RD Sharma Solutions Class 12 Maths Chapter 21 Ex 21.2

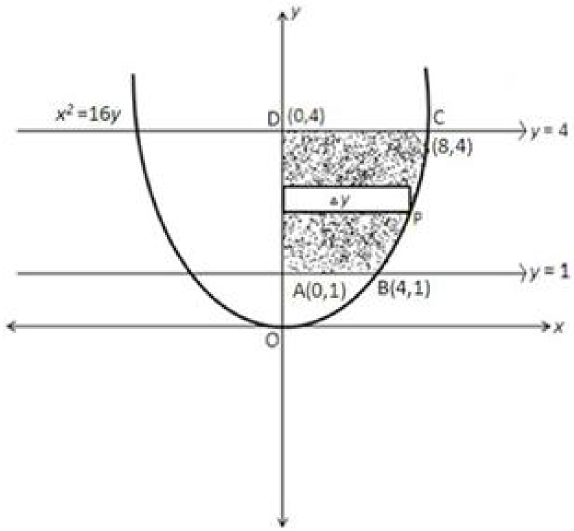
Areas of Bounded Regions Ex-21-2 Q2

To find region in first quadrant bounded by $y = 1$, $y = 4$ and y-axis and

$x^2 = 16y \qquad ---(1)$

Equation (1) represents a parabola with vertex $(0,0)$ and axes as y-axis.

A rough sketch of the curves is as under:-

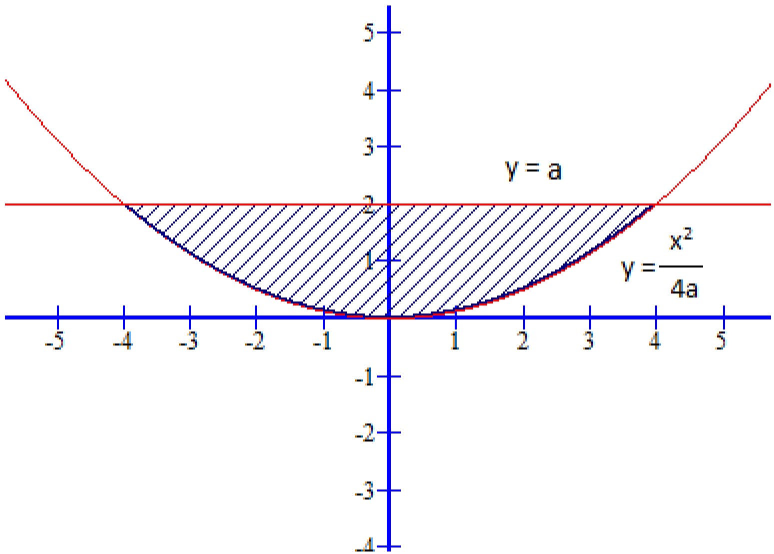


Shaded region is required area it is sliced in rectangles of area $x\Delta y$ which slides from $y = 1$ to $y = 4$, so

Required area = Region $ABCDA$

$$A = \int_1^4 x dy$$
$$= \int_1^4 4\sqrt{y} dy$$
$$= 4.\left[\frac{2}{3}y\sqrt{y}\right]_1^4$$
$$= 4.\left[\left(\frac{2}{3}.4\sqrt{4}\right) - \left(\frac{2}{3}.1.\sqrt{1}\right)\right]$$
$$= 4\left[\frac{16}{3} - \frac{2}{3}\right]$$
$$A = \frac{56}{3} \text{ sq. units}$$

Areas of Bounded Regions Ex-21-2 Q3



$$\text{Area of the region} = 2 \times \int_0^{2a} \left(a - \frac{x^2}{4a}\right) dx$$

$$= 2 \times \left[ax - \frac{x^3}{12a}\right]_0^{2a}$$

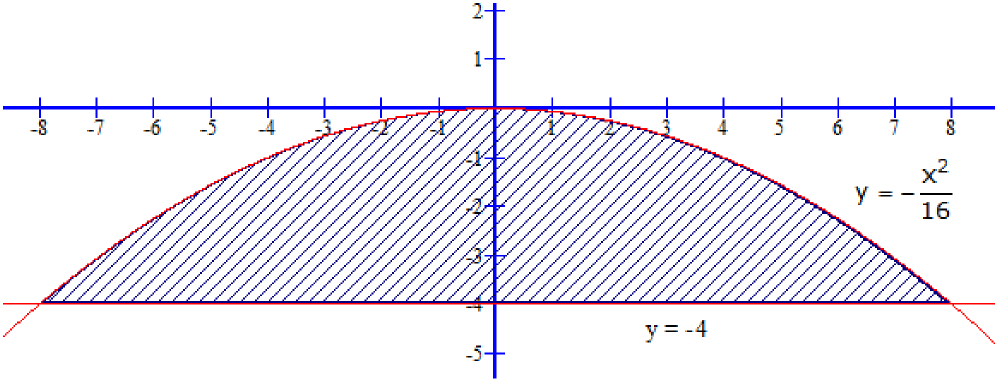
$$= 2\left[a(2a - 0) - \frac{(2a)^3 - 0^3}{12a}\right]$$

$$= 2\left[2a^2 - \frac{8a^3}{12a}\right]$$

$$= 2\left[\frac{16a^3}{12a}\right]$$

$$= \frac{8}{3}a^2 \text{ sq. units}$$

Areas of Bounded Regions Ex-21-2 Q4



$$\text{Area of the region} = 2\times\int_0^8\left[-\frac{x^2}{16}-(-4)\right]dx$$

$$= 2\times\left[-\frac{x^3}{48}+4x\right]_0^8$$

$$= 2\times\left[4x-\frac{x^3}{48}\right]_0^8$$

$$= 2\times\left[4(8-0)-\frac{(8)^3-0^3}{48}\right]$$

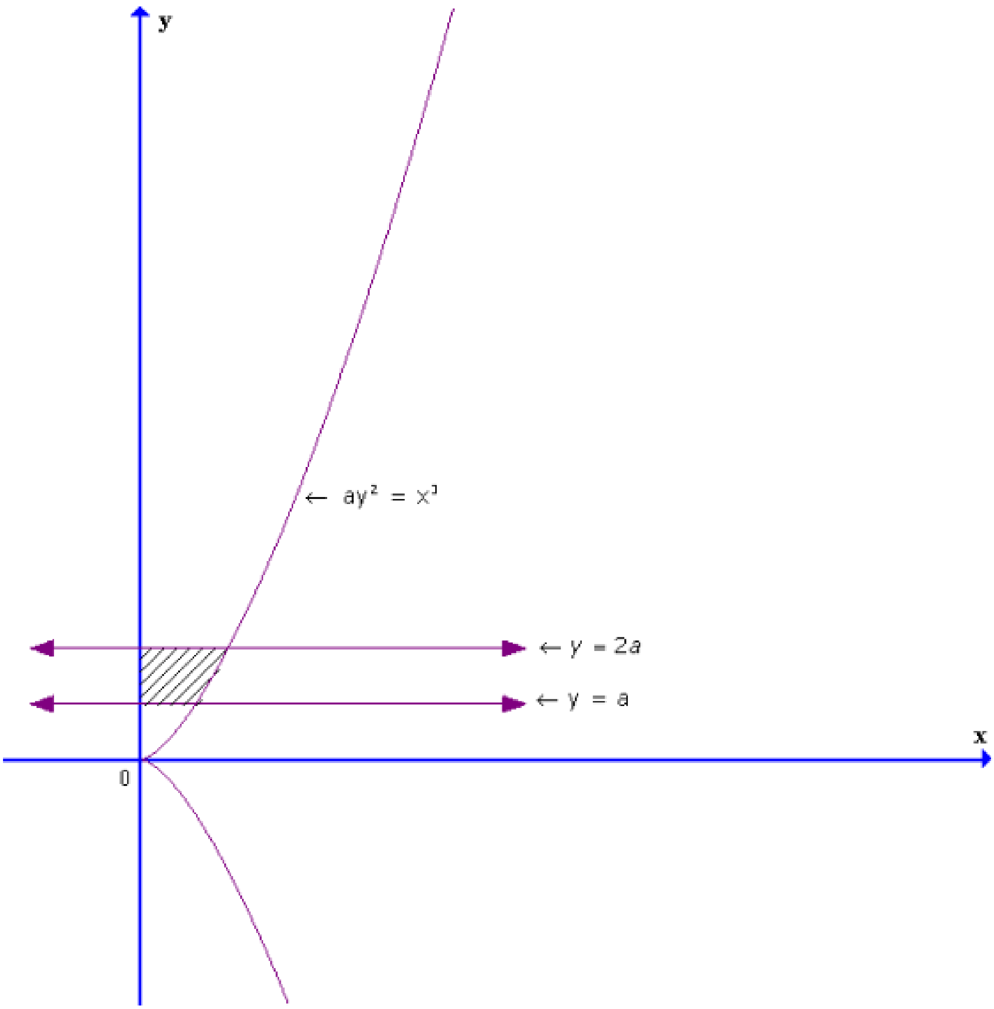
$$= 2\times\left[32-\frac{512}{48}\right]$$

$$= 2\times\left[32-\frac{32}{3}\right]$$

$$= 2\times\left[\frac{96-32}{3}\right]$$

$$= 2\times\frac{64}{3}=\frac{128}{3}\text{ sq. units}$$

Areas of Bounded Regions Ex-21-2 Q5



Area of the bounded region

$$= \int_a^{2a} \left(ay^2\right)^{\frac{1}{3}} \, dy$$

$$= a^{\frac{1}{3}} \int_a^{2a} y^{\frac{2}{3}} \, dy$$

$$= a^{\frac{1}{3}} \left[\frac{3}{5} y^{\frac{5}{3}}\right]_0^{2a}$$

$$= \frac{3}{5}\left(2^{\frac{5}{3}} - 1\right) a^2 \text{ sq units}$$