(1-t^2\right)^2dt$$

$$=\int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$

$$=\int_0^1\left[t^{\frac{1}{2}}+t^{\frac{9}{2}}-2t^{\frac{5}{2}}\right]dt$$

$$=\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{t^{\frac{11}{2}}}{\frac{11}{2}}-\frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$$

$$=\frac{2}{3}+\frac{2}{11}-\frac{4}{7}$$

$$=\frac{154+42-132}{231}$$

$$=\frac{64}{231}$$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1+\cos\theta = t^2$

Differentiating w.r.t. x, we get

$$-\sin\theta\, d\theta = 2t\, dt$$

$$\sin\theta\, d\theta = -2t\, dt$$

Now,

$x = 0 \Rightarrow t = \sqrt{2}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}}$$

$$= \int_{\sqrt{2}}^1 \frac{-2t dt}{t}$$

$$= -2\int_{\sqrt{2}}^1 dt$$

$$= -2\left[t\right]_{\sqrt{2}}^1$$

$$= -2\left[1-\sqrt{2}\right]$$

$$= 2\left[\sqrt{2}-1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}} = 2\left[\sqrt{2}-1\right]$$

Definite Integrals Ex 20.2 Q14

Let $3+4\sin x = t$

Differentiating w.r.t. x, we get

$$4\cos x dx = dt$$

$$\cos x dx = \frac{dt}{4}$$

Now,

$x = 0 \Rightarrow t = 3$

$x = \frac{\pi}{3} \Rightarrow t = 3+2\sqrt{3}$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx$$

$$= \int_3^{3+2\sqrt{3}} \frac{dt}{4t}$$

$$= \frac{1}{4}[\log t]_3^{3+2\sqrt{3}}$$

$$= \frac{1}{4}\left[\log\left(3+2\sqrt{3}\right) - \log 3\right]$$

$$= \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

Definite Integrals Ex 20.2 Q15

Let $\tan^{-1} x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx$$

$$= \int_0^{\frac{\pi}{4}} t^{\frac{1}{2}} dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[\left(\frac{\pi}{4}\right)^{\frac{3}{2}} - 0\right]$$

$$= \frac{1}{12}\pi^{\frac{3}{2}}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1} x}}{1+x^2} dx = \frac{1}{12}\pi^{\frac{3}{2}}$$

Definite Integrals Ex 20.2 Q16

$$\int_0^2 x\sqrt{x+2}\,dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2tdt$

When $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}\,dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2tdt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2 dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

Definite Integrals Ex 20.2 Q17

Let $x = \tan\theta$

Differentiating w.r.t. x, we get

$$dx = \sec^2\theta d\theta$$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta \qquad \left[\because \tan^2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}(\tan 2\theta)\sec^2\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta\sec^2\theta\, d\theta$$

Applying by parts, we get

$$= 2\left[\theta \int_0^{\frac{\pi}{4}} \sec^2\theta d\theta - \int_0^{\frac{\pi}{4}}\left(\sec^2\theta d\theta\right)\frac{d\theta}{d\theta}d\theta\right]$$

$$=2\left[\theta\tan\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan\theta d\theta]$$

$$= 2\left[\theta\tan\theta + \log(\cos\theta)\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - 0 - 0\right]$$

$$= 2\left[\frac{\pi}{4} + \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

$$\therefore \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = \frac{\pi}{2} - \log 2$$

Definite Integrals Ex 20.2 Q18

Let $\sin^2 x = t$

Differentiating w.r.t. x, we get

$$2\sin x\cos x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{4}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\times\frac{\pi}{4}$$

$$= \frac{\pi}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x\cos x}{1+\sin^4 x}dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.2 Q19

Putting $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{1-\tan^2\frac{x}{2}}{\sec^2\frac{x}{2}}$

$$\sin x = \frac{2\tan\frac{x}{2}}{\sec^2\frac{x}{2}}$$

$$\therefore \qquad I = \int_0^{\frac{\pi}{2}} \frac{1}{a\cos x + b\sin x}dx = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{a\left(1-\tan^2\frac{x}{2}\right)+2b\tan^2\frac{x}{2}}dx$$

Put $\tan\dfrac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

If $x = 0,\ t = 0$ and if $x = \dfrac{\pi}{2},\ t = 1$

$$\Rightarrow \qquad I = 2\int_0^1 \frac{dt}{a\left(1-t^2\right)+2bt}$$

$$= 2\int_0^1 \frac{dt}{-at^2+2bt+a}$$

$$= 2\int_0^1 \frac{dt}{-a\left[t^2-\frac{2b}{a}t-1\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{-\left[\left(t-\frac{b}{a}\right)^2-1-\frac{b^2}{a^2}\right]}$$

$$= \frac{2}{a}\int_0^1 \frac{dt}{\left(\frac{b^2}{a^2}+1\right)-\left(t-\frac{b}{a}\right)^2}$$

$$= \frac{2}{a}\left[\frac{1}{2\sqrt{\frac{b^2+a^2}{a^2}}}\left(\log\left|\frac{\frac{\sqrt{b^2+a^2}}{a^2}+\left(t-\frac{b}{a}\right)}{\frac{\sqrt{b^2+a^2}}{a^2}-\left(t-\frac{b}{a}\right)}\right|\right)\right]_0^1$$

$$= \frac{1}{\sqrt{b^2+a^2}}\log\left(\frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}}\right)$$

Definite Integrals Ex 20.2 Q20

We know that $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}\,dx = \int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\dfrac{5\left(1+\tan^2\frac{x}{2}\right)+4\left(2\tan\frac{x}{2}\right)}{1+\tan^2\frac{x}{2}}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1+\tan^2\frac{x}{2}}{\left(5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}\right)}\,dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}\,dx$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}+8\tan\frac{x}{2}}\,dx$$

$$= \int_0^1 \frac{2dt}{5 + 5t^2 + 8t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{1 + t^2 + \frac{8}{5}t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{1 - \frac{16}{25} + \frac{16}{25} + t^2 + \frac{8}{5}t} \qquad \left[\text{Adding and subtracting } \frac{16}{25}\right]$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{\left(\frac{3}{2}\right)^2 + \left(t + \frac{4}{5}\right)^2}$$

$$= \frac{2}{5}\left[\frac{5}{3}\tan^{-1}\left(t+\frac{4}{5}\right)\times\frac{5}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}\left(1+\frac{4}{5}\right)\times\frac{5}{3} - \tan^{-1}\frac{4}{5}\times\frac{5}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1} 3 - \tan^{-1}\frac{4}{3}\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}\left(\frac{3-\frac{4}{3}}{1+3\times\frac{4}{3}}\right)\right]_0^1 \qquad \left[\because \tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A - B}{1 + AB}\right)\right]$$

$$= \frac{2}{3}\left[\tan^{-1}\frac{\frac{5}{3}}{5}\right]$$

$$= \frac{2}{3}\tan^{-1}\frac{1}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{5 + 4\sin x}dx = \frac{2}{3}\tan^{-1}\frac{1}{3}$$

Definite Integrals Ex 20.2 Q21

We have,

$$\int_0^{\pi} \frac{\sin x}{\sin x + \cos x}\,dx$$

Let $\sin x = K\left(\sin x + \cos x\right) + L\frac{d}{dx}\left(\sin x + \cos x\right)$

$$= K\left(\sin x + \cos x\right) + L\left(\cos x - \sin x\right)$$

$$= \sin x\left(K - L\right) + \cos x\left(K + L\right)$$

Equating similar terms

$$K - L = 1$$

$$K + L = 0$$

$\Rightarrow K = \frac{1}{2}$ and $L = -\frac{1}{2}$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}\,dx = \frac{1}{2}\int_0^{\pi} dx + \left(\frac{-1}{2}\right)\int_0^{\pi} \frac{\cos x - \sin x}{\sin x + \cos x}\,dx$$

$$= \frac{1}{2}\left[x\right]_0^{\pi} - \frac{1}{2}\left(\log\left|\sin x + \cos x\right|\right)_0^{\pi} = \frac{\pi}{2} - \frac{1}{2}(0) = \frac{\pi}{2}$$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x}\,dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q22

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}},\ \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{3+2\sin x+\cos x}$$

$$= \frac{1}{3+2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= \frac{\left(1+\tan^2\frac{x}{2}\right)}{3\left(1+\tan^2\frac{x}{2}\right)+4\tan\frac{x}{2}+\left(1-\tan^2\frac{x}{2}\right)}$$

$$= \frac{\sec^2\frac{x}{2}dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

$$\therefore \int_0^{\pi}\frac{1}{3+2\sin x+\cos x}dx = \int_0^{\pi}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \int_0^{\pi} \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4}$$

$$= \int_0^{\infty} \frac{dt}{t^2 + 2t + 2}$$

$$= \int_0^{\infty} \frac{dt}{(t+1)^2 + 1}$$

$$= \left[\tan^{-1}(t+1)\right]_0^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}(0+1)$$

$$= \tan^{-1}(\infty) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}\left(\tan \frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{2\pi - \pi}{4}$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{1}{3 + 2\sin x + \cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q23

We have,

$$\int_0^1 1 \cdot \tan^{-1} x \, dx = \tan^{-1} x \int_0^1 dx - \int_0^1 \left(\int dx.\right) \frac{d}{dx}\left(\tan^{-1} x\right) dx$$

$$= \left[x \tan^{-1} x\right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx$$

$$= \left[x \tan^{-1} x - \frac{1}{2}\log\left(1+x^2\right)\right]_0^1$$

$$= \frac{\pi}{4} - \frac{1}{2}(\log 2 - 0)$$

$$= \frac{\pi}{4} - \frac{1}{2}\log 2$$

$$\therefore \int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2}\log 2$$

Definite Integrals Ex 20.2 Q24

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = \frac{x}{\sqrt{1-x^2}}, g = \sin^{-1} x$$

$$f = -\sqrt{1-x^2}, g' = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \sin^{-1} x - \int (-1) dx$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \sin^{-1} x + x$$

Hence

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ x - \sqrt{1-x^2} \sin^{-1} x \right\}_0^{\frac{1}{2}}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ \frac{1}{2} - \sqrt{1-(\frac{1}{2})^2} \sin^{-1} \frac{1}{2} \right\}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ \frac{1}{2} - \frac{\sqrt{3}}{2} \frac{\Pi}{6} \right\}$$

Definite Integrals Ex 20.2 Q25

$$I = \int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{2\sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}\right) dx$$

Let $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$x = 0 \Rightarrow t = -1$ and $x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = \sqrt{2}\int_{-1}^{0} \left(\frac{1}{\sqrt{1-t^2}}\right) dt$$

$$I = \sqrt{2}\left[\sin^{-1} t\right]_{-1}^{0}$$

$$I = \sqrt{2}\left[\sin^{-1}(0) - \sin^{-1}(-1)\right]$$

$$I = \frac{\pi}{\sqrt{2}}$$

Definite Integrals Ex 20.2 Q26

We have,

$$\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{4} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2 x \tan^3 x \, dx = \frac{1}{2}\int_0^1 t^3 \, dt = \frac{1}{2}\left[\frac{t^4}{4}\right]_0^1 = \frac{1}{8}$$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q27

We know that,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\frac{1}{5 + 3\cos x} = \frac{1}{5 + 3\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} = \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 3\left(1 - \tan^2 \frac{x}{2}\right)} = \frac{\sec^2 \frac{x}{2} dx}{8 + 2\tan^2 \frac{x}{2}}$$

$$\therefore \int_0^\pi \frac{dx}{5 + 3\cos x} dx = \frac{1}{2}\int_0^\pi \frac{\sec^2 \frac{x}{2}}{2^2 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \frac{1}{2}\int_0^\pi \left(\frac{\sec^2 \frac{x}{2} dx}{2^2 + \tan^2 \frac{x}{2}}\right) dx$$

$$= \int_0^\infty \frac{dt}{2^2 + t^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^\infty$$

$$= \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^\pi \frac{dx}{5 + 3\cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

Let $\tan x = t$

Differentiating w.r.t. x, we get

$$\sec^2 x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] \quad = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I = \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x\sec^2\frac{x}{2}}{2} + \tan\frac{x}{2} \right) dx$$

$$= \left[x\tan\left(\frac{x}{2}\right) - \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}\, dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}\, dx \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

Let $t = \tan^{-1} x$

$$dt = \frac{1}{1+x^2} dx$$

$$x = 0, t = 0$$

$$x = 1, t = \frac{\pi}{4}$$

$$I = \int_0^{\frac{\pi}{4}} t\,dt$$

$$= \left[\frac{t^2}{2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \frac{\pi^2}{16}$$

$$= \frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x}\, dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + 1 - (\cos x - \sin x)^2} \right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{4 - (\cos x - \sin x)^2} \right) dx$$

$$I = \frac{1}{4} \left[\log \left| \frac{2 + \sin x - \cos x}{2 - \sin x + \cos x} \right| \right]_0^{\pi/4}$$

$$I = -\frac{1}{4} \log \left(\frac{1}{3} \right)$$

$$I = \frac{1}{4} \log_e 3$$

Definite Integrals Ex 20.2 Q32

We have,

$$\int_0^1 x \tan^{-1} x\, dx = \tan^{-1} x \int_0^1 x dx - \int_0^1 \left(\int x dx \right) \frac{d}{dx} \left(\tan^{-1} x \right) dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{x^2}{1 + x^2} dx$$

$$= \left[\frac{x^2}{2} \tan^{-1} x \right]_0^1 - \frac{1}{2} \int_0^1 \frac{1 + x^2 - 1}{1 + x^2} dx$$

$$= \frac{1}{2} \left(\frac{\pi}{4} \right) - \frac{1}{2} \left[\int_0^1 dx - \int_0^1 \frac{dx}{1 + x^2} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[x - \tan^{-1} x \right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[1 - \frac{\pi}{4} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_0^1 x \tan^{-1} x dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

Let $I=\int \frac{1-x^2}{x^4+x^2+1}dx = -\int \frac{x^2-1}{x^4+x^2+1}dx.$

Then,

$$I=-\int \frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$ [Dividing the numerator and denominator by x^2]

$$\Rightarrow \quad I=-\int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2-1^2}dx$$

Let, $x+\frac{1}{x}=u$. Then, $d\left(x+\frac{1}{x}\right)=du \Rightarrow \left(1-\frac{1}{x^2}\right)dx=du$

$$\therefore \quad I=-\int \frac{du}{u^2-1^2}$$

$$\Rightarrow \quad I=-\frac{1}{2(1)}\log\left|\frac{u-1}{u+1}\right|+C$$

$$\Rightarrow \quad I=-\frac{1}{2}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right|+C=-\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|+C$$

$$\therefore \quad \int_0^1 \frac{1-x^2}{x^4+x^2+1}dx=\left[-\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|\right]_0^1=\left(-\frac{1}{2}\log\left|\frac{1}{3}\right|\right)-\left(-\frac{1}{2}\log|1|\right)=\log\sqrt{3}$$

$$=\log 3^{\frac{1}{2}}$$

$$=\frac{1}{2}\log 3$$

Definite Integrals Ex 20.2 Q34

Let $1 + x^2 = t$

Differentiating w.r.t. x, we get

$2x dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = 2$

$$\int_0^1 \frac{24x^3}{\left(1+x^2\right)^4} dx = \int_1^2 \frac{12\left(t-1\right)}{t^4} dt$$

$$= 12\int_1^2 \left(\frac{1}{t^3} - \frac{1}{t^4}\right) dt$$

$$= 12\left[-\frac{1}{2t^2} - \frac{1}{3t^3}\right]_1^2$$

$$= 12\left[-\frac{1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3}\right]$$

$$= 12\left[\frac{-3+1+12-8}{24}\right]$$

$$= \frac{12 \times 2}{24} = 1$$

$$\therefore \int_0^1 \frac{24x^3}{\left(1+x^2\right)^4} dx = 1$$

Definite Integrals Ex 20.2 Q35

Let $x - 4 = t^3$

Differentiating w.r.t. x, we get

$dx = 3t^2 dt$

Now, $x = 4 \Rightarrow t = 0$

$x = 12 \Rightarrow t = 2$

$$\therefore \int_4^{12} x\left(x-4\right)^{\frac{1}{3}} dx = \int_0^2 \left(t^3+1\right) t.3t^2 dt$$

$$= 3\int_0^2 \left(t^6 + 4t^3\right) dt$$

$$= 3\left[\frac{t^7}{7} + t^4\right]_0^2$$

$$= 3\left[\frac{128}{7} + 16\right]$$

$$= \frac{720}{7}$$

$$\therefore \int_4^{12} x\left(x-4\right)^{\frac{1}{3}} dx = \frac{720}{7}$$

Definite Integrals Ex 20.2 Q36

We have,

$$\int_0^{\frac{\pi}{2}} \underset{\text{I}}{x^2} \underset{\text{II}}{\sin x}\, dx$$

Using by parts, we get

$$x^2 \int \sin x dx - \int \left(\int \sin x dx \right) \frac{dx^2}{dx} . dx$$

$$= x^2 \cos x + \int \cos x . 2x dx$$

Again applying by parts

$$= x^2 \cos x + 2 \left[x \int \cos x dx - \int \left(\int \cos x dx \right) . \frac{dx}{dx} . dx \right]$$

$$= x^2 \cos x + 2 \left[x \sin x - \int \sin x dx \right]$$

$$= \left[x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{2}}$$

$$= \pi + 0 - 0 - 0 - 2$$

$$= \pi - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37

Let $x = \cos 2\theta$
Differentiating w.r.t. x, we get

$$dx = -2\sin 2\theta d\theta$$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}(-2\sin 2\theta)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}(\ 2\sin 2\theta)d\theta \qquad \left[\because \sin 2\theta = 2\sin\theta\cos\theta;\ \text{and}\ \ \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} (1-\cos 2\theta)d\theta$$

$$= 2\left[\theta - \frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = \int_0^1 \frac{-x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)^2} = -\int_0^1 \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2}$$

Let $x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2}dx = dt$

When $x = 0 \Rightarrow t = \infty$

$x = 1 \Rightarrow t = 2$

$$\therefore \int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = -\int_\infty^2 \frac{dt}{t^2} = \int_2^\infty \frac{dt}{t^2} = \left[-\frac{1}{t}\right]_2^\infty = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4\ dx$.

Therefore, $\int 5x^4 \sqrt{x^5 + 1}\ dx = \int \sqrt{t}\ dt = \frac{2}{3}dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3}\left(x^5 + 1\right)^{\frac{3}{2}}$

Hence, $\int_{-1}^{1} 5x^4 \sqrt{x^4 + 1}\ dx = \frac{2}{3}\left[\left(x^5 + 1\right)^{\frac{3}{2}}\right]_{-1}^{1}$

$$= \frac{2}{3}\left[\left(1^5 + 1\right)^{\frac{3}{2}} - \left((-1)^5 + 1\right)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Alternatively, first we transform the integral and then evaluate the transformed integral with new limits.

Let $t = x^5 + 1$. Then $dt = 5x^4\ dx$.

Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

Thus, as x varies from -1 to 1, t varies from 0 to 2

Therefore $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\ dx = \int_0^2 \sqrt{t}\ dt$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_0^2 = \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1 + 3\sin^2 x}\ dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x\left(\sec^2 x + 3\tan^2 x\right)}\ dx$$

Put $\tan x = t$

$\sec^2 x\ dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$$I = \int_0^{\infty} \frac{1}{\left(1 + t^2\right)\left(1 + 4t^2\right)}\ dt$$

$$I = -\frac{1}{3}\int_0^{\infty}\left[\frac{1}{\left(1 + t^2\right)} - \frac{1}{\left(1 + 4t^2\right)}\right] dt$$

$$I = -\frac{1}{3}\left[\tan^{-1} t - 2\tan^{-1} 2t\right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

Let $I = \int_0^{\frac{\pi}{4}} \sin^3 2t \cos 2t\, dt$. consider $\int \sin^3 2t \cos 2t\, dt$

Put $\sin 2t = u$ so that $2 \cos 2t\, dt = du$ or $\cos 2t\, dt = \frac{1}{2} du$

So $\int \sin^3 2t \cos 2t\, dt = \frac{1}{2} \int u^3\, du$

$$= \frac{1}{8}\left[u^4\right] = \frac{1}{8} \sin^4 2t = F(t) \text{ say}$$

Therefore, by the second fundamental theorem of integrals calculus

$I = F\left(\frac{\pi}{4}\right) - F(o) = \frac{1}{8}\left[\sin^4 \frac{\pi}{2} - \sin^4 0\right] = \frac{1}{8}$

Definite Integrals Ex 20.2 Q42

Let $5 - 4\cos\theta = t$

Differentiating w.r.t. x, we get

$$4\sin\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 1$

$\theta = \pi \Rightarrow t = 9$

$\therefore \int_0^{\pi} 5(5 - 4\cos\theta)^{\frac{1}{4}} \sin\theta d\theta$

$= \frac{5}{4}\int_1^9 t^{\frac{1}{4}} dt$

$= \frac{5}{4}\left[\frac{4}{5}.t^{\frac{5}{4}}\right]_1^9$

$= 3^{\frac{5}{2}} - 1$

$= 9\sqrt{3} - 1$

$\therefore \int_0^{\pi} 5(5 - 4\cos\theta)^{\frac{1}{4}} \sin\theta d\theta = 9\sqrt{3} - 1$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x, we get

$$2\sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\therefore \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta = \frac{1}{2}\int_0^{\sqrt{3}} t dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_0^{\sqrt{3}}$$

$$= \frac{3}{4}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

Definite Integrals Ex 20.2 Q44

Let $x^{\frac{2}{3}} = t$

Differentiating w.r.t. x, we get

$$\frac{3}{2}\sqrt{x}dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = \pi^{\frac{2}{3}} \Rightarrow t = \pi$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx$$

$$= \frac{2}{3}\int_0^{\pi} \cos^2 t\, dt$$

$$= \frac{1}{3}\int_0^{\pi} 1 + \cos 2t\, dt \qquad \left[\because 2\cos^2 t = t + \cos 2t\right]$$

$$= \frac{1}{3}\left[t + \frac{\sin 2t}{t}\right]_0^{\pi}$$

$$= \frac{1}{3}\left[\pi + 0 - 0 - 0\right] = \frac{\pi}{3}$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx = \frac{\pi}{3}$$

Definite Integrals Ex 20.2 Q45

Let $1+\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

When $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{dx}{x(1+\log x)^2}$$

$$= \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[-\frac{1}{t}\right]_1^{1+\log 2}$$

$$= 1 - \frac{1}{1+\log 2}$$

$$= \frac{\log 2}{1+\log 2}$$

$$\therefore \int_1^2 \frac{dx}{x(1+\log x)^2} = \frac{\log 2}{1+\log 2}$$

Definite Integrals Ex 20.2 Q46

We have,

$$\int_0^{\frac{\pi}{2}} \cos^5 x\,dx = \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 \cos x\,dx$$

Let $\sin x = t$

Differentiating w.r.t. x, we get

$\cos x dx = dt$

When $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 \cos x\,dx$$

$$= \int_0^1 \left(1 - t^2\right)^2 dt$$

$$= \int_0^1 \left(1 - 2t^2 + t^4\right) dt$$

$$= \left[t - \frac{2}{3}t^3 + \frac{t^5}{5}\right]_0^1$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{8}{15}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^5 x\,dx = \frac{8}{15}$$

Definite Integrals Ex 20.2 Q47

Let $I = \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)} dx$. We first find the anti derivative of the integrand.

Pu $30 - x^{\frac{3}{2}} = t$. Then $-\frac{3}{2}\sqrt{x}\ dx = dt$ or $\sqrt{x}\ dx = -\frac{2}{3}dt$

Thus, $\int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = -\frac{2}{3}\int \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right] = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right] = f(x)$

Therefore, by the second fundamental theorem of calculus, we have

$$I = F(9) - F(4) = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right]_4^9$$

$$= \frac{2}{3}\left[\frac{1}{(30 - 27)} - \frac{1}{30 - 8}\right] = \frac{2}{3}\left[\frac{1}{3} - \frac{1}{22}\right] = \frac{19}{99}$$

Definite Integrals Ex 20.2 Q48

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

When $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

Now,

$$\int_0^{\pi} \sin^3 x\,(1 + 2\cos x)(1 + \cos x)^2\,dx$$

$$= \int_0^{\pi} \sin^2 x\,(1 + 2\cos x)(1 + \cos x)^2 . \sin x dx$$

$$= -\int_{-1}^{1} (1 - t^2)(1 + 2t)(1 + t)^2\,dt \qquad [\sin^2 x = 1 - \cos^2 x]$$

$$= \int_{-1}^{1} (1 + 2t - t^2 - 2t^3)(1 + t^2 + 2t)dt$$

$$= \int_{-1}^{1} (1 - t^2 + 2t + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4)dt$$

$$= \int_{-1}^{1} (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5)dt$$

$$= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t6}{3}\right]_{-1}^{1}$$

$$= \left[2 + 0 + \frac{8}{3} - 0 - 2 - 0\right] = \frac{8}{3}$$

$$\therefore \int_0^{\pi} \sin^3 x\,(1 + 2\cos x)(1 + \cos x)^2\,dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49

$$I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx$$

Let $t = \sin x$

$$dt = \cos x dx$$

$$x = 0, t = 0$$

$$x = \frac{\pi}{2}, t = 1$$

$$I = \int_0^1 2t\tan^{-1}(t)dt$$

$$= 2\left[\frac{1}{2}t^2\tan^{-1}t - \frac{t}{2} + \frac{1}{2}\tan^{-1}t\right]_0^1$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx = 2\int_0^1 t \tan^{-1} t\, dt \qquad [\because \sin 2x = 2\sin x \cos x]$$

Using by parts

$$= 2\left\{\tan^{-1} t \int t dt - \int \left(\int t dt\right)\frac{d \tan^{-1} t}{dt} dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1} - \frac{1}{2}\int \frac{t^2}{1+t^2} dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left(\int dt - \int \frac{dt}{1+t^2}\right)\right\}$$

$$= 2\left[\frac{t^2}{2}\tan^{-1} t - \frac{1}{2}\left(t - \tan^{-1} t\right)\right]_0^1$$

$$= 2\left\{\frac{1}{2}\frac{\pi}{4} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right\}$$

$$= 2\left\{\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right\}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\cos^{-1} x\right)^2 \int_0^1 dx - \int_0^1 \left(\int dx\right) \frac{d\left(\cos^{-1} x\right)^2}{dx} dx$$

$$= \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx$$

Now,

Let $\cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$

When $x = 0 \Rightarrow t = \frac{\pi}{2}$

$x = 1 \Rightarrow t = 0$

$$\therefore \int_0^1 \frac{2x\cos^{-1} x}{\sqrt{1-x^2}} dx = -2\int_{\frac{\pi}{2}}^0 t\cos t\,dt = 2\int_0^{\frac{\pi}{2}} t\cos t\,dt$$

$$= 2\left[t\int \cos t\,dt - \int \left(\cos t\,dt\right)\frac{dt}{dt}.dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t - \int \sin t\,dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t + \cos t\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2} - 1\right]$$

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= 0 - 0 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= (\pi - 2)$$

$$\therefore \int_0^1 \left(\cos^{-1} x\right)^2 dx = (\pi - 2)$$

Definite Integrals Ex 20.2 Q53

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}}dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2\cos^2\frac{x}{2}}}{\left(2\sin^2\frac{x}{2}\right)^{\frac{3}{2}}}dx \qquad \left[\because 1+\cos x = 2\cos^2\frac{x}{2},\ 1-\cos x = 2\sin^2\frac{x}{2}\right]$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2}\cos\frac{x}{2}}{2\sqrt{2}\sin^3\frac{x}{2}}dx$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}\,cosec^2\frac{x}{2}\,dx \qquad \left[\because cosec^2\frac{x}{2} = \frac{1}{\sin^2\frac{x}{2}},\ \cot\frac{x}{2} = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right]$$

Let $\cot\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{-1}{2}cosec^2\frac{x}{2} = dt$$

Now, $x = \frac{\pi}{3} \Rightarrow t = \sqrt{3}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot\frac{x}{2}\,cosec^2\frac{x}{2}\,dx = -\int_{\sqrt{3}}^{1} t\,dt = -\left[\frac{t^2}{2}\right]_{\sqrt{3}}^{1} = \frac{-1}{2}[1-3]$$

$$= 1$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{3}{2}}}dx = 1$$

Definite Integrals Ex 20.2 Q54

Substitute $x^2 = a^2 \cos 2\theta$

Differentiating w.r.t. x, we get

$$2x\,dx = -2a^2 \sin 2\theta\, d\theta$$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_0^a x\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{a^2(1 - \cos 2\theta)}{a^2 - (1 - \cos 2\theta)}}\left(-a^2 \sin 2\theta\right)d\theta$$

$$= -a^2 \int_{\frac{\pi}{4}}^{0} \frac{\sin\theta}{\cos\theta} \sin 2\theta\, d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} 2\sin^2\theta\, d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} \left(1 - \cos 2\theta\right)d\theta$$

$$= a^2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$\therefore \int_0^a x\sqrt{\frac{a^2 - x^2}{a^2 + x^2}}dx = a^2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

Definite Integrals Ex 20.2 Q55

Let $x = a\cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2a\sin 2\theta$

Now, $x = -a \Rightarrow \theta = \frac{\pi}{2}$

$x = a \Rightarrow \theta = 0$

$$\therefore \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx = \int_{\frac{\pi}{2}}^{0}\sqrt{\frac{a\left(1-\cos 2\theta\right)}{a\left(1+\cos 2\theta\right)}}\left(-2\sin 2\theta\right)d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}}\frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta$$

$$\left[\begin{array}{l}\because 1-\cos 2\theta = 2\sin^2\theta \\ \quad 1+\cos 2\theta = 2\cos^2\theta \\ -\int_a^b f\left(x\right)dx = \int_b^a f\left(x\right)dx\end{array}\right]$$

$$= 2a\int_{0}^{\frac{\pi}{2}}\frac{\sin\theta.2\sin\theta\cos\theta}{\cos\theta}$$

$$= 4a\int_{0}^{\frac{\pi}{2}}s\text{ in}^2\theta d\theta$$

$$= 2a\int_{0}^{\frac{\pi}{2}}\left(1-\cos 2\theta\right)d\theta$$

$$= 2a\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2a\left[\frac{\pi}{2}-0-0+0\right] = \pi a$$

$$\therefore \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx = \pi a$$

Definite Integrals Ex 20.2 Q56

Let $\cos x = t$

Differentiating w.r.t. x, we get

$$-\sin x dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$$x = \frac{\pi}{2} \Rightarrow t = 0$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3\cos x + 2}$$

$$= -\int_1^0 \frac{tdt}{t^2 + 3t + 2}$$

$$= \int_0^1 \frac{tdt}{(t+2)(t+1)} \qquad \left[\because -\int_a^b f(x) = \int_b^a f(x)\right]$$

$$= \int_0^1 \left(-\frac{1}{t+1} + \frac{2}{t+2}\right) dt \qquad \text{[Applying partial fraction]}$$

$$= \left[-\log|1+t| + 2\log|t+2|\right]_0^1$$

$$= -\log 2 + 2\log 3 + 0 - 2\log 2$$

$$= 2\log 3 - 3\log 2$$

$$= \log \frac{9}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3\cos x + 2} = \log \frac{9}{8}$$

Definite Integrals Ex 20.2 Q57

$$I = \int_0^{\pi/2} \frac{\tan x}{1 + m^2 \tan^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x} dx$$

Put $\sin^2 x = t$ then $2\sin x\cos x \, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(1-t) + m^2 t} dt$$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(m^2-1)t + 1} dt$$

$$I = \frac{1}{2}\left[\frac{1}{m^2-1}\log\left|(m^2-1)t + 1\right|\right]_0^1$$

$$I = \frac{1}{2}\left[\frac{1}{m^2-1}\log\left|m^2\right| - \frac{1}{m^2-1}\ln|1|\right]$$

$$I = \frac{1}{2}\left[\frac{\log\left|m^2\right|}{m^2-1}\right]$$

$$I = \frac{1}{2}\left[\frac{2\log|m|}{m^2-1}\right]$$

$$I = \frac{\log|m|}{m^2-1}$$

Definite Integrals Ex 20.2 Q58

$$I = \int_0^{1/2} \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$$

Let $x = \sin u$

$dx = \cos u \, du$

$$I = \int_0^{\pi/6} \frac{1}{(1+\sin^2 u)}\,du$$

$$I = \int_0^{\pi/6} \frac{\sec^2 u}{(1+2\tan^2 u)}\,du$$

Let $\tan u = v$

$dv = \sec^2 u \, du$

$$I = \int_0^{1/\sqrt{3}} \frac{1}{(1+2v^2)}\,dv$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{2}v\right)\right]_0^{1/\sqrt{3}}$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$$

Definite Integrals Ex 20.2 Q59

$$I = \int_{1/3}^{1} \frac{(x - x^3)^{1/3}}{x^4} dx$$

$$I = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = t$

$$\frac{-2}{x^3} dx = dt$$

$$x = \frac{1}{3} \Rightarrow t = 8 \text{ and } x = 1 \Rightarrow t = 0$$

$$I = -\frac{1}{2}\int_{8}^{0} (t)^{1/3} dt$$

$$I = -\frac{1}{2}\left[\frac{t^{4/3}}{4/3}\right]_8^0$$

$$I = -\frac{1}{2}[0 - 12]$$

$$I = 6$$

Definite Integrals Ex 20.2 Q60

$$\int \sec^2 x \frac{\tan^2 x}{\tan^6 x + 2\tan^3 x + 1} dx$$

$$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$$

$$\int \frac{u^2}{u^6 + 2u^3 + 1} du$$

$$v = u^3 \rightarrow \frac{dv}{du} = 3u^2$$

$$\frac{1}{3}\int \frac{1}{v^2 + 2v + 1} dv$$

$$\frac{1}{3}\int \frac{1}{(v+1)^2} dv$$

$$-\frac{1}{3(v+1)}$$

$$-\frac{1}{3(u^3+1)}$$

$$-\frac{1}{3(\tan^3 x + 1)}$$

$$\left\{-\frac{1}{3(\tan^3 x + 1)}\right\}_0^{\frac{\pi}{4}}$$

$$\left\{-\frac{1}{6} + \frac{1}{3}\right\}$$

$$\frac{1}{6}$$

Definite Integrals Ex 20.2 Q61

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)}\tan^2 x\cos^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x\sin^2 x}\sin^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\sin^3 x dx$$

$$\cos x = t \rightarrow -\sin x = \frac{dt}{dx}$$

$$-\int_1^0 \sqrt{t}(1-t^2)dt$$

$$\int_0^1 (\sqrt{t} - t^{\frac{5}{2}})dt$$

$$\left\{\frac{2t^{\frac{3}{2}}}{3} - \frac{2t^{\frac{7}{2}}}{7}\right\}_0^1$$

$$\frac{2}{3} - \frac{2}{7}$$

$$\frac{8}{21}$$

Definite Integrals Ex 20.2 Q62

$$I = \int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^{n-1}} dx$$

Let $\cos\frac{x}{2} + \sin\frac{x}{2} = t$

$$\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx = 2dt$$

$x = 0 \Rightarrow t = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$

$$I = \int_1^{\sqrt{2}} \frac{2}{(t)^{n-1}} dt$$

$$I = \left[\frac{2t^{-n+2}}{-n+2}\right]_1^{\sqrt{2}}$$

$$I = \frac{2}{2-n}\left[\left(\sqrt{2}\right)^{2-n} - 1\right]$$

$$I = \frac{2}{2-n}\left[2^{1-\frac{n}{2}} - 1\right]$$