RD Sharma Solutions Class 12 Maths Chapter 21 Ex 21.1

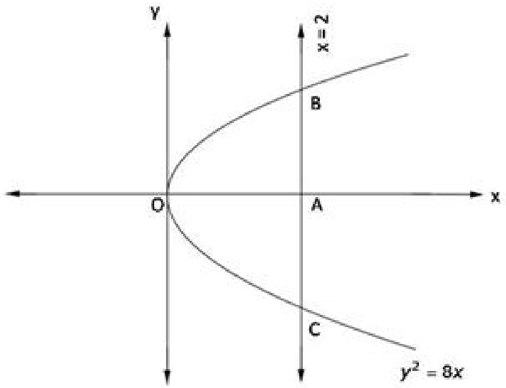
Areas of Bounded Regions Ex 21.1 Q1

Given equations are

$x = 2$ --- (1)

and $y^2 = 8x$ --- (2)

Equation (1) represents a line parallel to y-axis and equation (2) represents a parabola with vertex at origin and x-axis as its axis, A rough sketch is given as below:-



We have to find the area of shaded region . We sliced it in vertical rectangle width of rectangle = Δx,

Length = $(y - 0) = y$

Area of rectangle = $y \Delta x$

This rectangle can move horizontal from $x = 0$ to $x = 2$

Required area = Shaded region $OCBO$

$$= 2(\text{Shaded region } OABO)$$

$$= 2\int_0^2 y\, dx$$

$$= 2\int_0^2 \sqrt{8x}\, dx$$

$$= 2.2\sqrt{2}\int_0^2 \sqrt{x}\, dx$$

$$= 4\sqrt{2}\left[\frac{2}{3}x\sqrt{x}\right]_0^2$$

$$= 4\sqrt{2}\left[\left(\frac{2}{3}.2\sqrt{2}\right) - \left(\frac{2}{3}.0.\sqrt{0}\right)\right]$$

$$= 4\sqrt{2}\left(\frac{4\sqrt{2}}{3}\right)$$

Required area = $\frac{32}{3}$ square units

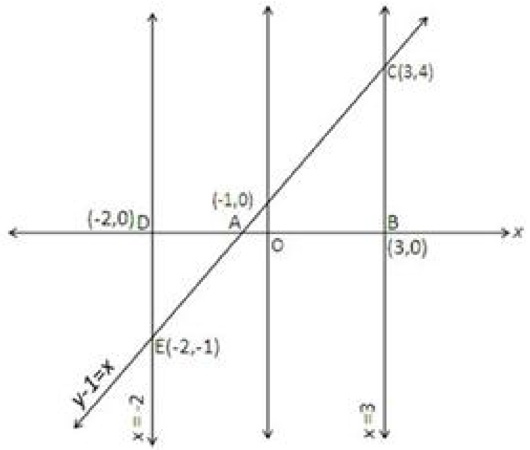
Areas of Bounded Regions Ex 21.1 Q2

To find area of region bounded by x-axis the ordinates $x = -2$ and $x = 3$ and

$y - 1 = x$ --- (1)

Equation (1) is a line that meets at axes at $(0,1)$ and $(-1,0)$.

A rough sketch of the curve is as under:-



Shaded region is required area.

Required area = Region $ABCA$ + Region $ADEA$

$$A = \int_{-1}^{3} y\,dx + \left|\int_{-2}^{-1} y\,dx\right|$$

$$= \int_{-1}^{3} (x+1)\,dx + \left|\int_{-2}^{-1} (x+1)\,dx\right|$$

$$= \left(\frac{x^2}{2} + x\right)_{-1}^{3} + \left|\left(\frac{x^2}{2} + x\right)_{-2}^{-1}\right|$$

$$= \left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right] + \left|\left(\frac{1}{2} - 1\right) - (2 - 2)\right|$$

$$= \left[\frac{15}{2} + \frac{1}{2}\right] + \left|-\frac{1}{2}\right|$$

$$= 8 + \frac{1}{2}$$

$$A = \frac{17}{2} \text{ sq. units}$$

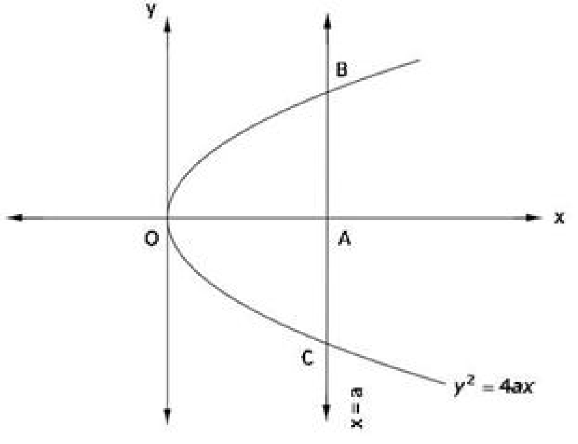
Areas of Bounded Regions Ex 21.1 Q3

We have to find the area of the region bounded by

$x = a$ $\quad ---(1)$

and $y^2 = 4ax$ $\quad ---(2)$

Equation (1) represents a line parallel to y-axis and equation (2) represents a parabola with vertex at origin and axis as x-axis. A rough sketch of the two curves is as below:-



We have to find the area of the shaded region. Now, we slice it in rectangles.
Width $= \Delta x$, Length $= y - 0 = y$

Area rectangle $= y\Delta x$

This approximating rectangle can move from $x = 0$ to $x = a$.

Required area = Region $OCBO$

$$= 2(\text{Region } OABO)$$

$$= 2\int_0^a \sqrt{4ax}\, dx$$

$$= 2.2\sqrt{a}\int_0^a \sqrt{x}\, dx$$

$$= 4\sqrt{a}.\left(\frac{2}{3}x\sqrt{x}\right)_0^a$$

$$= 4\sqrt{a}.\left(\frac{2}{3}a\sqrt{a}\right)$$

Required area $= \frac{8}{3}a^2$ square units

Areas of Bounded Regions Ex 21.1 Q4

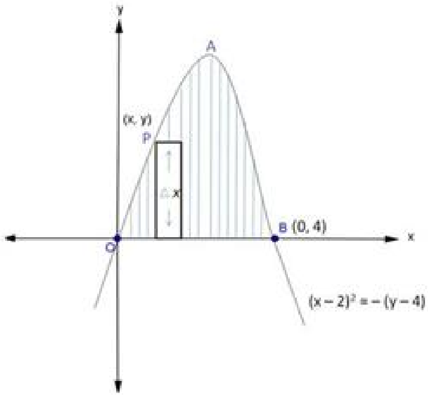
We have to find area bounded by x-axis and parabola

$y = 4x - x^2$

$$\Rightarrow \quad x^2 - 4x + 4 = -y + 4$$

$$\Rightarrow \quad (x-2)^2 = -(y-4) \qquad ---(1)$$

Equation (1) represents a downward parabola with vertex $(2,4)$ and passing through $(0,0)$ and $(0,4)$. A rough sketch is as below:-



the shaded region represents the required area. We slice the region in approximation rectangles with width $= \Delta x$, length $= y - 0 = y$

Area of rectangle $= y \Delta x$.

This approximation rectangle slide from $x = 0$ to $x = a$, so

Required area = Region $OABO$

$$= \int_0^4 \left(4x - x^2\right) dx$$

$$= \left(4\frac{x^2}{2} - \frac{x^3}{3}\right)_0^4$$

$$= \left(\frac{4 \times 16}{2} - \frac{64}{3}\right) - (0 - 0)$$

$$= \frac{64}{6}$$

Required area $= \dfrac{32}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q5

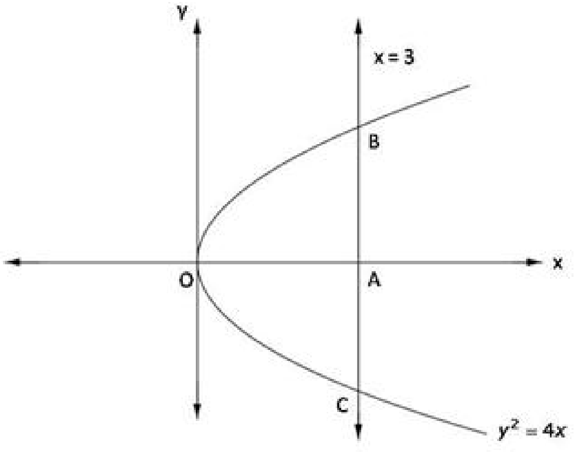
To find area bounded by

$y^2 = 4x$ ---(1)

and $x = 3$ ---(2)

Equation (1) represents a parabola with vertex at origin and axis as x-axis and equation (2) represents a line parallel to y-axis.

A rough sketch of the equations is as below:-



Shaded region represents the required area we slice this area with approximation rectangles with Width $= \Delta x$, length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle can slide from $x = 0$ to $x = 3$, so

$$\begin{aligned}
\text{Required area} &= \text{Region } OCBO \\
&= 2(\text{Region } OABO) \\
&= 2\int_0^3 y\,dx \\
&= 2\int_0^3 \sqrt{4x}\,dx \\
&= 4\int_0^3 \sqrt{x}\,dx \\
&= 4\left(\frac{2}{3}x\sqrt{x}\right)_0^3 \\
&= \frac{8}{3}.3\sqrt{3}
\end{aligned}$$

Required area $= 8\sqrt{3}$ square units

Areas of Bounded Regions Ex 21.1 Q6

We have to find the area enclosed by

$$y = 4 - x^2$$

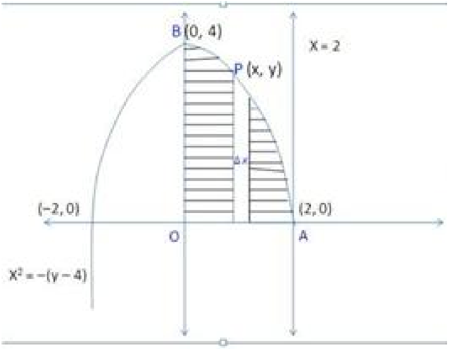
$$\Rightarrow \quad x^2 = -(y-4) \qquad ---(1)$$

$$x = 0 \qquad ---(2)$$

$$x = 2 \qquad ---(3)$$

Equation (1) represent a downward parabola with vertex at $(0,4)$ and passing through $(2,0), (-2,0)$. Equation (2) represents y-axis and equation (3) represents a line parallel to y-axis.

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice this region into approximation rectangles with Width $= \Delta x$, length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle move from $x = 0$ to $x = 2$, so

Required area = (Region $OABO$)

$$= \int_0^2 \left(4 - x^2\right) dx$$

$$= \left(4x - \frac{x^3}{3}\right)_0^2$$

$$= \left[4(2) - \frac{(2)^3}{3}\right] - [0]$$

$$= \left[\frac{24-8}{3}\right]$$

Required area $= \frac{16}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q7

We have to find area enclosed by x-axis and

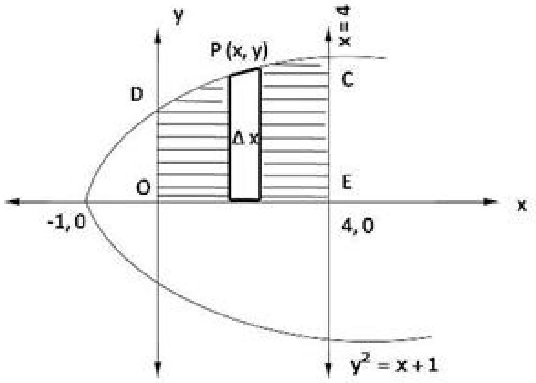
$$y = \sqrt{x+1}$$

$$\Rightarrow \quad y^2 = x+1 \qquad ---(1)$$

$$\text{and } x = 0 \qquad ---(2)$$

$$x = 4 \qquad ---(3)$$

Equation (1) represent a parabola with vertex at $(-1,0)$ and passing through $(0,1)$ and $(0,-1)$. Equation (2) is y-axis and equation (3) is a line parallel to y-axis passing through $(4,0)$. So rough sketch of the curve is as below:-



We slice the required region in approximation rectangle with its Width $= \Delta x$, and length $= y - 0 = y$

Area of rectangle $= y\Delta x$.

Approximation rectangle moves from $x = 0$ to $x = 4$. So

Required area = Shaded region

$$= (\text{Region } OECDO)$$

$$= \int_0^4 y\,dx$$

$$= \int_0^4 \sqrt{x+1}\,dx$$

$$= \left(\frac{2}{3}(x+1)\sqrt{x+1}\right)_0^4$$

$$= \frac{2}{3}\left[\left((4+1)\sqrt{4+1}\right) - \left((0+1)\sqrt{0+1}\right)\right]$$

Required area $= \frac{2}{3}\left[5\sqrt{5} - 1\right]$ square units

Thus, Required area $= \frac{2}{3}\left(5^{\frac{3}{2}} - 1\right)$ square units

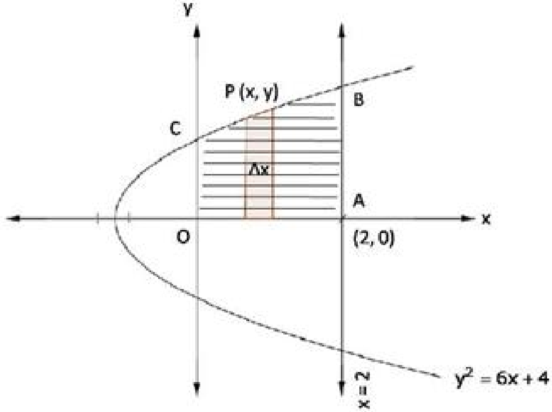
Areas of Bounded Regions Ex 21.1 Q8

We have to find area enclosed by x-axis

$x = 0,\ x = 2$ ---(1)

and $y^2 = 6x + 4$ ---(2)

Equation (1) represents y-axis and a line parallel to y-axis passing through (2,0) respectively. Equation (2) represents a parabola with vertex at $\left(-\frac{2}{3}, 0\right)$ and passes through the points $(0,2), (0,-2)$, so rough sketch of the curves is as below:-



Shaded region represents the required area. It is sliced in approximation rectangle with its Width $= \Delta x$, and length $= (y - 0) = y$

Area of rectangle $= y\Delta x$.

This approximation rectangle slide from $x = 0$ to $x = 2$, so

Required area = Region $OABCO$

$$= \int_0^2 \sqrt{6x + 4}\,dx$$

$$= \left\{\frac{2}{3}\frac{(6x+4)\sqrt{6x+4}}{6}\right\}_0^2$$

$$= \frac{1}{9}\left[\left((12 + 4)\sqrt{12 + 4}\right) - \left((0 + 4)\sqrt{0 + 4}\right)\right]$$

$$= \frac{1}{9}\left[16\sqrt{16} - 4\sqrt{4}\right]$$

$$= \frac{1}{9}(64 - 8)$$

Required area $= \frac{56}{9}$ square units

Areas of Bounded Regions Ex 21.1 Q9

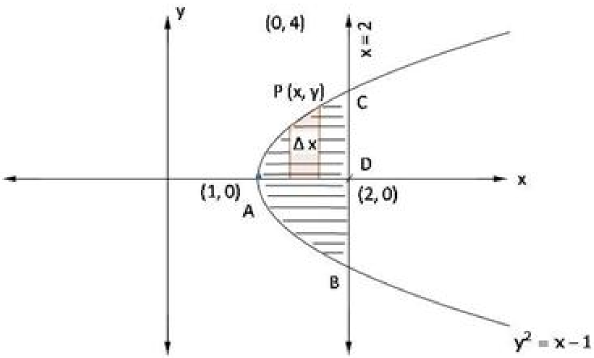
We have to find area enclosed by

$y^2 = x - 1$ ---(1)

and $x = 2$ ---(2)

Equation (1) is a parabola with vertex at (1,0) and axis as x-axis. Equation (2) represents a line parallel to y-axis passing through (2,0).

A rough sketch of curves is as below:-



Shaded region shows the required area. We slice it in approximation rectangle with its Width $= \Delta x$ and length $= y - 0 = y$

Area of the rectangle $= y\Delta x$.

This rectangle can slide from $x = 1$ to $x = 2$, so

Required area = Region $ABCA$

$$= 2\left(\text{Region } AOCA\right)$$

$$= 2\int_1^2 y dx$$

$$= 2\int_1^2 \sqrt{x-1}dx$$

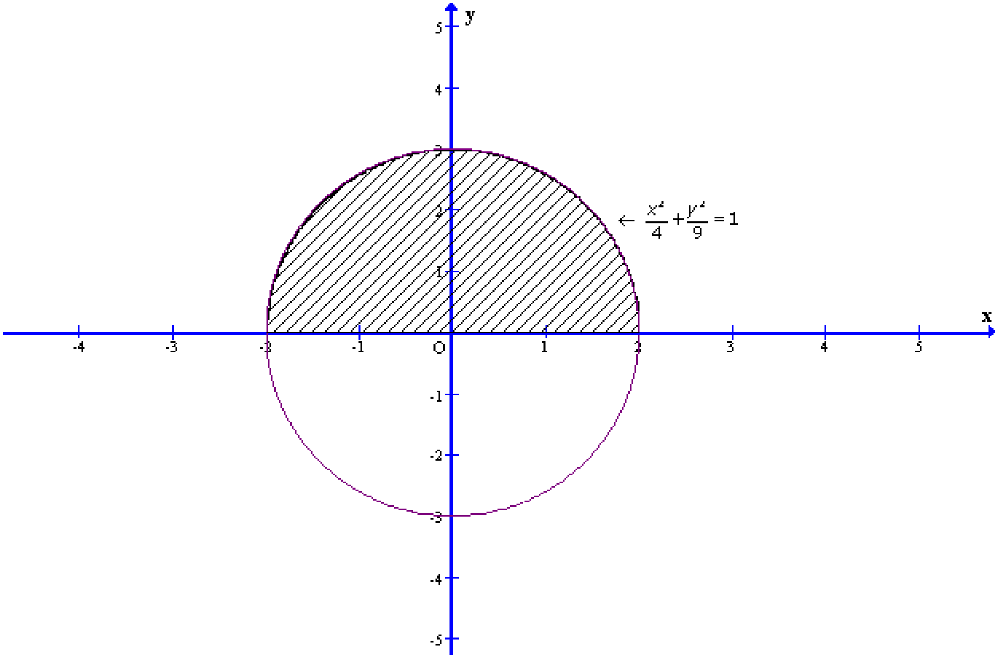
$$= 2\left(\frac{2}{3}(x-1)\sqrt{x-1}\right)_1^2$$

$$= \frac{4}{3}\left[\left((2-1)\sqrt{2-1}\right) - \left((1-1)\sqrt{1-1}\right)\right]$$

$$= \frac{4}{3}(1-0)$$

Required area $= \frac{4}{3}$ square units

Areas of Bounded Regions Ex 21.1 Q10



It can be observed that ellipse is symmetrical about x-axis.

Area bounded by ellipse $= 2\int_0^2 y\,dx$

$= 2\int_0^2 3\sqrt{1-\frac{x^2}{4}}\,dx$

$= 3\int_0^2 \sqrt{4-x^2}\,dx$

$= 3\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 3\left[1(0)+2\sin^{-1}(1)-0-2\sin^{-1}(0)\right]$

$= 3[\pi]$

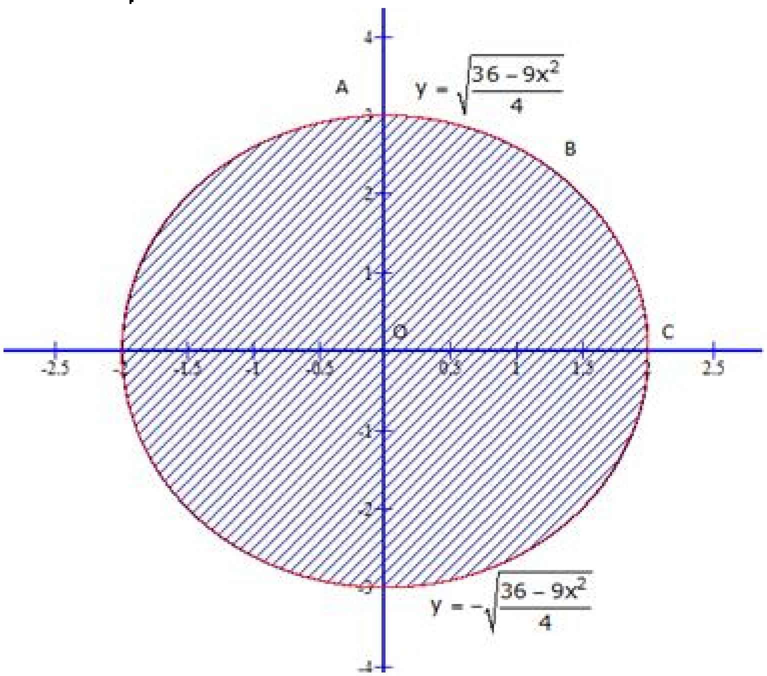
$= 3\pi$ sq. units

Areas of Bounded Regions Ex 21.1 Q11

$9x^2 + 4y^2 = 36$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$

$\Rightarrow y = \pm\sqrt{\frac{36 - 9x^2}{4}}$



Area of Sector OABCO =

$\int_0^2 \sqrt{\frac{36 - 9x^2}{4}}dx$

$= \frac{3}{2}\int_0^2 \sqrt{4 - x^2}dx$

$= \frac{3}{2}\left[\frac{x\sqrt{4 - x^2}}{2} + \frac{2^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= \frac{3}{2}\left[\frac{2\sqrt{4 - 2^2}}{2} + \frac{2^2}{2}\sin^{-1}\left(\frac{2}{2}\right)\right] - \frac{3}{2}\left[\frac{0\sqrt{4 - 0^2}}{2} + \frac{2^2}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]$

$= \frac{3}{2}.2.\frac{\pi}{2} - 0$

$= \frac{3\pi}{2}$ sq. units

Area of the whole figure = 4 × Ar. D OABCO

$= 4 \times \frac{3\pi}{2}$

= 6p sq. units

Areas of Bounded Regions Ex 21.1 Q12

We have to find area enclosed between the curve and x-axis.

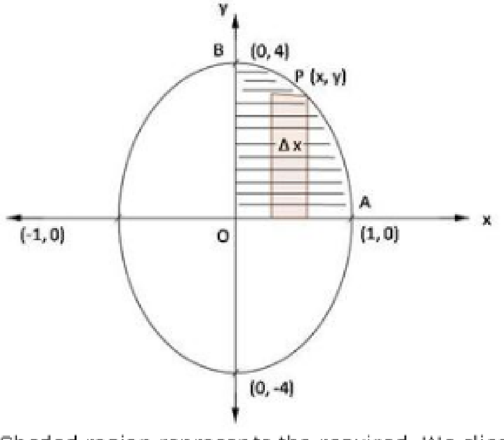
$y = 2\sqrt{1-x^2},\ x \in [0,1]$

$$\Rightarrow \quad y^2 + 4x^2 = 4,\ x \in [0,1]$$

$$\Rightarrow \quad \frac{x^2}{1} + \frac{y^2}{4} = 1,\ x \in [0,1] \qquad ---(1)$$

Equation (1) represents an ellipse with centre at origin and passes through $(\pm 1, 0)$ and $(0, \pm 2)$ and $x \in [0,1]$ as represented by region between y-axis and line $x = 1$.

A rough sketch of curves is as below:-



Shaded region represents the required. We slice it into approximation rectangles of Width $= \Delta x$ and length $= y$

Area of the rectangle $= y \Delta x$.

The approximation rectangle slides from $x = 0$ to $x = 1$, so

Required area = Region $OAPBO$

$$= \int_0^1 y\,dx$$

$$= \int_0^1 2\sqrt{1-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)\right]_0^1$$

$$= 2\left[\left(\frac{1}{2}\sqrt{1-1} + \frac{1}{2}\sin^{-1}(1)\right) - (0+0)\right]$$

$$= 2\left[0 + \frac{1}{2}\cdot\frac{\pi}{2}\right]$$

Required area $= \frac{\pi}{2}$ square units

Areas of Bounded Regions Ex 21.1 Q13

To find area under the curves

$y = \sqrt{a^2 - x^2}$

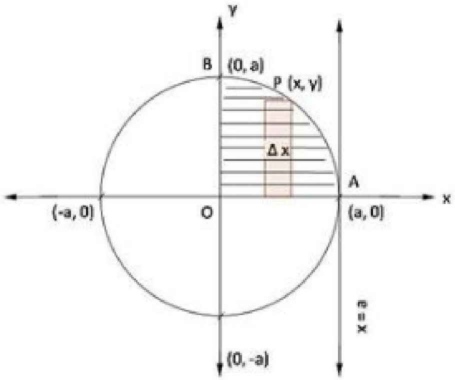
$\Rightarrow \quad x^2 + y^2 = a^2 \qquad ---(1)$

Between $x = 0 \qquad ---(2)$

$x = a \qquad ---(3)$

Equation (1) represents a circle with centre (0,0) and passes axes at $(0, \pm a)$ $(\pm a, 0)$ equation (2) represents y-axis and equation $x = a$ represent a line parallel to y-axis passing through (a,0).

A rough sketch of the curves is as below:-



Shaded region represents the required area. We slice it into approximation rectangles of Width $= \Delta x$ and length $= y - 0 = y$

Area of the rectangle $= y\Delta x$.

The approximation rectangle can slide from $x = 0$ to $x = a$, so

Required area = Region $OAPBO$

$$= \int_0^a y\,dx$$

$$= \int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}(1)\right) - (0)\right]$$

$$= \left[0 + \frac{a^2}{2} \cdot \frac{\pi}{2}\right]$$

Required area $= \frac{\pi}{4}a^2$ square units

Areas of Bounded Regions Ex 21.1 Q14

To find area bounded by x-axis and

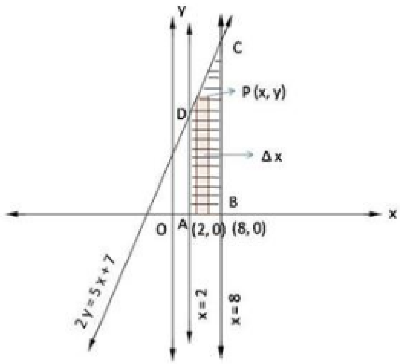
$2y = 5x + 7$ ---(1)

$x = 2$ ---(2)

$x = 8$ ---(3)

Equation (1) represents line passing through $\left(-\frac{7}{5}, 0\right)$ and $\left(0, \frac{7}{2}\right)$ equation (2), (3) shows line parallel to y-axis passing through $(2,0)$, $(8, 0)$ respectively.

A rough sketch of curves is as below:-



Shaded region represents the required area. We slice the region into approximation rectangles of Width $= \Delta x$ and length $= y$

Area of the rectangle $= y\Delta x$.

This approximation rectangle slides from $x = 2$ to $x = 8$, so

Required area = (Region $ABCDA$)

$$= \int_2^8 \left(\frac{5x + 7}{2}\right) dx$$

$$= \frac{1}{2}\left(\frac{5x^2}{2} + 7x\right)_2^8$$

$$= \frac{1}{2}\left[\left(\frac{5(8)^2}{2} + 7(8)\right) - \left(\frac{5(2)^2}{2} + 7(2)\right)\right]$$

$$= \frac{1}{2}\left[(160 + 56) - (10 + 14)\right]$$

$$= \frac{192}{2}$$

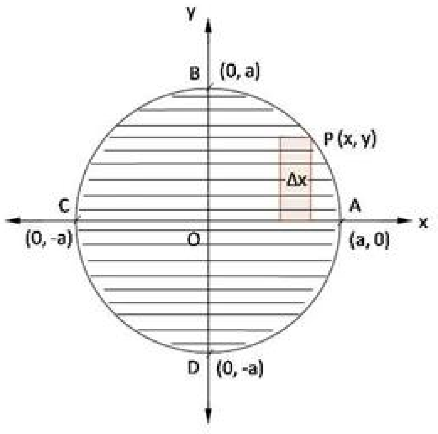
Required area = 96 square units

Areas of Bounded Regions Ex 21.1 Q15

We have to find the area of circle

$$x^2 + y^2 = a^2 \qquad ---(1)$$

Equation (1) represents a circle with centre $(0,0)$ and radius a, so it meets the axes at $(\pm a, 0), (0, \pm a)$. A rough sketch of the curve is given below:-



Shaded region is the required area. We slice the region $AOBA$ in rectangles of width Δx and length $= y - 0 = y$

Area of rectangle $= y \Delta x$.

This approximation rectangle can slide from $x = 0$ to $x = a$, so

Required area = Region $ABCDA$

$= 4\left(\text{Region } ABOA\right)$

$= 4\left(\int_0^a y dx\right)$

$= 4\int_0^a \sqrt{a^2 - x^2} dx$

$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= 4\left[\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right) - (0 + 0)\right]$

$= 4\left[0 + \frac{a^2}{2} \cdot \frac{\pi}{2}\right]$

$= 4\left(\frac{a^2\pi}{4}\right)$

Required area $= \pi a^2$ sq.units

Areas of Bounded Regions Ex 21.1 Q16

To find area enclosed by

$x = -2$, $x = 3$, $y = 0$ and $y = 1 + |x + 1|$

$\Rightarrow \quad y = 1 + x + 1$, if $x + 1 \ge 0$

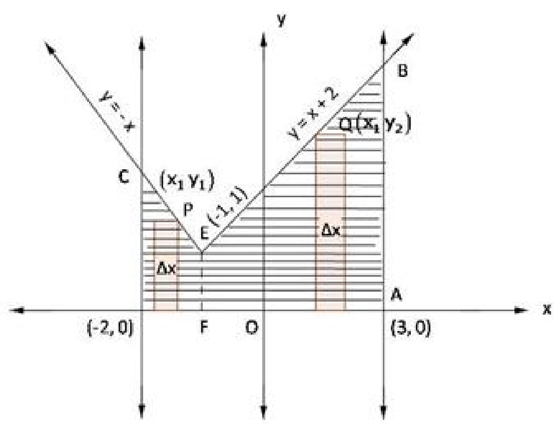
$\Rightarrow \quad y = 2 + x \qquad ---(1)$, if $x \ge -1$

And $\quad y = 1 - (x + 1)$, if $x + 1 < 0$

$\Rightarrow \quad y = 1 - x - 1$, if $x < -1$

$\Rightarrow \quad y = -x \qquad ---(2)$, if $x < -1$

So, equation (1) is a straight line that passes through $(0,2)$ and $(-1,1)$. Equation (2) is a line passing through $(-1,1)$ and $(-2,2)$ and it is enclosed by line $x = 2$ and $x = 3$ which are lines parallel to y-axis and pass through $(2,0)$ and $(3,0)$ respectively $y = 0$ is x-axis. So, a rough sketch of the curves is given as:-



Shaded region represents the required area.

So, required area = Region $(ABECDFA)$

Required area = (region $ABEFA$ + region $ECDFE$) $\qquad ---(1)$

region $ECDFE$ is sliced into approximation rectangle with width Δx and length y_1.
Area of those approximation rectangle is $y_1 \Delta x$ and these slids from $x = -2$ to $x = -1$.

Region $ABEFA$ is sliced into approximation rectangle with width Δx and length y_2.
Area of those rectangle is $y_2 \Delta x$ which slides from $x = -1$ to $x = 3$. So, using equation (1),

$$\text{Required area} = \int_{-2}^{-1} y_1 dx + \int_{-1}^{3} y_2 dx$$

$$= \int_{-2}^{-1} (-x)\, dx + \int_{-1}^{3} (x + 2)\, dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\left[\frac{1}{2} - \frac{4}{2}\right] + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{1}{2} - 2\right)\right]$$

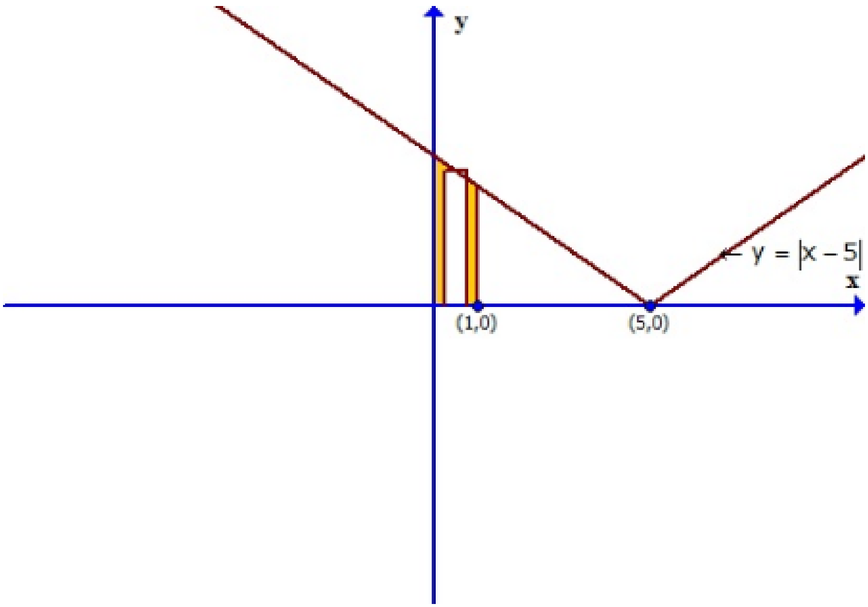
$$= \frac{3}{2} + \left(\frac{21}{2} + \frac{3}{2}\right)$$

$$= \frac{27}{2}$$

Required area $= \frac{27}{2}$ sq.units

Areas of Bounded Regions Ex 21.1 Q17

Consider the sketch of the given graph: $y = |x - 5|$



Therefore,

Required area $= \int_0^1 y\,dx$

$= \int_0^1 |x - 5|\,dx$

$= \int_0^1 -(x - 5)\,dx$

$= \left[\frac{-x^2}{2} + 5x\right]_0^1$

$= \left[-\frac{1}{2} + 5\right]$

$= \frac{9}{2}$ *sq. units*

Therefore, the given integral represents the area bounded by the curves, $x = 0, y = 0,\ x = 1$ *and* $y = -(x - 5)$.

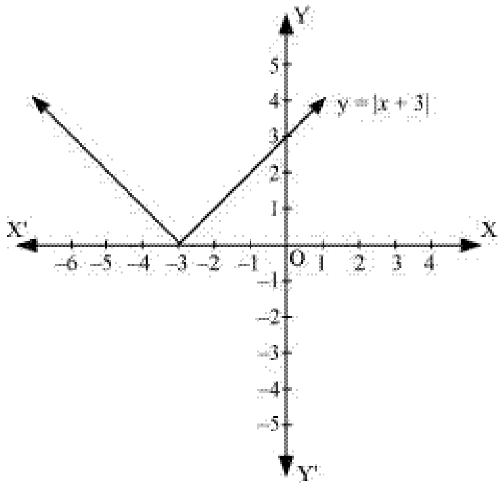
Areas of Bounded Regions Ex 21.1 Q18

The given equation is $y = |x+3|$

The corresponding values of x and y are given in the following table.

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.



It is known that, $(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$

$$\therefore \int_{-6}^{0} |(x+3)| dx = -\int_{-6}^{-3} (x+3) dx + \int_{-3}^{0} (x+3) dx$$

$$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

$$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right]$$

$$= 9$$

Areas of Bounded Regions Ex 21.1 Q19

We have,

$$y = |x+1| = \begin{cases} x+1, & \text{if } x+1 \geq 0 \\ -(x+1), & \text{if } x+1 < 0 \end{cases}$$

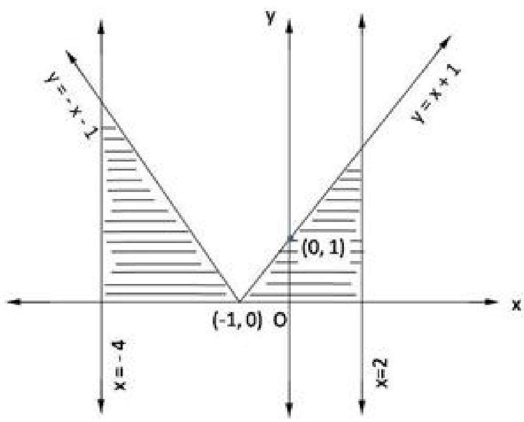
$$y = \begin{cases} (x+1), & \text{if } x \geq -1 \\ -x-1, & \text{if } x < -1 \end{cases}$$

$\Rightarrow \quad y = x + 1 \qquad (1)$

and $y = -x - 1 \qquad (2)$

Equation (1) represents a line which meets axes at $(0,1)$ and $(-1,0)$. Equation (2) represents a line passing through $(0,-1)$ and $(-1,0)$

A rough sketch is given below:-



$$\int_{-4}^{2} |x+1| dx = \int_{-4}^{-1} -(x+1)\, dx + \int_{-1}^{2} (x+1)\, dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-4}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2} - 1\right) - \left(\frac{16}{2} - 4\right)\right] + \left[\left(\frac{4}{2} + 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= -\left[\left(-\frac{1}{2} - 4\right)\right] + \left[4 + \frac{1}{2}\right]$$

$$= \frac{9}{2} + \frac{9}{2}$$

$$= \frac{18}{2}$$

Required area = 9 sq. unit

Areas of Bounded Regions Ex 21.1 Q20

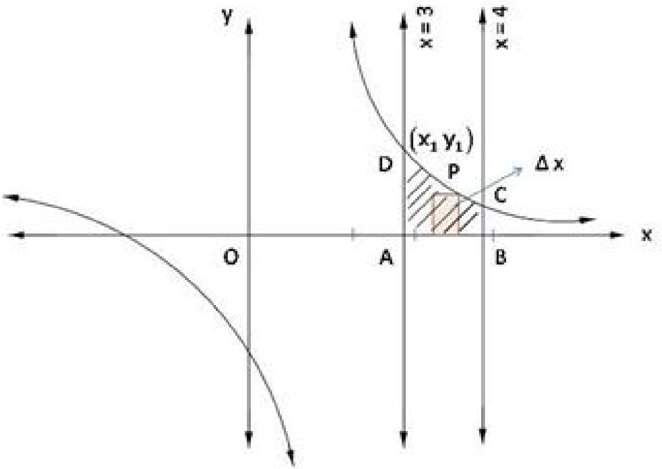
To find the area bounded by

x axis, $x = 3$, $x = 4$ and $xy - 3x - 2y - 10 = 0$

$\Rightarrow \quad y(x-2) = 3x + 10$

$\Rightarrow \quad y = \dfrac{3x+10}{x-2}$

A rough sketch of the curves is given below:-



Shaded region is required region.
It is sliced in rectangle with width $= \Delta x$ and length $= y$

Area of rectangle $= y\Delta x$

This approximation rectangle slide from $x = 3$ to $x = 4$. So,

Required area = Region $ABCDA$

$$= \int_3^4 y\,dx$$

$$= \int_3^4 \left(\frac{3x+10}{x-2}\right) dx$$

$$= \int_3^4 \left(3 + \frac{16}{x-2}\right) dx$$

$$= (3x)_3^4 + 16\{\log|x-2|\}_3^4$$

$$= (12-9) + 16(\log 2 - \log 1)$$

Required area $= (3 + 16\log 2)$ sq. units

Areas of Bounded Regions Ex 21.1 Q21

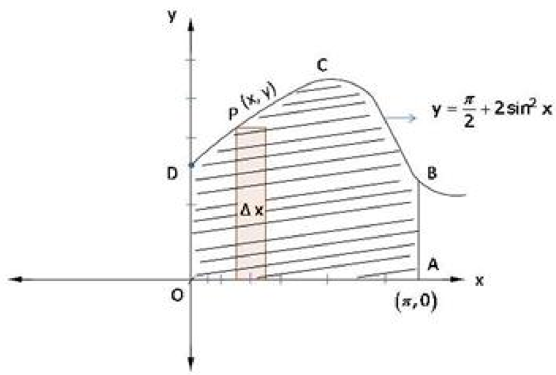
To find area bounded by $y = \frac{\pi}{2} + 2\sin^2 x$,

x-axis, $x = 0$ and $x = \pi$

A table for values of $y = \frac{\pi}{2} + 2\sin^2 x$ is:-

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\frac{\pi}{2} + 2\sin^2 x$	1.57	2.07	2.57	3.07	3.57	3.07	2.57	2.07	1.57

A rough sketch of the curves is given below:-



Shaded region represents required area. We slice it into rectangles of width $= \Delta x$ and length $= y$

Area of rectangle $= y\Delta x$

The approximation rectangle slides from $x = 0$ to $x = \pi$. So,

Required area = (Region $ABCDO$)

$$= \int_0^\pi y\,dx$$

$$= \int_0^\pi \left(\frac{\pi}{2} + 2\sin^2 x\right) dx$$

$$= \int_0^\pi \left(\frac{\pi}{2} + 1 - \cos 2x\right) dx$$

$$= \left[\frac{\pi}{2}x + x - \frac{\sin 2x}{2}\right]_0^\pi$$

$$= \left\{\left(\frac{\pi^2}{2} + \pi - \frac{\sin 2x}{2}\right) - (0)\right\}$$

$$= \frac{\pi^2}{2} + \pi$$

Required area $= \frac{\pi}{2}(\pi + 2)$ sq. units

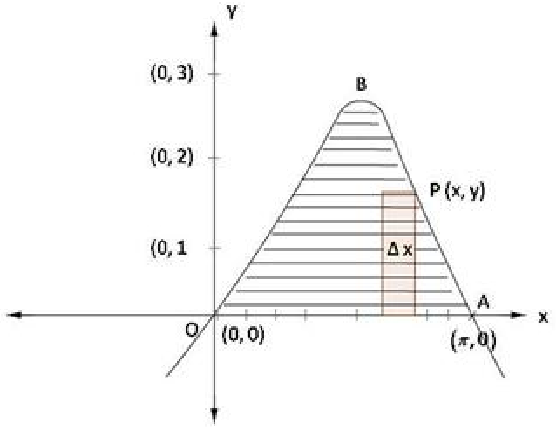
Areas of Bounded Regions Ex 21.1 Q22

To find area between by x-axis, $x = 0$, $x = \pi$ and

$$y = \frac{x}{\pi} + 2\sin^2 x \qquad ---(1)$$

The table for equation (1) is:-

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	0.66	1.25	1.88	2.5	1.88	1.25	0.66	0



Shaded region is the required area. We slice the area into rectangles with width $= \Delta x$, length $= y$

Area of rectangle $= y\Delta x$

The approximation rectangle slides from $x = 0$ to $x = \pi$. So,

Required area = (Region $ABOA$)

$$= \int_0^{\pi} y\,dx$$

$$= \int_0^{\pi} \left(\frac{x}{\pi} + 2\sin^2 x\right)dx$$

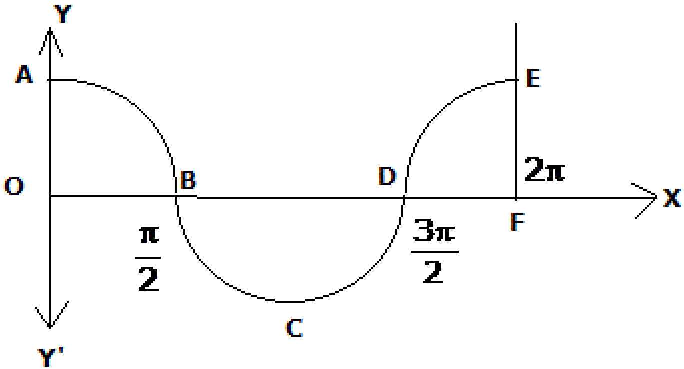
$$= \int_0^{\pi} \left(\frac{x}{\pi} + 1 - \cos 2x\right)dx$$

$$= \left[\frac{x^2}{2\pi} + x - \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \left(\frac{\pi^2}{2\pi} + \pi - 0\right) - (0)$$

Required area $= \frac{3\pi}{2}$ sq. units

Areas of Bounded Regions Ex 21.1 Q23



From the figure, we notice that

The required area= area of the region OABO + area of the region BCDB + area of the region DEFD

Thus, the reqd. area = $\int_0^{\pi/2} \cos x \, dx + \left|\int_{\pi/2}^{3\pi/2} \cos x \, dx\right| + \int_{3\pi/2}^{2\pi} \cos x \, dx$

$= [\sin x]_0^{\pi/2} + \left|[\sin x]_{\pi/2}^{3\pi/2}\right| + [\sin x]_{3\pi/2}^{2\pi}$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right| + \left[\sin 2\pi - \sin\frac{3\pi}{2}\right]$

$= 1 + 2 + 1 = 4$ square units

Areas of Bounded Regions Ex 21.1 Q24

To find area under the curve

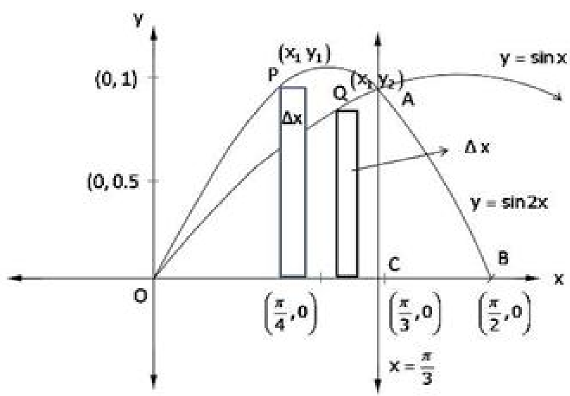
$y = \sin x$ $\quad ---(1)$

and $y = \sin 2x$ $\quad ---(2)$

between $x = 0$ and $x = \frac{\pi}{3}$.

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$Y = \sin x$	0	0.5	0.7	0.8	1
$Y = \sin 2x$	0	0.8	1	0.8	0

A rough sketch of the curve is given below:-



Area under curve $y = \sin 2x$

It is sliced in rectangles with width $= \Delta x$ and length $= y_1$

Area of rectangle $= y_1 \Delta x$

This approximation rectangle slides from $x = 0$ to $x = \frac{\pi}{3}$. So, Required area = Region $OPACO$

$A_1 = \int_0^{\frac{\pi}{3}} y_1 dx$

$= \int_0^{\frac{\pi}{3}} \sin 2x \, dx$

$= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{3}}$

$= -\left[-\frac{1}{4} - \frac{1}{2}\right]$

$A_1 = \frac{3}{4}$ sq.units

Area under curve $y = \sin x$:

It is sliced in rectangles with width Δx and langth y_2

Area of rectangle = $y_2 \Delta x$

This approximation rectangle slides from $x = 0$ to $x = \frac{\pi}{3}$. So,

Required area = Region $OQACO$

$= \int_0^{\frac{\pi}{3}} y_2 dx$

$= \int_0^{\frac{\pi}{3}} \sin x \, dx$

$= [-\cos x]_0^{\frac{\pi}{3}}$

$= -\left[\cos\frac{\pi}{3} - \cos 0\right]$

$= -\left(\frac{1}{2} - 1\right)$

$A_2 = \frac{1}{2}$ sq.units

So,

$A_2 : A_1 = \frac{1}{2} : \frac{3}{4}$

$A_2 : A_1 = 2 : 3$

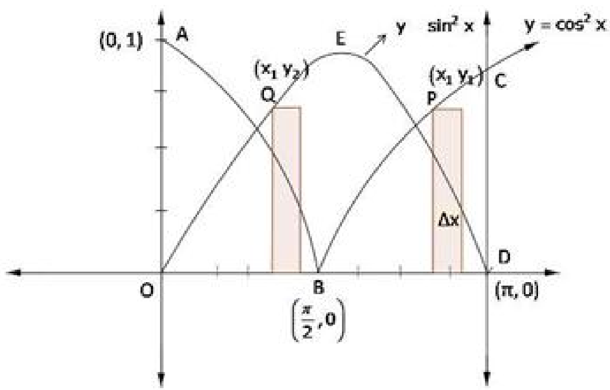
Areas of Bounded Regions Ex 21.1 Q25

To compare area under curves

$y = \cos^2 x$ and $y = \sin^2 x$ between $x = 0$ and $x = \pi$.

Table for $y = \cos^2 x$ and $y = \sin^2 x$ is

X	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$Y=\cos^2 x$	1	0.75	0.5	0.25	0	0.25	0.5	0.75	1
$Y=\sin^2 x$	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0



Area of region enclosed by $y = \cos^2 x$ and axis

A_1 = Region $OABO$ + Region $BCDB$

$= 2(\text{Region } BCDB)$

$= 2\int_{\frac{\pi}{2}}^{\pi} \cos^2 x \, dx$

$= 2\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \left[x - \frac{\sin 2x}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= \left[(\pi - 0) - \left(\frac{\pi}{2} - 0\right)\right]$

$= \pi - \frac{\pi}{2}$

$A_1 = \frac{\pi}{2}$ sq.units ---(1)

Area of region enclosed by $y = \sin^2 x$ and axis

A_2 = Region $OEDO$

$= \int_0^{\pi} \sin^2 x \, dx$

$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$

$= \frac{1}{2}\left[(\pi - 0) - (0)\right]$

$A_2 = \frac{\pi}{2}$ sq. units ---(2)

From equation (1) and (2),

$A_1 = A_2$

So,

Area enclosed by $y = \cos^2 x$ = Area enclosed by $y = \sin^2 x$

Areas of Bounded Regions Ex 21.1 Q26

The required area fig., of the region BOB'RFSB is enclosed by the ellipse and the lines $x = 0$ and $x = ae$.

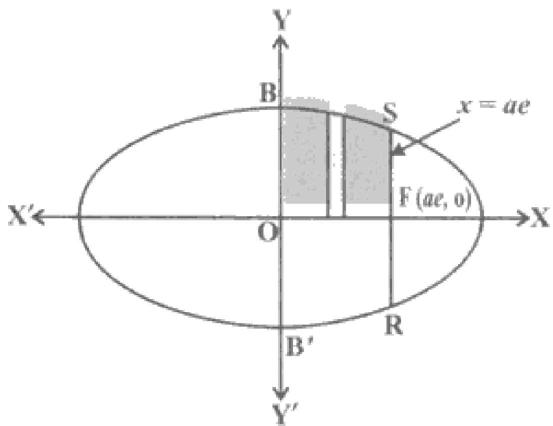
Note that the area of the region BOB'RFSB

$$= 2\int_0^{ae} y\,dx = 2\frac{b}{a}\int_0^{ae}\sqrt{a^2 - x^2}\,dx$$

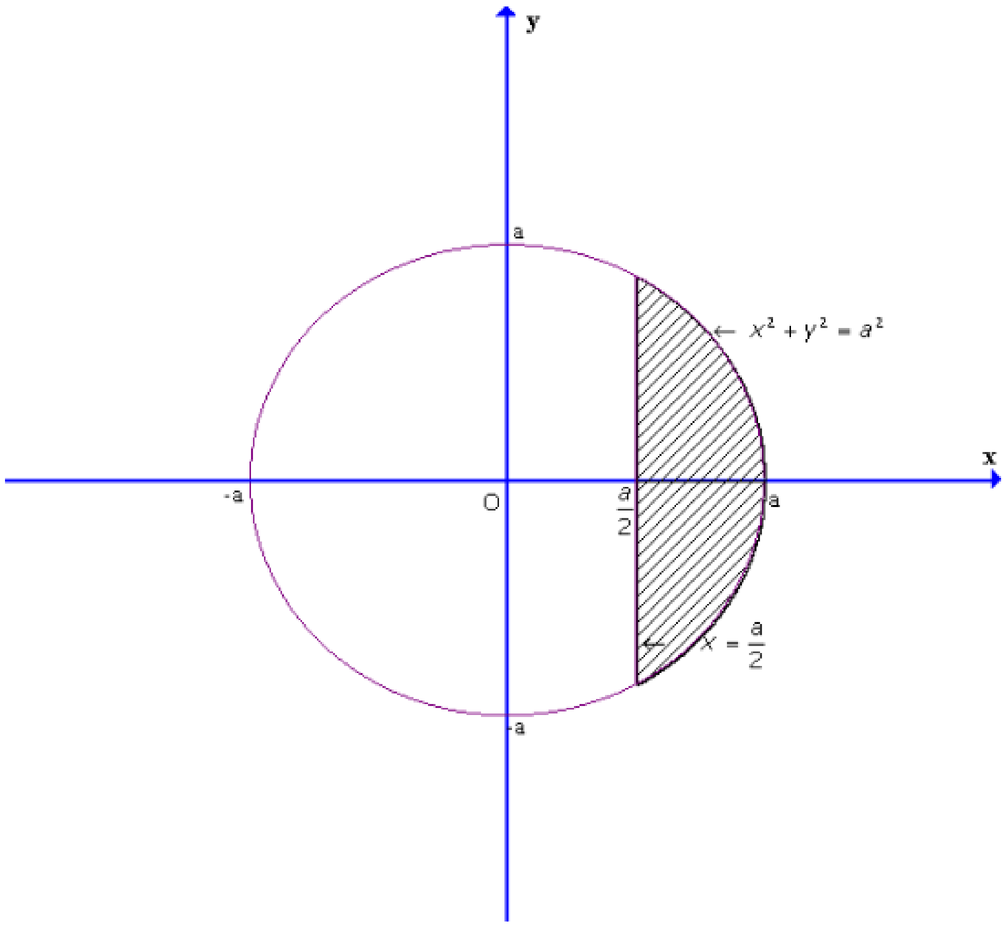
$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^{ae}$$

$$= \frac{2b}{2a}\left[ae\sqrt{a^2 - a^2e^2} + a^2\sin^{-1}e\right]$$

$$= ab\left[e\sqrt{1 - e^2} + \sin^{-1}e\right]$$



Areas of Bounded Regions Ex 21.1 Q27



Area of the minor segment of the circle

$$=2\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}dx$$

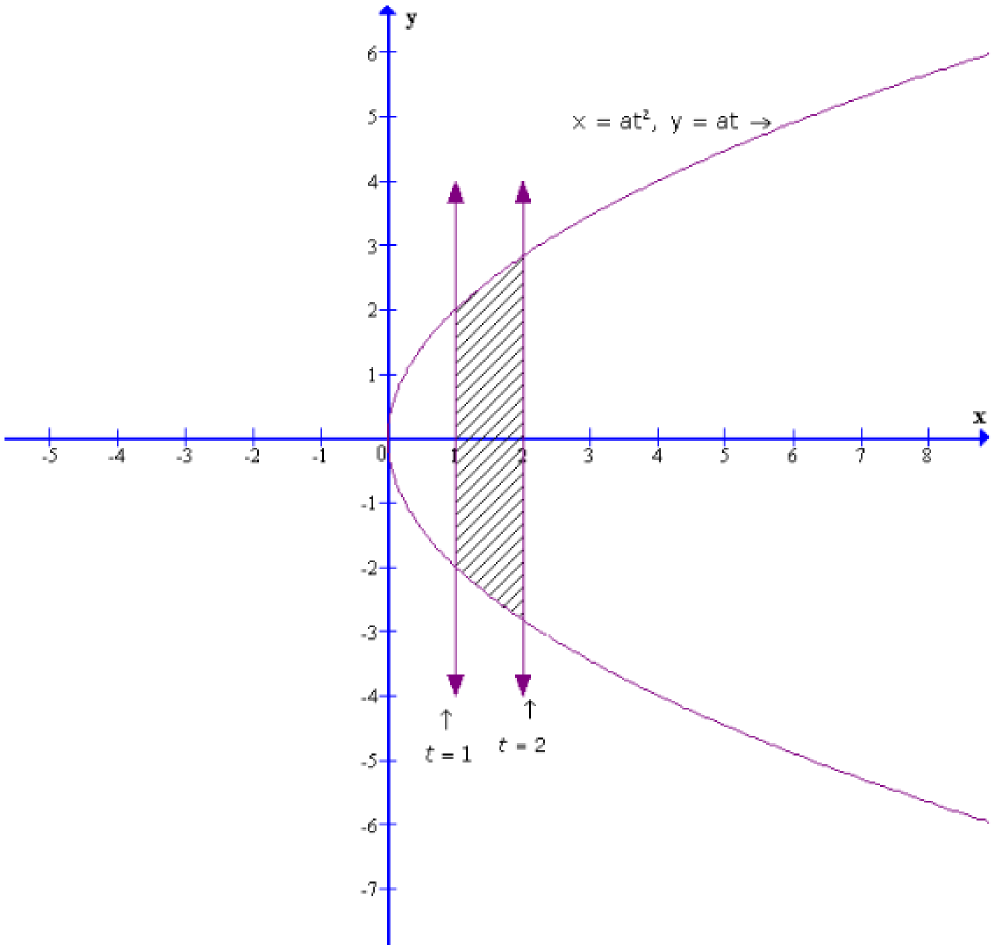
$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a}{2}(0) + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{2}\right) - \frac{a}{4}\sqrt{a^2 - \frac{a^2}{4}} - \frac{a^2}{2}\sin^{-1}\frac{a}{4}\right]$$

$$= 2\left[\frac{a^2}{2}\sin^{-1}\left(\frac{a}{2}\right) - \frac{a}{4}\sqrt{a^2 - \frac{a^2}{4}} - \frac{a^2}{2}\sin^{-1}\frac{a}{4}\right]$$

$$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right)\text{sq. units}$$

Areas of Bounded Regions Ex 21.1 Q28



Area of the bounded region

$= 2\int_1^2 y \frac{dx}{dt} dt$

$= 2\int_1^2 (2at)(2at) dt$

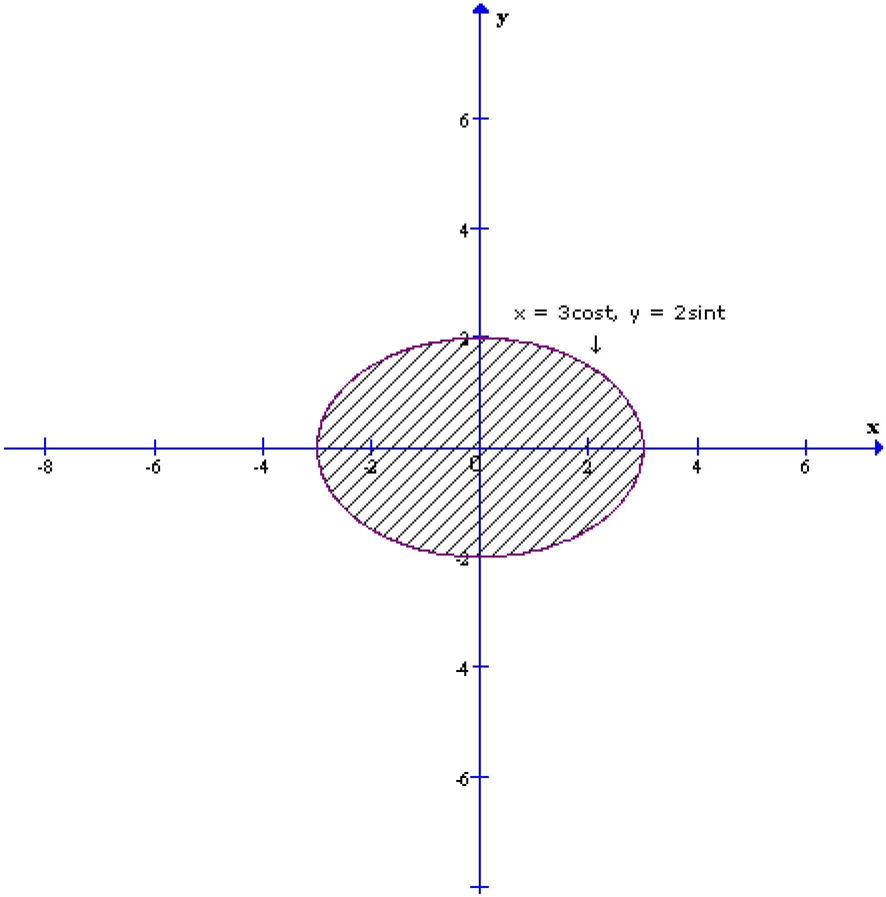
$= 8a^2 \int_1^2 t^2 dt$

$= 8a^2 \left[\frac{t^3}{3}\right]_1^2$

$= 8a^2 \left[\frac{8}{3} - \frac{1}{3}\right]$

$= \frac{56a^2}{3}$ sq. units

Areas of Bounded Regions Ex 21.1 Q29



Area of the bounded region

$$= 4\int_0^{\frac{\pi}{2}} 2\sin t\, dt$$

$$= -8[\cos t]_0^{\frac{\pi}{2}}$$

$$= -8[0-1]$$

= 8sq units

Note: Answer given in the book is incorrect.