RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.4

Differential Equations Ex 22.4 Q1

Here, $y = \log x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{x}$$

$$x\frac{dy}{dx} = 1$$

So, $y = \log x$ is a solutionof the equation

If $x = 1, \quad y = \log 1 = 0$

So,

$$y(1) = 0$$

Differential Equations Ex 22.4 Q2

Here, $y = e^x$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y$$

So, $y = e^x$ is a solution of the equation

If $x = 0, \quad y = e^0 = 1$

So,

$$y(0) = 1$$

Differential Equations Ex 22.4 Q3

Here, $y = \sin x$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x \quad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x$$

$$\frac{d^2y}{dx^2} = -y$$

$$\frac{d^2y}{dx^2} + y = 0$$

So, $y = \sin x$ is a solution of the equation.

Put $x = 0$ in equation (i),

$\Rightarrow$ $y = \sin 0$

$\Rightarrow$ $y = 0$

$\Rightarrow$ $y(0) = 0$

Put $x = 0$ in equation (ii),

$y' = \cos 0$

$y' = 1$

$\Rightarrow$ $y'(0) = 1$

Differential Equations Ex 22.4 Q4

Here, $y = e^x + 1$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x$$

$$\frac{dy}{dx} = y - 1 \quad \text{—(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

It is given differential equation. So,

$y = e^x + 1$ is a solutionof the equation

Put $x = 0$ in equation (i),

$\Rightarrow$ $y = e^0 + 1 = 2$

$y(0) = 2$

Put $x = 0$ in equation (ii),

$y' = e^0 = 1$

$y'(0) = 1$

Differential Equations Ex 22.4 Q5

Here, $y = e^{-x} + 2$ —(i)

Differentiating it with respect to x,

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dy}{dx} = -(y-2) \qquad \text{[Using equaton (i)]}$$

$$\frac{dy}{dx} + y = 2$$

It is given differential equation. So,

$y = e^{-x} + 2$ is a solutionof the equation

Put $x = 0$ in equation (i),

$$y = e^0 + 2$$
$$= 1 + 2$$
$$y = 3$$

So,

$$y(0) = 3$$

Differential Equations Ex 22.4 Q6

$$y = \sin x + \cos x \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \cos x - \sin x \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = -\sin x - \cos x$$

$$\frac{d^2y}{dx^2} = -(\sin x + \cos x)$$

$$\frac{d^2y}{dx^2} = -y \qquad \text{[Using equation (i)]}$$

$$\frac{d^2y}{dx^2} + y = 0$$

It is the given equation, so

$y = \sin x + \cos x$ is the solution of the given equation

Put $x = 0$ in equation (i),

$$y = \sin 0 + \cos 0$$
$$y = 0 + 1$$
$$y = 1$$

So,

$$y(0) = 1$$

Put $x = 0$ in equation (i),

$$\frac{dy}{dx} = \cos 0 - \sin 0$$
$$y' = 1$$

So,

$$y'(0) = 1$$

Differential Equations Ex 22.4 Q7

$$y = e^x + e^{-x} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x - e^{-x} \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + e^{-x}$$

$$\frac{d^2y}{dx^2} = y \qquad [\text{Using equation (i)}]$$

$$\frac{d^2y}{dx^2} - y = 0$$

It is the given equation, so

$y = e^x + e^{-x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$

$$y = 2$$

So,

$$y(0) = 2$$

Put $x = 0$ in equation (ii),

$$y' = e^0 - e^0$$

$$y' = 0$$

So,

$$y'(0) = 0$$

Differential Equations Ex 22.4 Q8

$$y = e^x + e^{2x} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = e^x + 2e^{2x} \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^x + 4e^{2x}$$

$$= (3-2)e^x + (6-2)e^{2x}$$

$$= 3e^x - 2e^x + 6e^{2x} - 2e^{2x}$$

$$= 3e^x + 6e^{2x} - 2e^x - 2e^{2x}$$

$$= 3\left(e^x + 2e^{2x}\right) - 2\left(e^x + e^{2x}\right)$$

$$\frac{d^2y}{dx^2} = 3\frac{dy}{dx} - 2y \qquad [\text{Using equation (i) and (ii)}]$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

It is the given equation, so

$y = e^x + 2e^{2x}$ is the solution of the given equation.

Put $x = 0$ in equation (i),

$$y = e^0 + e^0$$

$$y = 1 + 1$$

$$y = 2$$

So,

$$y(0)=2$$

Put $x=0$ in equation (ii),

$$\frac{dy}{dx}=e^0+2e^0$$
$$y'=1+2$$
$$y'=3$$

So,

$$y'(0)=3$$

Differential Equations Ex 22.4 Q9

$$y=xe^x+e^x \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=\left[x\times\frac{d}{dx}\left(e^x\right)+e^x\frac{d}{dx}(x)\right]+e^x$$
$$=xe^x+e^x(1)+e^x$$
$$\frac{dy}{dx}=xe^x+2e^x \qquad \text{---(ii)}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2}=x\frac{d}{dx}\left(e^x\right)+e^x\frac{d}{dx}(x)+2e^x$$
$$=(2-1)xe^x+(4-1)e^x$$
$$=2xe^x-xe^x+4e^x-e^x$$
$$=2xe^x+4e^x-xe^x-e^x$$
$$=2\left(xe^x+2e^x\right)-\left(xe^x+1\right)$$
$$\frac{d^2y}{dx^2}=2\frac{dy}{dx}-y \qquad \text{[Using equation (i) and (ii)]}$$
$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$$

It is the given equation, so

$y=xe^x+e^x$ is the solution of the given equation.

Put $y=0$ in equation (i),

$$y=0+e^0$$
$$y=1$$

So,

$$y(0)=1$$

Put $y=0$ in equation (ii),

$$\frac{dy}{dx}=0+2e^0$$
$$y'=2$$

So,

$$y'(0)=2$$