RD Sharma Solutions Class 12 Maths Chapter 20 Ex 20.5

Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 3$ and $f(x) = (x+4)$

$$h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$\Rightarrow I = \int_0^3 (x+4)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4+(h+4)+(2h+4)+-----+((n-1)h+4)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4n+h(1+2+3+-----+(n-1))\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[4n+h\left(\frac{n(n-1)}{2}\right)\right] \qquad \left[\because h\to 0\ \&\ h = \frac{3}{n} \Rightarrow n\to\infty\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{3}{n}\left[4n+\frac{3}{n}\frac{(n^2-1)}{2}\right]$$

$$\Rightarrow I = \lim_{n\to\infty} 12+\frac{9}{2}\left(1-\frac{1}{n}\right)$$

$$= 12+\frac{9}{2} = \frac{33}{2}$$

$$\therefore \int_0^3 (x+4)dx = \frac{33}{2}$$

Definite Integrals Ex 20.5 Q2

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0,\ b = 2$

$$\Rightarrow h = \frac{2}{n}\ \&\ f(x) = x + 3$$

Thus, we have,

$$I = \int_0^2 (x+3)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + ----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3 + (h+3) + (2h+3) + (3h+3) ----+(n-1)h + 3\right]$$

$$= \lim_{h\to 0} h\left[3n + h(1 + 2 + 3 + ----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[3n + h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n}\ \&\text{ if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[3n + \frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[6 + \frac{2}{n}n^2\left(1 - \frac{1}{n}\right)\right]$$

$$= 6 + 2 = 8$$

$$\therefore \int_0^2 (x+3)dx = 8$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 3$ and $f(x) = 3x-2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (3x-2)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\{3(1+h)-2\}+\{3(1+2h)-2\}+-----+\ \{3(1+(n-1)h)-2\}\right]$$

$$= \lim_{h\to 0} h\left[n+3h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[n+3h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \qquad \therefore \text{if } h\to 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{6}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{6}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 2+6 = 8$$

$$\therefore \int_1^3 (3x-2)dx = 8$$

Definite Integrals Ex 20.5 Q4

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a=-1,\ b=1$ and $f(x)=x+3$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_{-1}^{1}(x+3)dx$$

$$I = \lim_{h\to 0} h\left[f(-1)+f(-1+h)+f(-1+2h)+----+f(-1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+(2+h)+(2+2h)+----+\{(n-1)h+2\}\right]$$

$$= \lim_{h\to 0} h\left[2n+h(1+2+3+----)\right]$$

$$= \lim_{h\to 0} h\left[2n+h\frac{n(n-1)}{2}\right] \qquad \left[\because h=\frac{2}{n}\ \&\text{if } h\to 0 \Rightarrow n\to\infty\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 4+\frac{2n^2}{n^2}\left(1-\frac{1}{n}\right)$$

$$= 4+2 = 6$$

$$\therefore \int_{-1}^{1}(x+3)dx = 6$$

Definite Integrals Ex 20.5 Q5

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 5$

and $f(x) = (x+1)$

$\therefore h = \dfrac{5}{n} \Rightarrow nh = 5$

Thus, we have,

$$I = \int_0^5 (x+1)dx$$

$$I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1+(h+1)+(2h+1)+-----+((n-1)h+1)\right]$$

$$= \lim_{h\to 0} h\left[n+h(1+2+3+-----(n-1))\right]$$

$\because h = \dfrac{5}{n}$ and if $h \to 0,\ n \to \infty$

$$= \lim_{n\to\infty} \frac{5}{n}\left[n+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 5+\frac{25}{2n^2}\, n^2\left(1-\frac{1}{n}\right)$$

$$= 5+\frac{25}{2}$$

$$\therefore \int_0^5 (x+1)dx = \frac{35}{2}$$

Definite Integrals Ex 20.5 Q6

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1$, $b = 3$

and $f(x) = (2x+3)$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (2x+3)\,dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+3+\{2(1+h)+3\}+\{2(1+2h)+3\}----+2\{1+(n-1)+3\}\right]$$

$$= \lim_{h\to 0} h\left[5+(5+2h)+(5+4h)+-----5+2(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[5n+2h(1+2+3+-----(n-1))\right]$$

$$\because h = \frac{2}{n} \text{ and if } h\to 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[10+\frac{4}{n^2}\frac{n(n-1)}{2}\right] = 14$$

$$\therefore \int_1^3 (2x+3)\,dx = 14$$

Definite Integrals Ex 20.5 Q7

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f\left(a+(n-1)h\right)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 3,\ b = 5$

and $f(x) = (2-x)$

$\therefore\ h = \frac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$I = \int_3^5 (2-x)\,dx$

$= \lim_{h\to 0} h\left[f(3)+f(3+h)+f(3+2h)+----f\left(3+(n-1)h\right)\right]$

$= \lim_{h\to 0} h\left[(2-3)+\{2-(3+h)\}+\{2-(3+2h)\}+----\{2-(3+(n-1)h\}\right]$

$= \lim_{h\to 0} h\left[-1+(-1-h)+(-1-2h)+----\{-1-(n-1)h\}\right]$

$= \lim_{h\to 0} h\left[-n-h(1+2+----(n-1)h\right]$

$\because h = \frac{2}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$\therefore \lim_{n\to\infty} \frac{2}{n}\left[-n-\frac{2}{n}\frac{n(n-1)}{2}\right]$

$= \lim_{n\to\infty} -2-\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right) = -2-2 = -4$

$\therefore \int_3^5 (2-x)\,dx = -4$

Definite Integrals Ex 20.5 Q8

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0,\ b = 2$ and $f(x) = (x^2+1)$

$$\therefore\ h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_0^2 (x^2+1)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1+(h^2+1)+\{(2h)^2+1\}+----\{((n-1)h)^2+1\}\right]$$

$$= \lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----+(n-1)^2\right)\right]$$

$$\because h = \frac{2}{n}\ \&\ \text{if } h \Rightarrow 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 2+\frac{4}{3}\times 2 = \frac{14}{3}$$

$$\therefore \int_0^2 (x^2+1)dx = \frac{14}{3}$$

Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 2$ and $f(x) = x^2$

$\therefore h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2 x^2 dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+(1+h)^2+(1+2h)^2+-----+(1+(n-1)h\ )^2\right]$$

$$= \lim_{h\to 0} h\left[1+\left(1+2h+h^2\right)+\left(1+2\times 2h+2\times 2h^2\right)+----\left(1+2\times(n-1)h+(1-n)^2h^2\right)\right]$$

$$= \lim_{h\to 0} h\left[n+2h\{1+2+3---(n-1)\}+h^2\left\{1^2+2^2+3^2+---(n-1)^2\right\}\right]$$

$\because h = \dfrac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[n+\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 1+\frac{n^2}{n^2}\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+1+\frac{2}{6} = \frac{7}{3}$$

$$\therefore \int_1^2 x^2 dx = \frac{7}{3}$$

Definite Integrals Ex 20.5 Q10

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 2$, $b = 3$ and $f(x) = 2x^2 + 1$

$\therefore h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_2^3 (2x^2 + 1)dx$$

$$= \lim_{h\to 0} h\left[f(2) + f(2+h) + f(2+2h) + ----f(2+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(2\times 2^2 + 1)\{2(2+h)^2 + 1\} + \{2(2+2h)^2 + 1\} + ----- + \{2(2+(n-1)h)^2 + 1\}\right]$$

$$= \lim_{h\to 0} h\left[9n + 8h(1+2+3---) + 2h^2(1^2 + 2^2 + 3^2 + --)\right]$$

$\because h = \dfrac{1}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[9n + \frac{8}{n}\frac{n(n-1)}{2} + \frac{2}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 9 + \frac{4}{n^2}n^2\left(1-\frac{1}{n}\right) + \frac{1}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 9 + 4 + \frac{2}{3} = \frac{41}{3}$$

$$\therefore \int_2^3 (2x^2 + 1)dx = \frac{41}{3}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 2$ and $f(x) = x^2 - 1$

$\therefore\ h = \dfrac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2 \left(x^2-1\right)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(1^2-1\right)\left\{(1+h)^2-1\right\}+\left\{(1+2h)^2-1\right\}+----+\left\{(1+(n-1)h)^2-1\right\}\right]$$

$$= \lim_{h\to 0} h\left[0+2h(1+2+3+---)+h^2\left(1+2^2+3^2+--\right)\right]$$

$\because h = \dfrac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+\frac{2}{6} = \frac{4}{3}$$

$$\therefore \int_1^2 \left(x^2-1\right)dx = \frac{4}{3}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

where $h = \dfrac{b-a}{n}$

Here $a = 0,\ b = 2$ and $f(x) = x^2 + 4$

$\therefore h = \dfrac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 (x^2+4)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[4(h^2+4)+\{(2h)^2+4\}+----\{(n-1)h^2+4\}\right]$$

$\because h = \dfrac{2}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$= \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 8 + \frac{4}{3n^2}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 8 + \frac{4\times 2}{3} = \frac{32}{3}$$

$$\therefore \int_0^2 (x^2+4)dx = \frac{32}{3}$$

Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 4$ and $f(x) = x^2 - x$

$$h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$I = \int_1^4 \left(x^2 - x\right)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(1^2-1\right)+\left\{(1+h)^2-(1+h)\right\}+\left\{(1+2h)^2-(1+h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[0+\left(h+h^2\right)+\left\{2h+(2h)^2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[h+\left(1+2+3+---(n-1)\right)+h^2\left\{\left(1+2^2+3^2+---(n-1)^2\right\}\right]$$

$$\because h = \frac{3}{n} \ \&\ \text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{3}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{9}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{3}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= \frac{9}{2}+3 = \frac{27}{2}$$

$$\therefore \int_1^4 \left(x^2-x\right)dx = \frac{27}{2}$$

Definite Integrals Ex 20.5 Q14

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 1$ and $f(x) = 3x^2 + 5x$

$$h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_0^1 \left(3x^2 + 5x\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left\{0+\left(3h^2+5h\right)+\left\{3(2h)^2+5(2h)\right\}+----\right]\right.$$

$$= \lim_{h\to 0} h\left[\left\{3h^2\left(1+2^2+3^2+---(n-1)^2\right)\right\}+5h\left\{1+2+3+---(n-1)\right\}\right]$$

$$\because h = \frac{1}{n} \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{3}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{3}{n^3}\frac{n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{5}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \frac{3\times 2}{6}+\frac{5}{2} = \frac{7}{2}$$

$$\therefore \int_0^1 \left(3x^2+5x\right)dx = \frac{7}{2}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x)=e^x$$

Now

$$h=\frac{2}{n}$$

$$nh=2$$

Thus, we have

$$I=\int_0^2 e^x dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[1+e^h+e^{2h}+...+e^{(n-1)h}\right]$$

$$=\lim_{h\to 0} h\left\{\frac{(e^h)^n-1}{e^h-1}\right\}$$

$$=\lim_{h\to 0} h\left\{\frac{e^{nh}-1}{e^h-1}\right\}$$

$$=\lim_{h\to 0} h\left\{\frac{e^2-1}{e^h-1}\right\} \quad [nh=2]$$

$$=\lim_{h\to 0}\left\{\frac{e^2-1}{\frac{e^h-1}{h}}\right\}$$

$$=e^2-1$$

Definite Integrals Ex 20.5 Q16

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = a,\ b = b$ and $f(x) = e^x$

$$\therefore \quad h = \frac{b-a}{n} \Rightarrow nh = b-a$$

Thus, we have,

$$I = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[e^a + e^{a+h} + e^{a+2h} + ----e^{a+(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+e^{2h}+e^{3h}+----e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+\left(e^h\right)^2+\left(e^h\right)^3+----\left(e^h\right)^{n-1}\right]$$

$$= \lim_{h\to 0} he^a\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\} \qquad \left[\because a+ar+ar^2+----ar^{n-1} = a\left\{\frac{r^n-1}{r-1}\right\} \text{ if } r>1\right]$$

$$= \lim_{h\to 0} he^a n\left\{\frac{e^{nh}-1}{nh}\right\}\times\left(\frac{h}{e^{h-1}}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{e^\theta-1}{\theta} = 1 \quad \& \quad nh = b-a\right]$$

$$\therefore \lim_{h\to 0}\left(e^{b-a}-1\right) = e^b - e^a$$

$$\therefore \int_a^b e^x dx = e^b - e^a$$

Definite Integrals Ex 20.5 Q17

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right], \text{ where } h = \frac{b-a}{n}.$$

Since we have to find $\int_a^b \cos x dx$

We have, $\quad f(x) = \cos x$

$$\therefore \quad I = \int_a^b \cos x dx$$

$$\Rightarrow \quad I = \lim_{h\to 0} h\left[\cos a + \cos(a+h) + \cos(a+2h) + \ldots + \cos(a+(n-1)h)\right]$$

$$\Rightarrow \quad I = \lim_{h\to 0} h\left[\frac{\cos\left(a+(n-1)\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right] = \lim_{h\to 0} h\left[\frac{\cos\left(a+\frac{nh}{2}-\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$$

$$\Rightarrow \quad I = \lim_{h\to 0} h\left[\frac{\cos\left(a+\frac{b-a}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)}{\sin\frac{h}{2}}\right] \qquad [\because nh = b-a]$$

$$\Rightarrow \quad I = \lim_{h\to 0}\left[\frac{\frac{h}{2}}{\sin\frac{h}{2}}\times 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)\right]$$

$$\Rightarrow \quad I = \lim_{h\to 0}\left(\frac{\frac{h}{2}}{\sin\frac{h}{2}}\right)\times\lim_{h\to 0} 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{b-a}{2}\right)$$

$$\Rightarrow \quad I = \sin b - \sin a \qquad [\because 2\cos A\sin B = \sin(A-B) - \sin(A+B)]$$

Definite Integrals Ex 20.5 Q18

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0$, $b = \dfrac{\pi}{2}$ and $f(x) = \sin x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \sin x\, dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\sin 0+\sin h+\sin 2h+----\sin(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{nh}{2}-\frac{h}{2}\right)\times\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\sin\frac{\pi}{4}}{\sin\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\sin\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\sin\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\, dx = 1$$

Definite Integrals Ex 20.5 Q19

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = \frac{\pi}{2}$ and $f(x) = \cos x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \cos x\, dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\cos 0+\cos h+\cos 2h+----\cos(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{nh}{2}-\frac{h}{2}\right)\times\cos\frac{nh}{2}}{\cos\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\cos\frac{\pi}{4}}{\cos\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\cos\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\cos\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos x\, dx = 1$$

Definite Integrals Ex 20.5 Q20

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1,\ b = 4$ and $f(x) = 3x^2 + 2x$

$$I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(3+2)+\left\{3(1+h)^2+2(1-+h)\right\}+\left\{3(1+2h)^2+2(1+2h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[5+8h(1+2+3+--)+3h^2\left(1+2^2+3^2+---\right)\right]$$

$$\because h = \frac{3}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[5n+\frac{24}{n}\frac{n(n-1)}{2}+\frac{27}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty}\ 15+\frac{36}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{27}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 15+36+27 \quad = 78$$

$$\therefore \int_1^4\left(3x^2+2x\right)dx = 78$$

Definite Integrals Ex 20.5 Q21

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 2$ and $f(x) = 3x^2 - 2$

$$\therefore \quad h = \frac{2}{n} \quad \Rightarrow \quad nh = 2$$

Thus, we have,

$$I = \int_0^2 (3x^2 - 2)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-2+(3h^2-2)+\{3(2h)^2-2\}+----\right]$$

$$= \lim_{h\to 0} h\left[-2h+3h^2(1+2^2+3^2+----)\right]$$

$$\because h = \frac{2}{n} \quad \text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[-2n+\frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} -4+\frac{4}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = -4+8 = 4$$

$$\therefore \quad \int_0^2 (3x^2-2)dx = 4$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2$

$$\therefore \quad h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_0^2 (x^2+2)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+(h^2+2)+\{(2h)^2+2\}+----\right]$$

$$= \lim_{h\to 0} h\left[2h+h^2(1+2^2+3^2+----(n-1)^2\right]$$

$$\because h = \frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 4+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 4+\frac{8}{3} = \frac{20}{3}$$

$$\therefore \quad \int_0^2 (x^2+2)dx = \frac{20}{3}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=4,$ and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4 (x+e^{2x})\,dx = (4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[(0+e^0)+(h+e^{2h})+(2h+e^{2\cdot 2h})+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+(h+e^{2h})+(2h+e^{4h})+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\{1+2+\ldots(n-1)\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{(h(n-1)n)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{(e^8-1)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot(e^8-1)}{8} \qquad \left(\lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

$$=8+\frac{e^8-1}{2}$$

$$=\frac{15+e^8}{2}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Thus, we have,

$$I = \int_0^2 \left(x^2+x\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[0+\left(h^2+h\right)+\left\{(2h)^2+2h\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[\left\{h^2\left(1+2^2+3^2+----(n-1)^2\right\}+h\left\{1+2+3----(n-1)\right\}\right]$$

$$\because h = \frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \frac{8}{3}+2 = \frac{14}{3}$$

$$\therefore \int_0^2 \left(x^2+x\right)dx = \frac{14}{3}$$

Definite Integrals Ex 20.5 Q25

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2x + 1$

$$\therefore \quad h = \frac{2}{n} \quad \Rightarrow \; nh = 2$$

Thus, we have,

$$I = \int_0^2 \left(x^2 + 2x + 1\right)dx$$

$$= \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + ----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1 + \left(h^2 + 2h + 1\right) + \left\{(2h)^2 + 2\times 2h + 1\right\} + ----\right]$$

$$= \lim_{h\to 0} h\left[n + h^2\left(1 + 2^2 + 3^2 + ----(n-1)^2 + 2h\left(1 + 2 + 3 ----(n-1)\right)\right]$$

$$\because h = \frac{2}{n} \quad \text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[n + \frac{4}{n^2}\frac{n(n-1)(2n-1)}{6} + \frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2 + \frac{4}{3n^3}n^3\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right) + \frac{4}{n^2}n^2\left(1 - \frac{1}{n}\right)$$

$$= 2 + \frac{8}{3} + 4 \quad = \frac{26}{3}$$

$$\therefore \quad \int_0^2 \left(x^2 + 2x + 1\right)dx = \frac{26}{3}$$

Definite Integrals Ex 20.5 Q26

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 3$ and $f(x) = 2x^2 + 3x + 5$

$$\therefore \quad h = \frac{3}{n} \quad \Rightarrow nh = 3$$

Thus, we have,

$$I = \int_0^3 \left(2x^2 + 3x + 5\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[5+\left(2h^2+3h+5\right)+\left\{2(2h)^2+3\times 2h+5\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[5n+2h^2\left(1+2^2+3^2+----(n-1)^2+3h\left(1+2+3----(n-1)\right)\right)\right]$$

$$\because h = \frac{3}{n} \quad \& \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{18}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{9}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 15+\frac{9}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{27}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 15+18+\frac{27}{2} = \frac{93}{2}$$

$$\therefore \quad \int_0^3 \left(2x^2+3x+5\right)dx = \frac{93}{2}$$

Definite Integrals Ex 20.5 Q27

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+...+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=a,\ b=b,$ and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)...(a+2h)...a+(n-1)h\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+...+a}_{n\text{ times}}\right)+\left(h+2h+3h+...+(n-1)h\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+...+(n-1)\right)\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{(b-a)(b+a)}{2}$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

Definite Integrals Ex 20.5 Q28

Let $I = \int_0^5 (x+1)\,dx$

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)...f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=5$, and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\therefore \int_0^5 (x+1)\,dx = (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+...+f\left((n-1)\frac{5}{n}\right)\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+...\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1...1}_{n \text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+...(n-1)\frac{5}{n}\right]\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3...(n-1)\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$= 5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 5\left[1+\frac{5}{2}\right]$$

$$= 5\left[\frac{7}{2}\right]$$

$$= \frac{35}{2}$$

Definite Integrals Ex 20.5 Q29

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=2$, $b=3$, and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2dx=(3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

Definite Integrals Ex 20.5 Q30

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\dfrac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=x^2+x$$

Now

$$h=\frac{2}{n}$$

$$nh=2$$

Thus, we have

$$I=\int_1^3 (x^2+x)dx$$

$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+...+f(1+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[(1^2+1)+\left\{(1+h)^2+(1+h)\right\}+\left\{(1+2h)^2+(1+2h)\right\}+...\right]$$

$$=\lim_{h\to 0} h\left[\left(1^2+(1+h)^2+(1+2h)^2+...\right)+\left\{1+(1+h)+(1+2h)+...\right\}\right]$$

$$=\lim_{h\to 0} h\left[\left(n+2h(1+2+3+...)+h^2\left(1+2^2+3^3+...\right)\right)+\left(n+h(1+2+3+...)\right)\right]$$

$$=\lim_{h\to 0} h\left[\left(2n+3h(1+2+3+...+(n-1))+h^2\left(1+2^2+3^3+...+(n-1)^2\right)\right)\right]$$

$$\because h=\frac{2}{n} \ \& \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{h\to\infty}\frac{2}{n}\left[2n+\frac{9}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\frac{38}{3}$$

Definite Integrals Ex 20.5 Q31

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x)=x^2-x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_0^2 (x^2-x)dx$$
$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+...+f((n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[\{(0)^2-(0)\}+\{(h)^2-(h)\}+\{(2h)^2-(2h)\}+...\right]$$
$$=\lim_{h\to 0} h\left[((h)^2+(2h)^2+...)-\{(h)+(2h)+...\}\right]$$
$$=\lim_{h\to 0} h\left[h^2(1+2^2+3^3+...+(n-1)^2)-h\{1+2+3+...+(n-1)\}\right]$$
$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{k\to\infty}\frac{2}{n}\left[\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}-\frac{9}{n}\frac{n(n-1)}{2}\right]$$
$$=\frac{2}{3}$$

Definite Integrals Ex 20.5 Q32

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+...+f(a+(n-1)h)\right]$$

Where $h=\frac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=2x^2+5x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_1^3 (2x^2+5x)dx$$
$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+...+f(1+(n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[(2+5)+\{2(1+h)^2+5(1+h)\}+\{2(1+2h)^2+5(1+2h)\}+...\right]$$
$$=\lim_{h\to 0} h\left[(7n+9h(1+2+3+...)+2h^2(1+2^2+3^3+...))\right]$$
$$\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{k\to\infty}\frac{2}{n}\left[7n+\frac{18}{n}\frac{n(n-1)}{2}+\frac{8}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$
$$=\frac{112}{3}$$

Definite Integrals Ex 20.5 Q33

Given,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)\right],$$

where $h = \frac{b-a}{n}$

Here, $f(x) = 3x^2 + 1$, $a = 1$, $b = 3$. Therefore, $h = \frac{3-1}{n} = \frac{2}{n}$

$$\therefore I = \int_1^3 \left(3x^2 + 1\right)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \ldots + f(1+(n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3(1)^2 + 1 + 3(1+h)^2 + 1 + 3(1+2h)^2 + 1 + \ldots + 3(1+(n-1)h)^2 + 1\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3n + n + 6h\left(1+2+3+\ldots+(n-1)\right) + 3h^2\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{12}{n}\left(1+2+3+\ldots+(n-1)\right) + 3 \times \frac{4}{n^2}\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + \frac{24}{n^2} \times \frac{n(n-1)}{2} + \frac{24}{n^3} \times \frac{(n-1)(n)(2n-1)}{6}\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + 12\left(1 - \frac{1}{n}\right) + 4\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right)\right]$$

$$\Rightarrow I = 8 + 12 + 4 \times 2 = 28$$