RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.7

Differential Equations Ex 22.7 Q1

$$(x-1)\frac{dy}{dx} = 2xy$$

Separating the variables,

$$\int \frac{dy}{y} = \int \frac{2x}{x-1}dx$$

$$\int \frac{dy}{y} = \int \left(2 + \frac{2}{x-1}\right)dx$$

$$\log y = 2x + 2\log|x-1| + c$$

Differential Equations Ex 22.7 Q2

$$(x^2+1)dy = xydx$$

$$\int \frac{1}{y}dy = \int \frac{x}{x^2+1}dx$$

$$\int \frac{1}{y}dy = \frac{1}{2}\int \frac{2x}{x^2+1}dx$$

$$\log y = \frac{1}{2}\log|x^2+1| + \log c$$

$$y = \sqrt{x^2+1} \times c$$

Differential Equations Ex 22.7 Q3

$$\frac{dy}{dx} = (e^x+1)y$$

$$\int \frac{1}{y}dy = \int (e^x+1)dx$$

$$\log|y| = e^x + x + c$$

Differential Equations Ex 22.7 Q4

$$(x-1)\frac{dy}{dx} = 2x^3y$$

$$\frac{dy}{y} = \frac{2x^3}{x-1}dx$$

$$\int\frac{dy}{y} = 2\int\left(x^2+x+1+\frac{1}{x-1}\right)dx$$

$$\log|y| = 2\left(\frac{x^3}{3}+\frac{x^2}{2}+x+\log|x-1|\right)+c$$

$$\log|y| = \frac{2}{3}x^3+x^2+2x+2\log|x-1|+c$$

Differential Equations Ex 22.7 Q5

$$xy(y+1)dy = (x^2+1)dx$$

$$y(y+1)dy = \frac{x^2+1}{x}dx$$

$$\int(y^2+y)dy = \int\left(x+\frac{1}{x}\right)dx$$

$$\frac{y^3}{3}+\frac{y^2}{2} = \frac{x^2}{2}+\log|x|+c$$

Differential Equations Ex 22.7 Q6

$$5\frac{dy}{dx} = e^xy^4$$

$$5\int\frac{dy}{y^4} = \int e^xdx$$

$$5\left(\frac{y^{-4+1}}{-4+1}\right) = e^x+c$$

$$-\frac{5}{3y^3} = e^x+c$$

Differential Equations Ex 22.7 Q7

$$x\cos ydy = (xe^x\log x+e^x)dx$$

$$\int\cos ydy = \int e^x\left(\log x+\frac{1}{x}\right)dx$$

$$\sin y = e^x\log x+c$$

Since, $\int(f(x)+f'(x))e^xdx = e^xf(x)+c$

Differential Equations Ex 22.7 Q8

$$\frac{dy}{dx} = e^{x+y}+x^2e^y$$

$$= e^xe^y+x^2e^y$$

$$\frac{dy}{dx} = e^y(e^x+x^2)$$

$$\int e^{-y}dy = \int(e^x+x^2)dx$$

$$-e^{-y} = e^x+\frac{x^3}{3}+c$$

Differential Equations Ex 22.7 Q9

$$x\frac{dy}{dx}+y = y^2$$

$$x\frac{dy}{dx} = (y^2-y)$$

$$\frac{1}{y^2-y}dy = \frac{dx}{x}$$

$$\int\left(\frac{1}{y-1}-\frac{1}{y}\right)dy = \int\frac{dx}{x}$$

$$\log|y-1|-\log|y| = \log|x|+\log|c|$$

$$\log\left|\frac{y-1}{y}\right| = |xc|$$

$$y-1 = xyc$$

Differential Equations Ex 22.7 Q10

$$\left(e^y+1\right)\cos x dx+e^y \sin x dy=0$$
$$\left(e^y+1\right)\cos x dx=-e^y \sin x dy$$
$$\int \frac{\cos x}{\sin x}dx=-\int \frac{e^y}{e^y+1}dy$$
$$\int \cot x dx=-\int \frac{e^y}{e^y+1}dy$$
$$\log|\sin x|=-\log\left|e^y+1\right|+\log|c|$$
$$\sin x=\frac{c}{e^y+1}$$
$$\sin x\left(e^y+1\right)=c$$

Differential Equations Ex 22.7 Q11

$$x\cos^2 y dx=y\cos^2 x dy$$
$$\frac{x}{\cos^2 x}dx=\frac{y}{\cos^2 y}dy$$
$$\int x\sec^2 x dx=\int y\sec^2 y dy$$
$$x\times\int \sec^2 x-\int\left(1\times\int \sec^2 x dx\right)dx=y\int \sec^2 y dy-\int\left(1\times\int \sec^2 y dy\right)dy$$
$$x\tan x-\int \tan x dx=y\tan y-\int \tan y dy+c$$
$$x\tan x-\log|\sec x|=y\tan y-\log|\sec y|+c$$

Differential Equations Ex 22.7 Q12

$$xy dy=(y-1)(x+1)dx$$
$$\frac{y}{y-1}dy=\frac{x+1}{x}dx$$
$$\int\left(1+\frac{1}{y-1}\right)dy=\int\left(1+\frac{1}{x}\right)dx$$
$$y+\log|y-1|=x+\log|x|+c$$
$$y-x=\log|x|-\log|y-1|+c$$

Differential Equations Ex 22.7 Q13

$$x\frac{dy}{dx}+\cot y=0$$
$$x\frac{dy}{dx}=-\cot y$$
$$\int \tan y dy=-\int \frac{dx}{x}$$
$$\log|\sec y|=-\log|x|+\log|c|$$
$$\sec y=\frac{c}{x}$$
$$x\sec y=c$$
$$x=c\cos y$$

Differential Equations Ex 22.7 Q14

$$\frac{dy}{dx}=\frac{xe^x\log x+e^x}{x\cos y}$$
$$\int \cos y dy=\int e^x\left(\log x+\frac{1}{x}\right)dx$$
$$\sin y=e^x\log x+c$$

Since, $\int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)+c$

Differential Equations Ex 22.7 Q15

$$\frac{dy}{dx}=e^{x+y}+e^y x^3$$
$$\frac{dy}{dx}=e^y\left(e^x+x^3\right)$$
$$\int e^{-y}dy=\int\left(e^x+x^3\right)dx$$
$$-e^{-y}=e^x+\frac{x^4}{4}+c_1$$
$$e^x+\frac{x^4}{4}+e^{-y}=c$$

Differential Equations Ex 22.7 Q16

$$y\sqrt{1+x^2}+x\sqrt{1+y^2}\frac{dy}{dx}=0$$

$$x\sqrt{1+y^2}\frac{dy}{dx}=-y\sqrt{1+x^2}$$

$$\int\frac{\sqrt{1+y^2}}{y}dy=-\int\frac{\sqrt{1+x^2}}{x}dx$$

$$\int\frac{y\sqrt{1+y^2}}{y^2}dy=-\int\frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $1+y^2=t^2$

$\Rightarrow$ $2ydy=2tdt$

$1+x^2=v^2$

$\Rightarrow$ $2xdx=2vdv$

$$\int\frac{t\times tdt}{t^2-1}=-\int\frac{v\times vdv}{v^2-1}$$

$$\int\frac{t^2dt}{t^2-1}=-\int\frac{v^2dv}{v^2-1}$$

$$\int\left(1+\frac{1}{t^2-1}\right)dt=\int\left(1+\frac{1}{v^2-1}\right)dv$$

$$t+\frac{1}{2}log\left|\frac{t-1}{t+1}\right|=-v-log\left|\frac{v-1}{v+1}\right|+c$$

$$\sqrt{1+y^2}+\frac{1}{2}log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right|=-\sqrt{1+x^2}-\frac{1}{2}log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|+c$$

$$\sqrt{1+y^2}+\sqrt{1+x^2}+\frac{1}{2}log\left|\frac{\sqrt{y^2+1}-1}{\sqrt{y^2+1}+1}\right|+\frac{1}{2}log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|=c$$

Differential Equations Ex 22.7 Q17

$$\sqrt{1+x^2}dy+\sqrt{1+y^2}dx=0$$

$$\sqrt{1+x^2}dy=-\sqrt{1+y^2}dx$$

$$\int\frac{dy}{\sqrt{1+y^2}}=-\int\frac{dx}{\sqrt{1+x^2}}$$

$$log\left|y+\sqrt{1+y^2}\right|=-log\left|x+\sqrt{1+x^2}\right|=log|c|$$

$$\left(y+\sqrt{1+y^2}\right)\left(x+\sqrt{1+x^2}\right)=c$$

Differential Equations Ex 22.7 Q18

$$\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$$

$$\sqrt{\left(1+x^2\right)+y^2\left(1+x^2\right)}=-xy\frac{dy}{dx}$$

$$\sqrt{\left(1+x^2\right)\left(1+y^2\right)}=-xy\frac{dy}{dx}$$

$$\frac{ydy}{\sqrt{1+y^2}}=-\frac{\sqrt{1+x^2}}{x}dx$$

$$\int\frac{ydy}{\sqrt{1+y^2}}=-\int\frac{x\sqrt{1+x^2}}{x^2}dx$$

Let $1+y^2=t^2$

$\Rightarrow$ $2ydy=2tdt$

$1+x^2=v^2$

$\Rightarrow$ $2xdx=2vdv$

$$\int\frac{tdt}{t}=-\int\frac{v\times vdv}{v^2-1}$$

$$\int dt=-\int\frac{v^2}{v^2-1}dv$$

$$-\int dt=\int\left(1+\frac{1}{v^2-1}\right)dv$$

$$-t=v+\frac{1}{2}\log\left|\frac{v-1}{v+1}\right|+c_1$$

$$-\sqrt{1+y^2}=\sqrt{1+x^2}+\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|+c_1$$

$$\sqrt{1+x^2}+\sqrt{1+y^2}+\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|=c$$

Differential Equations Ex 22.7 Q19

$$\frac{dy}{dx}=\frac{e^x\left(\sin^2x+\sin x2x\right)}{y\left(2\log y+1\right)}$$

$$y\left(2\log y+1\right)dy=e^x\left(\sin^2x+\sin 2x\right)dx$$

$$\int\left(2y\log y+y\right)dy=\int e^x\left(\sin^2x+\sin 2x\right)dx$$

$$2\left[\log y\times\int ydy-\int\left(\frac{1}{2}\int ydy\right)dy\right]+\frac{y^2}{2}=e^x\sin^2x+c$$

Using integration by parts and

$$\int\left(f(x)+f'(x)\right)e^xdxdy+\frac{y^2}{2}=e^x\sin^2x+c$$

$$y^2\log y-\frac{y^2}{2}+\frac{y^2}{2}=e^x\sin^2x+c$$

$$y^2\log y=e^x\sin^2x+c$$

Differential Equations Ex 22.7 Q20

$$\frac{dy}{dx}=\frac{x\left(2\log x+1\right)}{\sin y+y\cos y}$$

$$\int\left(\sin y+y\cos y\right)dy=\int\left(2x\log x+x\right)dx$$

$$\int\sin ydy+\int y\cos ydy=2\int x\log xdx+\int xdx$$

$$\int\sin ydy+\left\{y\times\left(\int\cos ydy\right)-\int\left(1\times\int\cos ydy\right)dy\right\}=2\left\{\log x\int xdx-\int\left(\frac{1}{x}\int xdx\right)dx\right\}+\int xdx+c$$

$$\int\sin ydy+y\sin y-\int\sin ydy=x^2\log x-2\int\frac{x}{2}dx+\int xdx+c$$

$$y\sin y=x^2\log x+c$$

Differential Equations Ex 22.7 Q21

$$\left(1-x^2\right)dy + xydx = xy^2dx$$
$$\left(1-x^2\right)dy = dx\left(xy^2 - xy\right)$$
$$\left(1-x^2\right)dy = xy\left(y-1\right)dx$$
$$\int \frac{dy}{y\left(y-1\right)} = \int \frac{xdx}{1-x^2}$$
$$\int \left(\frac{1}{y-1} - \frac{1}{y}\right)dy = \frac{1}{2}\int \frac{2x}{1-x^2}dx$$
$$\int \left(\frac{1}{y-1} - \frac{1}{y}\right)dy = -\frac{1}{2}\int \frac{-2x}{1-x^2}dx$$
$$\log\left|y-1\right| - \log\left|y\right| = -\frac{1}{2}\log\left|1-x^2\right| + c$$

Differential Equations Ex 22.7 Q22

$$\tan ydx + \sec^2 y \tan xdy = 0$$
$$\tan ydx = -\sec^2 y \tan xdy$$
$$-\frac{dx}{\tan x} = \frac{\sec^2 ydy}{\tan y}$$
$$-\int \cot xdx = \int \frac{\sec^2 ydy}{\tan y}$$
$$-\log\left|\sin x\right| = \log\left|\tan y\right| + \log\left|c\right|$$
$$\frac{1}{\sin x} = c\tan y$$
$$\sin x \tan y = c_1$$

Differential Equations Ex 22.7 Q23

$$\left(1+x\right)\left(1+y^2\right)dx + \left(1+y\right)\left(1+x^2\right)dy = 0$$
$$\left(1+x\right)\left(1+y^2\right)dx = -\left(1+y\right)\left(1+x^2\right)dy$$
$$\frac{\left(1+y\right)dy}{\left(1+y^2\right)} = -\frac{\left(1+x\right)}{\left(1+x^2\right)}dx$$
$$\int \left(\frac{1}{1+y^2} + \frac{y}{1+y^2}\right)dy = -\int \left[\frac{1}{1+x^2} + \frac{x}{1+x^2}\right]dx$$
$$\int \frac{1}{1+y^2}dy + \frac{1}{2}\int \frac{2y}{1+y^2}dy = -\int \frac{1}{1+x^2}dx - \frac{1}{2}\int \frac{2x}{1+x^2}dx$$
$$\tan^{-1}\left(y\right) + \frac{1}{2}\log\left|1+y^2\right| = -\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + c$$
$$\tan^{-1}x + \tan^{-1}y + \frac{1}{2}\log\left|\left(1+y^2\right)\left(1+x^2\right)\right| = c$$

Differential Equations Ex 22.7 Q24

$$\tan y\frac{dy}{dx} = \sin\left(x+y\right) + \sin\left(x-y\right)$$
$$\tan y\frac{dy}{dx} = 2\sin\left\{\frac{\left(x+y\right)+\left(x-y\right)}{2}\right\}\cos\left\{\frac{\left(x+y\right)-\left(x-y\right)}{2}\right\}$$
$$= 2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)$$
$$\tan y\frac{dy}{dx} = 2\sin x\cos y$$
$$\frac{\tan y}{\cos y}dy = 2\sin xdx$$
$$\int \sec y\tan ydy = 2\int \sin xdx$$
$$\sec y = -2\cos x + c$$
$$\sec y + 2\cos x = c$$

Differential Equations Ex 22.7 Q25

$$\cos x \cos y \frac{dy}{dx} = -\sin x \sin y$$

$$\frac{\cos y}{\sin y} dy = -\frac{\sin x}{\cos x} dx$$

$$\int \cot y dy = -\int \tan x dx$$

$$\log \sin y = \log \cos x + \log c$$

$$\sin y = c \cos x$$

Differential Equations Ex 22.7 Q26

$$\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$$

$$\frac{dy}{dx} = -\cos x \tan y$$

$$\frac{dy}{\tan y} = -\cos x dx$$

$$\int \cot y dy = -\int \cos x dx$$

$$\log|\sin y| = -\sin x + c$$

$$\sin x + \log|\sin y| = c$$

Differential Equations Ex 22.7 Q27

$$x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$$

$$x\sqrt{1-y^2}dx = -y\sqrt{1-x^2}dy$$

$$\frac{ydy}{\sqrt{1-y^2}} = -\frac{xdx}{\sqrt{1-x^2}}$$

$$\frac{1}{-2}\int \frac{-2y}{\sqrt{1-y^2}}dy = \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx$$

$$-\frac{1}{2}2\times\sqrt{1-y^2} = \frac{1}{2}\times 2\sqrt{1-x^2} + c_1$$

$$\sqrt{1-y^2} + \sqrt{1-x^2} = c$$

Differential Equations Ex 22.7 Q28

$$y\left(1+e^x\right)dy = (y+1)e^x dx$$

$$\frac{ydy}{y+1} = \frac{e^x dx}{1+e^x}$$

$$\int\left(1-\frac{1}{y+1}\right)dy = \int\left(\frac{e^x}{1+e^x}\right)dx$$

$$y - \log|y+1| = \log\left|1+e^x\right| + c$$

Differential Equations Ex 22.7 Q29

$$(y + xy)dx + (x - xy^2)dy = 0$$
$$y(1+x)dx = (xy^2 - x)dy$$
$$y(1+x)dx = x(y^2 - 1)dy$$
$$\frac{(y^2 - 1)dy}{y} = \frac{1+x}{x}dx$$
$$\int\left(y - \frac{1}{y}\right)dy = \int\left(\frac{1}{x} + 1\right)dx$$
$$\frac{y^2}{2} - \log|y| = \log|x| + x + c_1$$
$$\frac{y^2}{2} - x - \log|y| - \log|x| = c_1$$
$$\log|x| + x + \log|y| - \frac{y^2}{2} = c$$

Differential Equations Ex 22.7 Q30

$$\frac{dy}{dx} = 1 - x + y - xy$$
$$= (1 - x) + y(1 - x)$$
$$\frac{dy}{dx} = (1 - x)(1 + y)$$
$$\int\frac{dy}{1+y} = \int(1 - x)dx$$
$$\log|y + 1| = x - \frac{x^2}{2} + c$$

Differential Equations Ex 22.7 Q31

$$(y^2 + 1)dx - (x^2 + 1)dy = 0$$
$$(y^2 + 1)dx = (x^2 + 1)dy$$
$$\int\frac{dy}{y^2 + 1} = \int\frac{dx}{x^2 + 1}$$
$$\tan^{-1}y = \tan^{-1}x + c$$

Differential Equations Ex 22.7 Q32

$$dy + (x + 1)(y + 1)dx = 0$$
$$dy = -(x + 1)(y + 1)dx$$
$$\int\frac{dy}{y + 1} = -\int(x + 1)dx$$
$$\log|y + 1| = -\frac{x^2}{2} - x + c$$
$$\log|y + 1| + \frac{x^2}{2} + x = c$$

Differential Equations Ex 22.7 Q33

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$
$$\int\frac{dy}{1 + y^2} = \int(1 + x^2)dx$$
$$\tan^{-1}y = x + \frac{x^3}{3} + c$$
$$\tan^{-1}y - x - \frac{x^3}{3} = c$$

Differential Equations Ex 22.7 Q34

$$(x-1)\frac{dy}{dx} = 2x^3y$$

$$\frac{dy}{y} = \frac{2x^3dx}{x-1}$$

$$\int\frac{dy}{y} = 2\int\left(x^2 + x + 1 + \frac{1}{x-1}\right)dx$$

$$\log|y| = \log e^{\left(\frac{2}{3}x^3+x^2+2x\right)} + \log|x-1|^2 + \log|c|$$

$$y = c|x-1|^2 e^{\left(\frac{2}{3}x^3+x^2+2x\right)}$$

Differential Equations Ex 22.7 Q35

$$\frac{dy}{dx} = e^{x+y} + e^{-x+y}$$

$$= e^x \times e^y + e^{-x} \times e^y$$

$$\frac{dy}{dx} = e^y\left(e^x + e^{-x}\right)$$

$$\frac{dy}{e^y} = \left(e^x + e^{-x}\right)dx$$

$$\int e^{-y}dy = \int\left(e^x + e^{-x}\right)dx$$

$$-e^{-y} = e^x - e^{-x} + c$$

$$e^{-x} - e^{-y} = e^x + c$$

Differential Equations Ex 22.7 Q36

$$\frac{dy}{dx} = \left(\cos^2 x - \sin^2 x\right)\cos^2 y$$

$$\frac{dy}{\cos^2 y} = \left(\cos^2 x - \sin^2 x\right)dx$$

$$\int\sec^2 y\,dy = \int\cos 2x\,dx$$

$$\tan y = \frac{\sin 2x}{2} + c$$

Differential Equations Ex 22.7 Q37(i)

$$\left(xy^2 + 2x\right)dx + \left(x^2y + 2y\right)dy = 0$$

$$\left(x^2y + 2y\right)dy = -\left(xy^2 + 2x\right)dx$$

$$y\left(x^2 + 2\right)dy = -x\left(y^2 + 2\right)dx$$

$$\frac{y}{y^2+2}dy = -\frac{x}{x^2+2}dx$$

$$\int\frac{2y}{y^2+2}dy = -\int\frac{2x}{x^2+2}dx$$

$$\log\left|y^2+2\right| = -\log\left|x^2+2\right| + \log|c|$$

$$\left|y^2+2\right| = \left|\frac{c}{x^2+2}\right|$$

$$y^2 + 2 = \frac{c}{x^2+2}$$

Differential Equations Ex 22.7 Q37(ii)

Consider the given equation

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2y^2 = 0$$

$$\Rightarrow \frac{\log y dy}{y^2} = \frac{-x^2 dx}{\operatorname{cosec} x}$$

$$\Rightarrow -\frac{\log y dy}{y^2} = x^2 \sin x dx$$

Integrating on both the sides,

$$\Rightarrow -\int \frac{\log y dy}{y^2} = \int x^2 \sin x dx$$

Using integration by parts on both sides

$$\Rightarrow \frac{\log y + 1}{y} = -x^2 \cos x + 2(x \sin x + \cos x) + C$$

$$\Rightarrow \frac{\log y + 1}{y} + x^2 \cos x - 2(x \sin x + \cos x) = C$$

Differential Equations Ex 22.7 Q38(i)

$$xy\frac{dy}{dx} = 1 + x + y + xy$$

$$= (1 + x) + y(1 + x)$$

$$xy\frac{dy}{dx} = (1 + x)(1 + y)$$

$$\int \frac{ydy}{y+1} = \int \frac{1+x}{x} dx$$

$$\int \left(1 - \frac{1}{y+1}\right) dy = \int \left(\frac{1}{x} + 1\right) dx$$

$$y - \log|y + 1| = \log|x| + x + \log|c|$$

$$y = \log|cx(y+1)| + x$$

Differential Equations Ex 22.7 Q38(ii)

$$y(1 - x^2)\frac{dy}{dx} = x(1 + y^2)$$

$$\frac{ydy}{(1+y^2)} = \frac{xdx}{1-x^2}$$

$$-\int \frac{2ydy}{(1+y^2)} = \int \frac{-2x}{(1-x^2)} dx$$

$$-\log|1+y^2| = \log|1-x^2| + \log|c_1|$$

$$\log|c| = \log|1-x^2| + \log|1+y^2|$$

$$c = (1-x^2)(1+y^2)$$

Differential Equations Ex 22.7 Q38(iii)

$$ye^{x/y}\,dx = (xe^{x/y} + y^2)dy$$

$$ye^{x/y}\,dx - xe^{x/y}dy = y^2dy$$

$$(ydx - xdy)e^{x/y} = y^2dy$$

$$\left(\frac{ydx - xdy}{y^2}\right)e^{x/y} = dy$$

$$e^{x/y}\,d\left(\frac{x}{y}\right) = dy$$

Integrating on both the sides we get,

$e^{x/y} = y + C$, which is the required solution.

Differential Equations Ex 22.7 Q38(iv)

$$(1 + y^2)\tan^{-1}x\,dx + 2y(1 + x^2)dy = 0$$

$$(1 + y^2)\tan^{-1}x\,dx = -2y(1 + x^2)dy$$

$$-\frac{\tan^{-1}x}{2(1 + x^2)}dx = \frac{y}{(1 + y^2)}dy$$

Integrating on both the sides

$$\int -\frac{\tan^{-1}x}{2(1 + x^2)}dx = \int \frac{y}{(1 + y^2)}dy$$

$$-\left(\tan^{-1}x\left(\frac{1}{2}\tan^{-1}x\right) - \int \frac{1}{(1 + x^2)}\left(\frac{1}{2}\tan^{-1}x\right)dx\right) = \frac{1}{2}\ln(y^2 + 1) + C$$

$$-\frac{1}{4}(\tan^{-1}x)^2 = \frac{1}{2}\ln(y^2 + 1) + C_1$$

$$\frac{1}{2}(\tan^{-1}x)^2 + \ln(y^2 + 1) = C$$

Differential Equations Ex 22.7 Q39

$$\frac{dy}{dx} = y\tan 2x,\ y(0) = 2$$

$$\int \frac{dy}{y} = \int \tan 2x\,dx$$

$$\log|y| = \frac{1}{2}\log|\sec 2x| + \log c$$

$$y = \sqrt{\sec 2x}\,c \qquad \text{---(i)}$$

Put $x = 0, y = 2$

$$2 = \sqrt{\sec 0} \times c$$

$$2 = c$$

Put $c = 2$ in equation (i),

$$y = 2\sqrt{\sec 2x}$$

$$y = \frac{2}{\sqrt{\cos 2x}}$$

Differential Equations Ex 22.7 Q40

$$2x\frac{dy}{dx} = 3y,\ y(1) = 2$$

$$\int \frac{2dy}{y} = \int \frac{3dx}{x}$$

$$2\log|y| = 3\log|x| + \log c$$

$$y^2 = x^3c \qquad \text{---(i)}$$

Put $x = 1,\ y = 2$

$$4 = c$$

Put $c = 4$ in equation (i),

$$y^2 = 4x^3$$

Differential Equations Ex 22.7 Q41

$$xy\frac{dy}{dx} = y + 2,\ y(2) = 0$$

$$\frac{ydy}{y+2} = \frac{dx}{x}$$

$$\int\left(1 - \frac{2}{y+2}\right)dy = \int\frac{dx}{x}$$

$$y - 2\log|y+2| = \log|x| + \log|c| \qquad \text{---(i)}$$

Put $y = 0,\ x = 2$

$$0 - 2\log 2 = \log 2 + \log c$$

$$-3\log 2 = \log c$$

$$\log\left(\frac{1}{8}\right) = \log c$$

$$c = \frac{1}{8}$$

Put $c = \frac{1}{8}$ in equation (i),

$$y - 2\log|y+2| = \log\left|\frac{x}{8}\right|$$

Differential Equations Ex 22.7 Q42

$$\frac{dy}{dx} = 2e^x y^3,\ y(0) = \frac{1}{2}$$

$$\int\frac{dy}{y^3} = \int 2e^x dx$$

$$-\frac{1}{2y^2} = 2e^x + c \qquad \text{---(i)}$$

Put $x = 0,\ y = \frac{1}{2}$

$$-\frac{4}{2} = 2e^0 + c$$

$$-2 = 2 + c$$

$$c = -4$$

Put $c = -4$ in equation (i),

$$-\frac{1}{2y^2} = 2e^x - 4$$

$$-1 = 4e^x y^2 - 8y^2$$

$$-1 = -y^2\left(8 - 4e^x\right)$$

$$y^2\left(8 - 4e^x\right) = 1$$

Differential Equations Ex 22.7 Q43

$$\frac{dr}{dt} = -rt,\ r(0) = r_0$$

$$\int \frac{dr}{r} = -\int t dt$$

$$\log|r| = -\frac{t^2}{2} + c \qquad ---(i)$$

Put $t = 0,\ r = r_0$ inequation (i),

$$\log|r_0| = 0 + c$$

$$\log|r_0| = c$$

Now,

$$\log|r| = -\frac{t^2}{2} + \log|r_0|$$

$$\frac{r}{r_0} = e^{-\frac{t^2}{2}}$$

$$r = r_0 e^{-\frac{t^2}{2}}$$

Differential Equations Ex 22.7 Q44

$$\frac{dy}{dx} = y \sin 2x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \sin 2x dx$$

$$\log|y| = -\frac{\cos 2x}{2} + c \qquad ---(i)$$

Put $y = 1,\ x = 0$

$$\log|1| = -\frac{\cos 0}{2} + c$$

$$0 = -\frac{1}{2} + c$$

$$c = \frac{1}{2}$$

So,

$$\log|y| = -\frac{\cos 2x}{2} + \frac{1}{2}$$

$$= \frac{1 - \cos 2x}{2}$$

$$\log|y| = \sin^2 x$$

$$y = e^{\sin^2 x}$$

Differential Equations Ex 22.7 Q45(i)

$$\frac{dy}{dx} = y \tan x,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int \tan x dx$$

$$\log|y| = \log|\sec x| + c \qquad ---(i)$$

Put $y = 1,\ x = 0$

$$0 = \log(1) + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = \log|\sec x|$$

$$y = \sec x \qquad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Differential Equations Ex 22.7 Q45(ii)

$$2x\frac{dy}{dx} = 5y,\ y(1) = 1$$

$$\int \frac{2dy}{y} = \int \frac{5dx}{x}$$

$$2\log|y| = 5\log|x| + c \qquad ---(i)$$

Put $x = 1,\ y = 1$

$$2\log(1) = 5\log(1) + c$$

$$0 = c$$

Put $c = 0$ in equation (i),

$$2\log|y| = 5\log|x|$$

$$y^2 = |x|^5$$

$$y = |x|^{\frac{5}{2}}$$

Differential Equations Ex 22.7 Q45(iii)

$$\frac{dy}{dx} = 2e^{2x}y^2,\ y(0) = -1$$

$$\int \frac{dy}{y^2} = \int 2e^{2x}dx$$

$$-\frac{1}{y} = \frac{2e^{2x}}{2} + c$$

$$-\frac{1}{y} = e^{2x} + c \qquad \text{---(i)}$$

Put $y = -1,\ x = 0$

$$1 = e^0 + c$$

$$1 = 1 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$-\frac{1}{y} = e^{2x}$$

$$y = -e^{-2x}$$

Differential Equations Ex 22.7 Q45(iv)

$$\cos y\frac{dy}{dx} = e^x,\ y(0) = \frac{\pi}{2}$$

$$\int \cos y dy = \int e^x dx$$

$$\sin y = e^x + c \qquad \text{---(i)}$$

Put $x = 0,\ y = \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = e^0 + c$$

$$1 = 1 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$\sin y = e^x$$

$$y = \sin^{-1}\left(e^x\right)$$

Differential Equations Ex 22.7 Q45(v)

$$\frac{dy}{dx} = 2xy,\ y(0) = 1$$

$$\int \frac{dy}{y} = \int 2x dx$$

$$\log|y| = 2\frac{x^2}{2} + c$$

$$\log|y| = x^2 + c \qquad ---(i)$$

Put $x = 0,\ y = 1$

$$\log(1) = 0 + c$$

$$0 = 0 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$\log y = x^2$$

$$y = e^{x^2}$$

Differential Equations Ex 22.7 Q45(vi)

$$\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2, y(0) = 1$$

$$= (1 + x^2)(1 + y^2)$$

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2) dx$$

$$\tan^{-1} y = x + \frac{x^3}{3} + c \qquad ---(i)$$

Put $x = 0, y = 1$

$$\tan^{-1} y = x + \frac{x^3}{3} + c$$

$$c = \frac{\pi}{4}$$

Put $c = \frac{\pi}{4}$ in equation (i)

$$\tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$$

Differential Equations Ex 22.7 Q45(vii)

$$xy\frac{dy}{dx}=(x+2)(y+2),\ y(1)=-1$$

$$\frac{ydy}{(y+2)}=\frac{(x+2)}{x}dx$$

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$

$$y-x-2\log(y+2)-2\log x=c$$

Put $x=1,\ y=-1$

$$-1-1-2\log(-1+2)-2\log 1=c$$

$$\Rightarrow -2=c$$

Thus, we have

$$y-x-2\log(y+2)-2\log x=-2$$

Differential Equations Ex 22.7 Q45(viii)

$$\frac{dy}{dx}=1+x+y^2+xy^2$$

$$\frac{dy}{dx}=(1+x)(1+y^2)$$

$$\frac{1}{(1+y^2)}dy=(1+x)dx$$

Integrating on both the sides we get

$$\int\frac{1}{(1+y^2)}dy=\int(1+x)dx$$

$$\tan^{-1}y=x+\frac{x^2}{2}+C\ \ldots(i)$$

Put $y=0$, $x=0$ then

$$\tan^{-1}0=0+0+C$$

$$C=0$$

From(i) we have

$$\tan^{-1}y=x+\frac{x^2}{2}$$

$$y=\tan\left(x+\frac{x^2}{2}\right)$$

Differential Equations Ex 22.7 Q45(ix)

$$2(y+3) - xy\frac{dy}{dx} = 0$$

$$2(y+3) = xy\frac{dy}{dx}$$

$$\frac{2}{x}dx = \frac{y}{y+3}dy$$

Integrating on both the sides we get

$$\int\frac{2}{x}dx = \int\frac{y}{y+3}dy$$

$$2\ln|x| = y + 3 - 3\ln|y+3| + C.....(i)$$

Put x = 1 and y = -2 in eq (i)

$$2\ln|1| = -2 + 3 - 3\ln|-2+3| + C$$

$$0 = 1 - 0 + C$$

$$C = -1$$

From eq (i) we have

$$2\ln|x| = y + 3 - 3\ln|y+3| - 1$$

$$\ln(|x|)^2 = y + 2 - \ln(|y+3|)^3$$

$$\ln(|x|)^2 + -\ln(|y+3|)^3 = y + 2$$

$$x^2(y+3)^3 = e^{y+2}$$

Differential Equations Ex 22.7 Q46

$$x\frac{dy}{dx} + \cot y = 0,\ y = \frac{\pi}{4} \text{ at } x = \sqrt{2}$$

$$x\frac{dy}{dx} = -\cot y$$

$$\frac{dy}{\cot y} = -\frac{dx}{x}$$

$$\int \tan y\,dy = -\int\frac{dx}{x} + c$$

$$\log|\sec y| = -\log|x| + c \qquad \text{---(i)}$$

Put $x = \sqrt{2},\ y = \frac{\pi}{4}$

$$\log\left|\sec\frac{\pi}{4}\right| = -\log\left|\sqrt{2}\right| + c$$

$$\log\left|\sqrt{2}\right| = -\frac{1}{2}\log 2 + c$$

$$\frac{1}{2}\log 2 = -\frac{1}{2}\log 2 + c$$

$$\log 2 = c$$

Put c in equation (i),

$$\log|\sec y| = -\log|x| + \log 2$$

$$\sec y = \frac{2}{x}$$

$$x = \frac{2}{\sec y}$$

$$x = 2\cos y$$

Differential Equations Ex 22.7 Q47

$$\left(1+x^2\right)\frac{dy}{dx}+\left(1+y^2\right)=0,\ y=1 \text{ at } x=0$$

$$\left(1+x^2\right)\frac{dy}{dx}=-\left(1+y^2\right)$$

$$\int\frac{dy}{\left(1+y^2\right)}=-\int\frac{dx}{1+x^2}$$

$$\tan^{-1}y=-\tan^{-1}x+c \qquad \text{---(i)}$$

Put $x=0,\ y=1$

$$\tan^{-1}(1)=-\tan^{-1}0+c$$

Put c in equation (1),

$$\tan^{-1}y=-\tan^{-1}x+\frac{\pi}{4}$$

$$\tan^{-1}y=\left(\frac{\pi}{4}-\tan^{-1}x\right)$$

$$y=\tan\left(\frac{\pi}{4}-\tan^{-1}x\right)$$

$$y=\frac{\tan\frac{\pi}{4}-\tan\left(\tan^{-1}x\right)}{1+\tan\frac{\pi}{4}\tan\left(\tan^{-1}x\right)}$$

$$y=\frac{1-x}{1+x}$$

$$y+xy=1-x$$

$$x+y=1-xy$$

Differential Equations Ex 22.7 Q48

$$\frac{dy}{dx}=\frac{2x\,(\log x+1)}{\sin y+y\cos y},\ y=0 \text{ at } x=1$$

$$\int(\sin y+y\cos y)dy=\int 2x\,(\log x+1)dx$$

$$\Rightarrow \quad \int\sin y\,dy+\int y\cos y\,dy=\int 2x\log x\,dx+2\int x\,dx$$

$$\Rightarrow \quad -\cos y+\left[y\times\int\cos y\,dy-\int\left(1\times\int\cos y\,dy\right)dy\right]=2\left[\log x\int x\,dx-\int\left(\frac{1}{x}\int x\,dx\right)dx\right]+x^2+c$$

$$\Rightarrow \quad -\cos y+y\sin y-\int\sin y\,dy=2\frac{x^2}{2}\log x-2\int\frac{x}{2}dx+x^2+c$$

$$\Rightarrow \quad -\cos y+y\sin y+\cos y=x^2\log x-\frac{x^2}{2}+x^2+c$$

$$y\sin y=x^2\log x+\frac{x^2}{2}+c \qquad \text{---(i)}$$

Put $y=0,\ x=1$

$$0=0+\frac{1}{2}+c$$

$$c=-\frac{1}{2}$$

Put $c=-\frac{1}{2}$ in equation (i),

$$y\sin y=x^2\log x+\frac{x^2}{2}-\frac{1}{2}$$

$$2y\sin y=2x^2\log x+x^2-1$$

Differential Equations Ex 22.7 Q49

$$e^{\frac{dy}{dx}} = x + 1$$

$$\frac{dy}{dx} = \log(x+1),\ y = 3 \text{ at } x = 0$$

$$\int dy = \int \log(x+1)\,dx$$

$$y = \log|x+1| \times \int 1 \times dx - \int\left(\frac{1}{x+1} \times \int 1dx\right)dx + c$$

Using integration by parts

$$y = x\log|x+1| - \int \frac{x}{x+1}dx + c$$

$$y = x\log|x+1| - \left(\int\left(1 - \frac{1}{x+1}\right)dx\right) + c$$

$$= x\log|x+1| - (x - \log|x+1|) + c$$

$$y = x\log|x+1| - x + \log|x+1| + c$$

$$y = (x+1)\log|x+1| - x + c \qquad \text{---(i)}$$

Put $y = 3$ and $x = 0$

$$3 = 0 - 0 + c$$

$$c = 3$$

Put $c = 3$ in equation (i),

$$y = (x+1)\log|x+1| - x + 3$$

Differential Equations Ex 22.7 Q50

$$\cos y dy + \cos x \sin y dx = 0$$
$$\cos y dy = -\cos x \sin y dx$$
$$\frac{\cos y}{\sin y} dy = -\cos x dx$$
$$\int \cot y dy = -\int \cos x dx$$
$$\log|\sin y| = -\sin x + c \quad \text{---(i)}$$

Put $y = \frac{\pi}{2}$ and $x = \frac{\pi}{2}$

$$\log\left|\sin\frac{\pi}{2}\right| = -\sin\frac{\pi}{2} + c$$
$$0 = -1 + c$$
$$c = 1$$

Put $c = 1$ in equation (1),

$$\log|\sin y| = 1 - \sin x$$
$$\log|\sin y| + \sin x = 1$$

Differential Equations Ex 22.7 Q51

$$\frac{dy}{dx} = -4xy^2, \; y = 1 \text{ when } x = 0$$
$$\int \frac{dy}{y^2} = -4\int x dx$$
$$-\frac{1}{y} = -4\frac{x^2}{2} + c \quad \text{---(i)}$$

Put $y = 1$ and $x = 0$

$$-1 = 0 + c$$
$$c = -1$$

Plut $c = -1$ in equation (i),

$$-\frac{1}{y} = -2x^2 - 1$$
$$\frac{1}{y} = 2x^2 + 1$$
$$y = \frac{1}{2x^2 + 1}$$

Differential Equations Ex 22.7 Q52

The differential equation of the curve is:

$$y' = e^x \sin x$$
$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$
$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x \, dx \qquad ...(1)$$

Let $I = \int e^x \sin x \, dx$.

$$\Rightarrow I = \sin x \int e^x dx - \int\left(\frac{d}{dx}(\sin x) \cdot \int e^x dx\right) dx$$

$$\Rightarrow I = \sin x \cdot e^x - \int \cos x \cdot e^x dx$$
$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot \int e^x dx - \int\left(\frac{d}{dx}(\cos x) \cdot \int e^x dx\right) dx\right]$$
$$\Rightarrow I = \sin x \cdot e^x - \left[\cos x \cdot e^x - \int(-\sin x) \cdot e^x dx\right]$$
$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$
$$\Rightarrow 2I = e^x(\sin x - \cos x)$$
$$\Rightarrow I = \frac{e^x(\sin x - \cos x)}{2}$$

Differential Equations Ex 22.7 Q53

The differential equation of the given curve is:

$$xy\frac{dy}{dx} = (x+2)(y+2)$$
$$\Rightarrow \left(\frac{y}{y+2}\right) dy = \left(\frac{x+2}{x}\right) dx$$
$$\Rightarrow \left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$$

Integrating both sides, we get:

$$\int\left(1 - \frac{2}{y+2}\right) dy = \int\left(1 + \frac{2}{x}\right) dx$$
$$\Rightarrow \int dy - 2\int \frac{1}{y+2} dy = \int dx + 2\int \frac{1}{x} dx$$
$$\Rightarrow y - 2\log(y+2) = x + 2\log x + C$$
$$\Rightarrow y - x - C = \log x^2 + \log(y+2)^2$$
$$\Rightarrow y - x - C = \log\left[x^2(y+2)^2\right] \qquad ...(1)$$

Differential Equations Ex 22.7 Q54

Let the rate of change of the volume of the balloon be k (where k is a constant)

$$\Rightarrow \frac{dv}{dt} = k$$

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k \qquad \left[\text{Volume of sphere} = \frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi r^2\, dr = k\, dt$$

Integrating both sides, we get:

$$4\pi \int r^2 dr = k \int dt$$

$$\Rightarrow 4\pi \cdot \frac{r^3}{3} = kt + C$$

$$\Rightarrow 4\pi r^3 = 3(kt + C) \qquad ...(1)$$

Now, at $t = 0$, $r = 3$:

$$4\pi \times 3^3 = 3\ (k \times 0 + C)$$

$$108\pi = 3C$$

$$C = 36\pi$$

At $t = 3$, $r = 6$:

$$4\pi \times 6^3 = 3\ (k \times 3 + C)$$

$$864\pi = 3\ (3k + 36\pi)$$

$$3k = -288\pi - 36\pi = 252\pi$$

$$k = 84\pi$$

Substituting the values of k and C in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$

$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after t seconds is $(63t + 27)^{\frac{1}{3}}$.

Differential Equations Ex 22.7 Q55

Let p, t, and r represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of $r\%$ per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100}\int dt$$

$$\Rightarrow \log p = \frac{rt}{100} + k$$

$$\Rightarrow p = e^{\frac{rt}{100}+k} \quad \text{...(1)}$$

It is given that when $t = 0$, $p = 100$.

$\Rightarrow 100 = e^k$ ... (2)

Now, if $t = 10$, then $p = 2 \times 100 = 200$.

$$200 = e^{\frac{r}{10}+k}$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot e^k$$

$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot 100 \quad \text{(From (2))}$$

$$\Rightarrow e^{\frac{r}{10}} = 2$$

$$\Rightarrow \frac{r}{10} = \log_e 2$$

$$\Rightarrow \frac{r}{10} = 0.6931$$

$$\Rightarrow r = 6.931$$

Hence, the value of r is 6.93%.

Differential Equations Ex 22.7 Q56

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$$

$$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int\frac{dp}{p} = \frac{1}{20}\int dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20}+C} \qquad ...(1)$$

Now, when $t = 0$, $p = 1000$.

$1000 = e^C$ ... (2)

Differential Equations Ex 22.7 Q57

Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$

$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$

$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int\frac{dy}{y} = k\int dt$$

$$\Rightarrow \log y = kt + C \qquad ...(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$$\log y_0 = C$$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$
$$\Rightarrow \log y - \log y_0 = kt$$
$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$
$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad ...(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y_0$$
$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \qquad ...(3)$$

Substituting this value in equation (2), we get:

$$k \cdot 2 = \log\left(\frac{11}{10}\right)$$
$$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2}\log\left(\frac{11}{10}\right) \cdot t = \log\left(\frac{y}{y_0}\right)$$
$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad ...(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1.

$y = 2y_0$ at $t = t_1$

From equation (4), we get:

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Differential Equations Ex 22.7 Q58

Consider the given equation

$$\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$$

$$\Rightarrow \frac{dy}{(1+y)} = \frac{-\cos x dx}{(2+\sin x)}$$

Integrating both the sides,

$$\Rightarrow \int\frac{dy}{(1+y)} = \int\frac{-\cos x dx}{(2+\sin x)}$$

$$\Rightarrow \log(1+y) = -\log(2+\sin x) + \log C$$

$$\Rightarrow \log(1+y) + \log(2+\sin x) = \log C$$

$$\Rightarrow \log(1+y)(2+\sin x) = \log C$$

$$\Rightarrow (1+y)(2+\sin x) = C \ldots(1)$$

Given that $y(0) = 1$

$$\Rightarrow (1+1)(2+\sin 0) = C$$

$$\Rightarrow C = 4$$

Substituting the value of C in equation (1), we have,

$$\Rightarrow (1+y)(2+\sin x) = 4$$

$$\Rightarrow (1+y) = \frac{4}{(2+\sin x)}$$

$$\Rightarrow y = \frac{4}{(2+\sin x)} - 1 \ldots(2)$$

We need to find the value of $y\left(\frac{\pi}{2}\right)$

Substituting the value of $x=\frac{\pi}{2}$ in equation (2), we get,

$$y = \frac{4}{\left(2+\sin\frac{\pi}{2}\right)} - 1$$

$$\Rightarrow y = \frac{4}{(2+1)} - 1$$

$$\Rightarrow y = \frac{4}{3} - 1$$

$$\Rightarrow y = \frac{1}{3}$$

**Note* : Answer given in the book is incorrect.