RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.9

Differential Equations Ex 22.9 Q1

Here, $x^2 dy + y(x+y)dx = 0$

$$\frac{dy}{dx} = -\frac{y(x+y)}{x^2}$$

It is homogeneous equation

Put $y = vx$

and,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = -\frac{vx(x+vx)}{x^2}$$

$$v + x\frac{dv}{dx} = -v - v^2$$

$$x\frac{dv}{dx} = -2v - v^2$$

$$\int \frac{1}{v^2+2v}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{v^2+2v+1-1}dv = -\int \frac{dx}{x}$$

$$\int \frac{1}{(v+1)^2-(1)^2}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\log\left|\frac{v+1-1}{v+1+1}\right| = -\log|x| + \log|c|$$

$$\log\left|\frac{v}{v+2}\right|^{\frac{1}{2}} = -\log\left|\frac{c}{x}\right|$$

$$\frac{v}{v+2} = \frac{c^2}{x^2}$$

$$\frac{\frac{y}{x}}{\frac{y}{x}+2} = \frac{c^2}{x^2}$$

$$\frac{y}{y+2x} = \frac{c^2}{x^2}$$

$$yx^2 = (y+2x)c^2$$

Differential Equations Ex 22.9 Q2

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

It is homogeneous equation

Put $\quad y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x}{vx + x}$$

$$v + x\frac{dv}{dx} = \frac{v - 1}{v + 1}$$

$$x\frac{dv}{dx} = \frac{v - 1}{v + 1} - v$$

$$= \frac{v - 1 - v^2 - v}{v + 1}$$

$$x\frac{dv}{dx} = -\frac{\left(1 + v^2\right)}{v + 1}$$

$$\int \frac{v + 1}{v^2 + 1}dv = -\int \frac{dx}{x}$$

$$\int \frac{v}{v^2 + 1}dv + \int \frac{1}{v^2 + 1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v}{v^2 + 1}dv + \int \frac{1}{v^2 + 1}dv = -\int \frac{dx}{x}$$

$$\frac{1}{2}\text{og}\left|v^2 + 1\right| + \tan^{-1} v = -\log|x| + \log|c|$$

$$\log\left|\frac{y^2 + x^2}{x^2}\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log\left|x^2 + y^2\right| - 2\log|x| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log\left|\frac{c}{x}\right|$$

$$\log\left|x^2 + y^2\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = 2\log(c)$$

$$\log\left|x^2 + y^2\right| + 2\tan^{-1}\left(\frac{y}{x}\right) = k$$

Differential Equations Ex 22.9 Q3

Here, $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$.

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - \frac{v}{1}$$

$$= \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\int \frac{2v}{1 + v^2}dv = -\int \frac{dx}{x}$$

$$\log\left|1 + v^2\right| = -\log|x| + \log|c|$$

$$1 + v^2 = \frac{c}{x}$$

$$1 + \frac{y^2}{x^2} = \frac{c}{x}$$

$$x^2 + y^2 = cx$$

Differential Equations Ex 22.9 Q4

Here, $\frac{xdy}{dx} = x + y,\ x \neq 0$

$$\frac{dy}{dx} = \frac{x + y}{x}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{x + vx}{x}$$

$$v + x\frac{dv}{dx} = 1 + v$$

$$\int dv = \int \frac{dx}{x}$$

$$v = \log|x| + c$$

$$\frac{y}{x} = \log|x| + c$$

$$y = x\log|x| + cx$$

Differential Equations Ex 22.9 Q5

Here, $\left(x^2 - y^2\right)dx - 2xydy = 0$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2}{2v}$$

$$\int \frac{2v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\frac{1}{-3}\int \frac{-6v}{1 - 3v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{-6v}{1 - 3v^2} = -3\int \frac{dx}{x}$$

$$\log\left|1 - 3v^2\right| = -3\log|x| + \log|c|$$

$$1 - 3v^2 = \frac{c}{x^3}$$

$$x^3\left(1 - \frac{3y^2}{x^2}\right) = c$$

$$\frac{x^3\left(x^2 - 3y^2\right)}{x^2} = c$$

$$x\left(x^2 - 3y^2\right) = c$$

Differential Equations Ex 22.9 Q6

$$\frac{dy}{dx} = \frac{x + y}{x - y}$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{1 + v}{1 - v}$$

$$x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{1 - v}$$

$$\frac{1 - v}{1 + v^2}dv = \frac{dx}{x}$$

$$\int \frac{1 - v}{1 + v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{1}{1 + v^2}dv - \frac{1}{2}\int \frac{2v}{1 + v^2}dv = \int \frac{dx}{x}$$

$$\tan^{-1} v - \frac{1}{2}\log\left(1 + v^2\right) = \log x + c$$

$$\tan^{-1}\frac{y}{x} = \frac{1}{2}\log\left(x^2 + y^2\right) + c$$

Differential Equations Ex 22.9 Q7

Here, $2xy\frac{dy}{dx} = x^2 + y^2$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

It is a homogeneous equation

Put $y = vx$

and, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\frac{2v}{1-v^2}dv = \frac{dx}{x}$$

$$\int\frac{-2v}{1-v2}dv = -\int\frac{dx}{x}$$

$$\log\left|1-v^2\right| = -\log\left|x\right| + \log c$$

$$\left(1-v^2\right) = \frac{c}{x}$$

$$x\left(1-\frac{y^2}{x^2}\right) = c$$

$$\frac{x\left(x^2-y^2\right)}{x^2} = c$$

$$x^2 - y^2 = cx$$

Differential Equations Ex 22.9 Q8

Consider the given differential equation

$$x^2\frac{dy}{dx}=x^2-2y^2+xy$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2-2y^2+xy}{x^2}$$

This is a homogeneous differential equation.

Substituting $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we have

$$v+x\frac{dv}{dx}=\frac{x^2-2v^2\times x^2+x\times v\times x}{x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=1-2v^2+v$$

$$\Rightarrow x\frac{dv}{dx}=1-2v^2$$

$$\Rightarrow \frac{dv}{1-2v^2}=\frac{dx}{x}$$

$$\Rightarrow \frac{dv}{v^2-\frac{1}{2}}=-2\frac{dx}{x}$$

$$\Rightarrow \int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow \int\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2-v^2}=2\int\frac{dx}{x}$$

$$\Rightarrow \frac{\sqrt{2}}{2}\log\left(\frac{\frac{1}{\sqrt{2}}+v}{\frac{1}{\sqrt{2}}-v}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{\frac{1}{\sqrt{2}}+\frac{y}{x}}{\frac{1}{\sqrt{2}}-\frac{y}{x}}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=2\log x+\log C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\log x^2+\log C$$

$$\Rightarrow \log\left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=\log Cx^2$$

$$\Rightarrow \left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)^{\frac{1}{\sqrt{2}}}=Cx^2$$

$$\Rightarrow \left(\frac{x+y\sqrt{2}}{x-y\sqrt{2}}\right)=\left(Cx^2\right)^{\sqrt{2}}$$

Differential Equations Ex 22.9 Q9

Here, $xy\frac{dy}{dx} = x^2 - y^2$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{xvx}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{v} - v$$

$$x\frac{dv}{dx} = \frac{1 - v^2 - v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1 - 2v^2}{v}$$

$$\frac{v}{1 - 2v^2}dv = \frac{dx}{x}$$

$$\int \frac{-4v}{1 - 2v^2}dv = -4\int \frac{dx}{x}$$

$$\log\left|1 - 2v^2\right| = -4\log|x| + \log c$$

$$\left(1 - 2\frac{y^2}{x^2}\right) = \frac{c}{x^4}$$

$$\left(\frac{x^2 - 2y^2}{x^2}\right) = \frac{c}{x^4}$$

$$x^2\left(x^2 - 2y^2\right) = c$$

Differential Equations Ex 22.9 Q10

Here, $ye^{\frac{x}{y}}dx = \left(xe^{\frac{x}{y}} + y\right)dy$

$$\frac{dx}{dy} = \frac{xe^{\frac{x}{y}} + y}{ye^{\frac{x}{y}}}$$

It is a homogeneous equation

Put $x = vy$

and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{vye^{\frac{vy}{y}} + y}{ye^{\frac{vy}{y}}}$$

$$v + y\frac{dv}{dy} = \frac{ve^{v} + 1}{e^{v}}$$

$$y\frac{dv}{dy} = \frac{ve^{v} + 1}{e^{v}} - v$$

$$y\frac{dv}{dy} = \frac{ve^{v} + 1 - ve^{v}}{e^{v}}$$

$$y\frac{dv}{dy} = \frac{1}{e^{v}}$$

$$\int evdv = \int\frac{dy}{y}$$

$$e^{v} = \log|y| + c$$

$$e^{\frac{x}{y}} = \log y + c$$

Differential Equations Ex 22.9 Q11

Here, $x^2\frac{dy}{dx} = x^2 + xy + y^2$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

It is a homogeneous equaton

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + xvx + v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = 1 + v + v^2 - v^2$$

$$x\frac{dv}{dx} = 1 + v^2$$

$$\int\frac{dv}{1 + v^2} = \int\frac{dx}{x}$$

$$\tan^{-1}v = \log|x| + c$$

$$\tan^{-1}\frac{y}{x} = \log|x| + c$$

Differential Equations Ex 22.9 Q12

Here, $\left(y^2 - 2xy\right)dx = \left(x^2 - 2xy\right)dy$

$$\frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - 2xvx}{x^2 - 2xvx}$$

$$v + x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 2v}{1 - 2v} - v$$

$$= \frac{v^2 - 2v - v + 2v^2}{1 - 2v}$$

$$x\frac{dv}{dx} = \frac{3v^2 - 3v}{1 - 2v}$$

$$\frac{1 - 2v}{3\left(v^2 - v\right)}dv = \frac{dx}{x}$$

$$\frac{-}{3}\frac{(2v - 1)}{v^2 - v}\ dv = \frac{dx}{x}$$

$$\int\frac{2v - 1}{v^2 - v}dv = -3\int\frac{dx}{x}$$

$$\log\left|v^2 - v\right| = -3\log|x| + \log c$$

$$v^2 - v = \frac{c}{x^3}$$

$$\frac{y^2}{x^2} - \frac{y}{x} = \frac{c}{x^3}$$

$$y^2 - xy = \frac{c}{x}$$

$$x\left(y^2 - xy\right) = c$$

Differential Equations Ex 22.9 Q13

Here, $2xydx + \left(x^2 + 2y^2\right)dy = 0$

$$\frac{dy}{dx} = \frac{2xy}{x^2 + 2y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2xvx}{x^2 + 2v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{2v}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{2v}{1 + 2v^2} - v$$

$$= \frac{2v - v - 2v^3}{1 + 2v^2}$$

$$x\frac{dv}{dx} = \frac{v - 2v^3}{1 + 2v^2}$$

$$\int \frac{1 + 2v^2}{v - 2v^3}dv = \int \frac{dx}{x} \qquad ---(i)$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1 + 2v^2}{v\left(1 - 2v^2\right)}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A}{v} + \frac{Bv + c}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v\left(1 - 2v^2\right)} = \frac{A\left(1 - 2v^2\right) + (Bv + c)v}{v\left(1 - 2v^2\right)}$$

$$1 + 2v^2 = A - 2Av^2 + Bv^2 + cv$$

$$1 + 2v^2 = v^2(-2A + B) + cv + A$$

Comparing the coefficients of like powers of v,

$A = 1$

$c = 0$

$-2A + B = 2$

$-2 + B = 0$

$B = 4$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} + \frac{4v}{1 - 2v^2}$$

$$\frac{1 + 2v^2}{v - 2v^3} = \frac{1}{v} - \frac{(-4v)}{\left(1 - 2v^2\right)}$$

Differential Equations Ex 22.9 Q14

Here, $3x^2dy = \left(3xy + y^2\right)dx$

$$\frac{dy}{dx} = \frac{3xy + y^2}{3x^2}$$

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{3xvx + v^2x^2}{3x^2}$$

$$v + x\frac{dv}{dx} = \frac{3v + v^2}{3}$$

$$x\frac{dv}{dx} = \frac{3v + v^2}{3} - v$$

$$x\frac{dv}{dx} = \frac{3v + v^2 - 3v}{3}$$

$$x\frac{dv}{dx} = \frac{v^2}{3}$$

$$3\int\frac{1}{v^2}dv = \int\frac{dx}{x}$$

$$3\left(-\frac{1}{v}\right) = \log|x| + c$$

$$-\frac{3x}{y} = \log|x| + c$$

Differential Equations Ex 22.9 Q15

Here, $\frac{dy}{dx} = \frac{x}{2y + x}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x}{2vx + x}$$

$$v + x\frac{dv}{dx} = \frac{1}{2v + 1}$$

$$x\frac{dv}{dx} = \frac{1}{2v + 1} - v$$

$$x\frac{dv}{dx} = \frac{1 - 2v^2 - v}{2v + 1}$$

$$\int \frac{2v + 1}{1 - v - 2v^2}dv = \int \frac{dx}{x}$$

$$-\int \frac{2v + 1}{2v^2 + v - 1}dv = \int \frac{dx}{x}$$

$$\frac{1}{2}\int \frac{4v + 2}{2v^2 + v - 1}dv = -\int \frac{dx}{x}$$

$$\int \frac{4v + 1 + 1}{2v^2 + v - 1}dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1}dv + \int \frac{1}{2v^2 + v - 1}dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int \frac{1}{v^2 + \frac{v}{2} - \frac{1}{2}}dv = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int \frac{dv}{v^2 + 2v\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} = -2\int \frac{dx}{x}$$

$$\int \frac{4v + 1}{2v^2 + v - 1}dv + \frac{1}{2}\int \frac{dv}{\left(v + \frac{1}{4}\right)^2 - \left(\frac{3}{4}\right)^2} = -2\int \frac{dx}{x}$$

$$\log\left|2v^2 + v - 1\right| + \frac{1}{2} \times \frac{1}{2\left(\frac{3}{4}\right)}\log\left|\frac{v + \frac{1}{4} - \frac{3}{4}}{v + \frac{1}{4} + \frac{3}{4}}\right| = -2\log\left|x\right| + \log c$$

Differential Equations Ex 22.9 Q16

Here, $(x+2y)dx-(2x-y)dy=0$

$$\frac{dy}{dx}=\frac{(x+2y)}{(2x-y)}$$

It is a homogeneous equation

Put $y=vx$

$$\therefore \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

So,

$$v+x\frac{dv}{dx}=\frac{x+2vx}{2x-vx}$$

$$v+x\frac{dv}{dx}=\frac{1+2v}{2-v}$$

$$x\frac{dv}{dx}=\frac{1+2v}{2-v}-\frac{v}{1}$$

$$x\frac{dv}{dx}=\frac{1+2v-2v+v^2}{2-v}$$

$$x\frac{dv}{dx}=\frac{1+v^2}{2-v}$$

$$\frac{2-v}{1+v^2}=\frac{dx}{x}$$

$$\int\frac{2-v}{1+v^2}dv=\int\frac{dx}{x}$$

$$\int\frac{2}{1+v^2}dv-\int\frac{v}{1+v^2}dv=\int\frac{dx}{x}$$

$$2\tan^{-1}v-\frac{1}{2}\log\left|1+v^2\right|=\log|x|+\log c$$

$$2\tan^{-1}v=\log xc+\log\left|1+v^2\right|^{\frac{1}{2}}$$

$$e^{2\tan^{-1}v}=\left(1+v^2\right)^{\frac{1}{2}}xc$$

$$e^{2\tan^{-1}\frac{y}{x}}=\left\{\frac{\left(y^2+x^2\right)^{\frac{1}{2}}}{x}\right\}xc$$

$$e^{2\tan^{-1}\frac{y}{x}}=\left(y^2+x^2\right)^{\frac{1}{2}}c$$

Differential Equations Ex 22.9 Q17

Here, $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sqrt{\frac{v^2x^2}{x^2} - 1}$$

$$v + x\frac{dv}{dx} = v - \sqrt{v^2 - 1}$$

$$x\frac{dv}{dx} = v - \sqrt{v^2 - 1} - v$$

$$x\frac{dv}{dx} = -\sqrt{v^2 - 1}$$

$$\int \frac{dv}{\sqrt{v^2 - 1}} - \int \frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = -\log|x| + \log c$$

$$\left(\frac{y}{x} + \sqrt{v^2 - 1}\right) = \frac{c}{x}$$

$$y + \sqrt{y^2 - x^2} = c$$

Differential Equations Ex 22.9 Q18

$$\frac{dy}{dx} = \frac{y}{x}\left\{\log\left(\frac{y}{x}\right) + 1\right\}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x}\left\{\log\left(\frac{vx}{x}\right) + 1\right\}$$

$$v + x\frac{dv}{dx} = v\log v + v$$

$$x\frac{dv}{dx} = v\log v$$

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$$\log\log v = \log|x| + \log c$$

$$\log v = xc$$

$$\log\frac{y}{x} = xc$$

$$\frac{y}{x} = e^{xc}$$

$$y = xe^{xc}$$

Differential Equations Ex 22.9 Q19

$$\frac{dy}{dx}=\frac{y}{x}+\sin\left(\frac{y}{x}\right)$$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

So,

$$v+x\frac{dv}{dx}=v+\sin v$$

$$x\frac{dv}{dx}=\sin v$$

$$\operatorname{cosec} v\,dv=\frac{dx}{x}$$

$$\int \operatorname{cosec} v\,dv=\int\frac{dx}{x}$$

$$\log\tan\frac{v}{2}=\log x+\log c$$

$$\tan\frac{v}{2}=Cx$$

$$\tan\frac{y}{2x}=Cx$$

Differential Equations Ex 22.9 Q20

Here, $y^2dx + \left(x^2 - xy + y^2\right)dy = 0$

$$\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - xvx + v^2x^2}$$

$$v + x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2}$$

$$x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2} - \frac{v}{1}$$

$$= \frac{-v^2 - v + v^2 - v^3}{1 - v + v^2}$$

$$x\frac{dv}{dx} = \frac{-v - v^3}{v^2 - v + 1}$$

$$\frac{v^2 - v + 1}{-v\left(1 + v^2\right)}dv = \frac{dx}{x}$$

$$\left(\frac{1}{1 + v^2} - \frac{1}{v}\right)dv = \frac{dx}{x}$$

$$-\int\frac{1}{v}dv + \int\frac{1}{1 + v^2}dv = \int\frac{dx}{x}$$

$$-\log|v| + \tan^{-1}v = \log|x| + \log c$$

$$\log\left|\frac{x}{y}\right| + \tan^{-1}\left(\frac{y}{x}\right) = \log c$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log xc - \log\frac{x}{y}$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log\left(\frac{xcy}{x}\right)$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \log(cy)$$

$$e^{\tan^{-1}\left(\frac{y}{x}\right)} = cy$$

Differential Equations Ex 22.9 Q21

Here, $\left[x\sqrt{x^2+y^2}-y^2\right]dx+xydy=0$

$$\frac{dy}{dx}=\frac{\left[y^2-x\sqrt{x^2+y^2}\right]}{xy}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{\left[v^2x^2-x\sqrt{x^2+v^2x^2}\right]}{xvx}$$

$$v+x\frac{dv}{dx}=\frac{\left[v^2-\sqrt{1+v^2}\right]}{v}$$

$$x\frac{dv}{dx}=\frac{v^2-\sqrt{1+v^2}}{v}-v$$

$$=\frac{v^2-\sqrt{1+v^2}-v^2}{v}$$

$$x\frac{dv}{dx}=\frac{-\sqrt{1+v^2}}{v}$$

$$\int\frac{v}{\sqrt{1+v^2}}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v}{\sqrt{1+v^2}}dv=-\int\frac{dx}{x}$$

Let $1+v^2=t$

$$2vdv=dt$$

$$\frac{1}{2}\int\frac{1}{\sqrt{t}}dt=-\int\frac{dx}{x}$$

$$\frac{1}{2}\times2\sqrt{t}=-\log|x|+\log c$$

$$\sqrt{1+v^2}=\log\left|\frac{c}{x}\right|$$

$$\frac{\sqrt{x^2+y^2}}{x}=\log\left|\frac{c}{x}\right|$$

$$\sqrt{x^2+y^2}=x\log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q22

Here, $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v dv = -\int \frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q23

Here, $\frac{y}{x}\cos\left(\frac{y}{x}\right)dx - \left\{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{\frac{y}{x}\cos\left(\frac{y}{x}\right)}{\frac{x}{y}\sin\left(\frac{y}{x}\right) + \cos\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{\frac{vx}{x}\cos\left(\frac{vx}{x}\right)}{\frac{x}{vx}\sin\left(\frac{vx}{x}\right) + \cos\left(\frac{vx}{x}\right)}$$

$$= \frac{v\cos v}{\frac{1}{v}\sin v + \cos v}$$

$$v + x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{v^2\cos v}{\sin v + v\cos v} - v$$

$$x\frac{dv}{dx} = \frac{v^2\cos v - v\sin v - v^2\cos v}{\sin v + v\cos v}$$

$$x\frac{dv}{dx} = \frac{-v\sin v}{\sin v + v\cos v}$$

$$\frac{\sin v + v\cos v}{v\sin v}dv = -\frac{dx}{x}$$

$$\int\left(\frac{1}{v} + \cot v\right)dv = -\log|x| + \log c$$

$$\log|v| + \log|\sin v| = \log\left|\frac{c}{x}\right|$$

$$\log|v\sin v| = \log\left|\frac{c}{x}\right|$$

$$|v\sin v| = \left|\frac{c}{x}\right|$$

$$\left|x\left(\frac{y}{x}\right)\sin\left(\frac{y}{x}\right)\right| = |c|$$

$$\left|y\sin\frac{y}{x}\right| = c$$

Differential Equations Ex 22.9 Q24

Here, $xy\log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2\log\left(\frac{x}{y}\right)\right\}dy = 0$

$$\frac{dy}{dx} = \frac{x^2\log\left(\frac{x}{y}\right) - y^2}{xy\log\left(\frac{x}{y}\right)}$$

It is a homogeneous equation

Put $x = vy$

and $\frac{dx}{dy} = v + y\frac{dv}{dy}$

So,

$$v + y\frac{dv}{dy} = \frac{v^2y^2\log\left(\frac{vy}{y}\right) - y^2}{vy\log\left(\frac{vy}{y}\right)}$$

$$v + y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v}$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1}{v\log v} - v$$

$$y\frac{dv}{dy} = \frac{v^2\log v - 1 - v^2\log v}{v\log v}$$

$$y\frac{dv}{dy} = \frac{-1}{v\log v}$$

$$\int v\log v\,dv = -\int\frac{dy}{y}$$

$$\log v \times \int v\,dv - \int\frac{1}{v}\times\int v\,dv\,dv = -\log|y| + \log c$$

Itegrating it by parts

$$\frac{v^2}{2}\log v \int\frac{1}{v}\times\frac{v^2}{2}dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{1}{2}\int v\,dv = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\log v - \frac{v^2}{4} = \log\left|\frac{c}{y}\right|$$

$$\frac{v^2}{2}\left[\log v - \frac{1}{2}\right] = \log\left|\frac{c}{y}\right|$$

Differential Equations Ex 22.9 Q25

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

Here it is a homogeneous equation

Put $x=vy$

And

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

So,

$$\frac{dx}{dy}=-\frac{e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}{\left(1+e^{\frac{x}{y}}\right)}$$

$$v+y\frac{dv}{dy}=-\frac{e^{\frac{vy}{y}}\left(1-\frac{vy}{y}\right)}{\left(1+e^{\frac{vy}{y}}\right)}$$

$$=-\frac{e^{v}(1-v)}{\left(1+e^{v}\right)}$$

$$y\frac{dv}{dy}=-\frac{e^{v}(1-v)}{\left(1+e^{v}\right)}-v$$

$$=\frac{-e^{v}(1-v)-v\left(1+e^{v}\right)}{\left(1+e^{v}\right)}$$

$$\frac{\left(1+e^{v}\right)}{-e^{v}(1-v)-v\left(1+e^{v}\right)}dv=\frac{dy}{y}$$

$$x+ye^{x/y}=c$$

Differential Equations Ex 22.9 Q26

Here, $\left(x^2+y^2\right)\frac{dy}{dx}=8x^2-3xy+2y^2$

$$\frac{dy}{dx}=\frac{8x^2-3xy+2y^2}{x^2+y^2}$$

It is a homogeneous equation

Put $\quad y=vx$

and $\quad \frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=\frac{8x^2-3xvx+2v^2x^2}{x^2+v^2x^2}$$

$$v+x\frac{dv}{dx}=\frac{8-3v+2v^2}{1+v^2}$$

$$x\frac{dv}{dx}=\frac{8-3v+2v^2}{1+v^2}-v$$

$$=\frac{8-3v+2v^2-v-v^3}{1+v^2}$$

$$x\frac{dv}{dx}=\frac{8-4v+2v^2-v^3}{1+v^2}$$

$$\frac{1+v^2}{8-4v+2v^2-v^3}dv=\frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv=\frac{dx}{x}$$

$$\frac{1+v^2}{4(2-v)+v^2(2-v)}dv=\frac{dx}{x}$$

$$\int\frac{1+v^2}{\left(4+v^2\right)(2-v)}dv=\int\frac{dx}{x} \qquad \text{---(A)}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)}=\frac{Av+B}{4+v^2}+\frac{c}{2-v}$$

$$\frac{1+v^2}{\left(4+v^2\right)(2-v)}=\frac{(Av+B)(2-v)+c\left(4+v^2\right)}{\left(4+v^2\right)(2-v)}$$

$$1+v^2=2Av-Av^2+2B-Bv+4c+cv^2$$

$$1+v^2=v^2(-A+c)+v(2A-B)+2B+4c$$

Comparing the coefficients of like powers of v

$$-A+c=1 \qquad \text{---(i)}$$

$$2A-B=0$$

$\Rightarrow \quad B=2A \qquad$ ---(ii)

$$2B+4c=1 \qquad \text{---(iii)}$$

Solving equation (i), (ii) and (iii)

$$A=-\frac{3}{8}, B=-\frac{3}{4}, C=\frac{5}{8}$$

Using equation (A)

$$\int \frac{\left(-\frac{3}{8}v - \frac{3}{4}\right)}{4+v^2}dv + \frac{5}{8}\int \frac{1}{2-v}dv = \int \frac{dx}{x}$$

$$-\frac{3}{8}\int \frac{v+2}{4+v^2}dv + \frac{5}{8}\int \frac{1}{2-v}dv = \int \frac{dx}{x}$$

$$-\frac{3}{8}\int \frac{v}{4+v^2}dv - \frac{3}{8}\int \frac{1}{4+v^2}dv + \frac{5}{8}\int \frac{1}{2-v}dv = \int \frac{dx}{x}$$

$$-\frac{3}{16}\log\left|4+v^2\right| - \frac{3}{8}\tan^{-1}\frac{v}{2} - \frac{5}{8}\log|2-v| = \log|x| + loc$$

$$-\left[\log\left|4+v^2\right|^{\frac{3}{16}} + \log e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)} + \log(2-v)^{\frac{5}{8}}\right] = \log|xc|$$

$$\left(4+v^2\right)^{\frac{3}{16}} \times e^{\frac{3}{8}\tan^{-1}\left(\frac{v}{2}\right)} \times (2-v)^{\frac{5}{8}} = \frac{c}{x}$$

$$\frac{\left(4x^2+y^2\right)^{\frac{3}{16}}}{x^{\frac{3}{8}}} \times e^{\frac{3}{8}\tan^{-1}\left(\frac{y}{2x}\right)} \frac{(2x-y)^{\frac{5}{8}}}{x^{\frac{5}{8}}} = \frac{c}{x}$$

$$\left(4x^2+y^2\right)^{\frac{3}{16}} \times (2x-y)^{\frac{5}{8}} = c\, e^{\frac{-3}{8}\tan^{-1}\left(\frac{y}{2x}\right)}$$

Differential Equations Ex 22.9 Q27

Here, $\left(x^2-2xy\right)dy + \left(x^2-3xy+2y^2\right)dx = 0$

$$\frac{dy}{dx} = \frac{x^2-3xy+2y^2}{2xy-x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2-3xvx+2v^2x^2}{2xvx-x^2}$$

$$x\frac{dv}{dx} = \frac{1-3v+2v^2}{2v-1} - v$$

$$x\frac{dv}{dx} = \frac{1-3v+2v^2-2v^2+v}{2v-1}$$

$$x\frac{dv}{dx} = \frac{1-2v}{2v-1}$$

$$\frac{2v-1}{1-2v}dv = \frac{dx}{x}$$

$$\frac{1-2v}{1-2v}dv = -\int\frac{dx}{x}$$

$$\int dv = -\int\frac{dx}{x}$$

$$v = -\log|x| + C$$

$$y/x + \log x = C$$

Differential Equations Ex 22.9 Q28

Here, $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

$$\frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\cos^2\left(\frac{vx}{x}\right)}{x}$$

$$v + x\frac{dv}{dx} = v - \cos^2 v$$

$$x\frac{dv}{dx} = v - \cos^2 v - v$$

$$x\frac{dv}{dx} = -\cos^2 v$$

$$\frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\int \sec^2 v dv = -\int \frac{dx}{x}$$

$$\tan v = -\log|x| + \log c$$

$$\tan\frac{y}{x} = \log\left|\frac{c}{x}\right|$$

Differential Equations Ex 22.9 Q29

Here, $x\frac{dy}{dx} - y = 2\sqrt{y^2 - x^2}$

$$\frac{dy}{dx} = \frac{2\sqrt{y^2 - x^2} + y}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2\sqrt{v^2x^2 - x^2} + vx}{x}$$

$$v + x\frac{dv}{dx} = 2\sqrt{v^2 - 1} + v$$

$$x\frac{dv}{dx} = 2\sqrt{v^2 - 1}$$

$$\int \frac{dv}{\sqrt{v^2 - 1}} = 2\int \frac{dx}{x}$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = 2\log|x| + \log|c|$$

$$\log\left|v + \sqrt{v^2 - 1}\right| = \log\left|cx^2\right|$$

$$v + \sqrt{v^2 - 1} = \left|cx^2\right|$$

$$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} - 1} = \left|cx^2\right|$$

$$\left(y + \sqrt{y^2 - x^2}\right) = cx^3$$

Differential Equations Ex 22.9 Q30

Here, $x\cos\left(\frac{y}{x}\right)(ydx + xdy) = y\sin\left(\frac{y}{x}\right)(xdy - ydx)$

$$yx\cos\left(\frac{y}{x}\right) + x^2\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = xy\sin\left(\frac{y}{x}\right) - y^2\sin\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \left(\frac{-y^2\sin\left(\frac{y}{x}\right) - xy\cos\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right) - xy\sin\left(\frac{y}{x}\right)}\right)$$

$$\frac{dy}{dx} = \frac{-xy\cos\left(\frac{y}{x}\right) - y^2\sin\left(\frac{y}{x}\right)}{x^2\cos\left(\frac{y}{x}\right) - xy\sin\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{-xvx\cos\left(\frac{vx}{x}\right) - v^2x^2\sin\left(\frac{vx}{x}\right)}{x^2\cos\left(\frac{vx}{x}\right) - xvx\sin\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{-v\cos v - v^2\sin v}{\cos v - v\sin v} - v$$

$$x\frac{dv}{dx} = \frac{-v\cos v - v^2\sin v - v\cos v + v^2\sin v}{\cos v - v\sin v}$$

$$x\frac{dv}{dx} = \frac{-2v\cos v}{\cos v - v\sin v}$$

$$\int\frac{\cos v - v\sin v}{v\cos v}dv = -2\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v} - \tan v\right)dv = -2\int\frac{dx}{x}$$

Differential Equations Ex 22.9 Q31

Here, $\left(x^2 + 3xy + y^2\right)dx - x^2dy = 0$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2 + 3xvx + v^2x^2}{x^2}$$

$$v + x\frac{dv}{dx} = 1 + 3v + v^2$$

$$x\frac{dv}{dx} = 1 + 2v + v^2$$

$$x\frac{dv}{dx} = (v+1)^2$$

$$\int\frac{1}{(v+1)^2}dv = \int\frac{dx}{x}$$

$$-\frac{1}{v+1} = \log|x| - c$$

$$\frac{x}{x+y} + \log|x| = c$$

Differential Equations Ex 22.9 Q32

Here, $(x-y)\frac{dy}{dx} = x+2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$x\frac{dv}{dx} = \frac{1+2v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$

$$\frac{1-v}{v^2+v+1}dv = \frac{dx}{x}$$

$$-\frac{v-1}{v^2+v+1}dv = \frac{dx}{x}$$

$$\frac{1}{2}\times\frac{2v-2}{v^2+v+1}dv = \frac{-dx}{x}$$

$$\int\frac{(2v+1)-3}{v^2+v+1}dv = -\int\frac{2dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv - \int\frac{3}{v^2+2v\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1} = -2\int\frac{dx}{x}$$

$$\int\frac{2v+1}{v^2+v+1}dv - \int\frac{3}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -2\int\frac{dx}{x}$$

$$\log\left|v^2+v+1\right| - 3\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = -2\log|x|+c$$

$$\log\left|y^2+xy+x^2\right| = 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right)+c$$

Differential Equations Ex 22.9 Q33

$$\left(2x^2y + y^3\right)dx + \left(xy^2 + 3x^3\right)dy = 0$$

$$\frac{dy}{dx} = \frac{2x^2y + y^3}{3x^3 - xy^2}$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{2x^2vx + v^3x^3}{3x^3 - xv^2x^2}$$

$$x\frac{dv}{dx} = \frac{2v + v^3}{3 - v^2} - v$$

$$= \frac{2v + v^3 - 3v + v^3}{3 - v^2}$$

$$x\frac{dv}{dx} = \frac{2v^3 - v}{3 - v^2}$$

$$\int \frac{3 - v^2}{2v^3 - v}dv = \int \frac{dx}{x} \qquad \text{---(i)}$$

$$\frac{3 - v^2}{v\left(2v^2 - 1\right)} = \frac{A}{(v)} + \frac{Bv + c}{\left(2v^2 - 1\right)}$$

$$3 - v^2 = A\left(2v^2 - 1\right) + (Bv + c)(v)$$

$$= 2Av^2 - A + Bv^2 + cv$$

$$3 - v^2 = (2A + B)v^2cv - A$$

Comparing the coefficient of like powers of v

$A = -3$

$C = 0$

and $\quad 2A + B = -1$

$\Rightarrow \quad 2(-3) + B = -1$

$\Rightarrow \quad B = 5$

So,

$$\int \frac{-3}{v}dv + \int \frac{5v}{2v^2 - 1}dv = \int \frac{dx}{x}$$

$$-3\int \frac{1}{v}dv + \frac{5}{4}\int \frac{4v}{2v^2 - 1}dv = \int \frac{dx}{x}$$

$$-3\log|v| + \frac{5}{4}\log\left|2v^2 - 1\right| = \log|x| + \log|c|$$

$$-12\log|v| + 5\log\left|2v^2 - 1\right| = 4\log|x| + 4\log|c|$$

$$\frac{\left|2v^2-1\right|^5}{v^{12}} = x^4c^4$$

$$\frac{\left|2y^2-x^2\right|^5}{x^{10}} = x^4c^4\left(\frac{y}{x}\right)^{12}$$

$$\left|2y^2-x^2\right|^5 = x^{14}c^4\frac{y^{12}}{x^{12}}$$

$$x^2c^4y^{12} = \left|2y^2-x^2\right|^5$$

Differential Equations Ex 22.9 Q34

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

$$\frac{dy}{dx} = \frac{y - x\sin\left(\frac{y}{x}\right)}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - x\sin\left(\frac{vx}{x}\right)}{x}$$

$$x\frac{dv}{dx} = v - \sin v - v$$

$$\int \text{cosec}\,v\,dv = -\int\frac{dx}{x}$$

$$\log\left|\text{cosec}\,v + \cot v\right| = -\log\frac{c}{x}$$

$$\log\left|\text{cosec}\,v + \cot v\right| = \log\frac{x}{c}$$

$$\text{cos sec}\left(\frac{y}{x}\right) + \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$\frac{\left(1+\cos\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{x}{c}$$

$$x\sin\left(\frac{y}{x}\right) = c\left(1+\cos\frac{y}{x}\right)$$

Differential Equations Ex 22.9 Q35

$$ydx + \left\{x\log\left(\frac{y}{x}\right)\right\}dy - 2xdy = 0$$

$$y + x\log\left(\frac{y}{x}\right)\frac{dy}{dx} - 2x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{2x - x\log\left(\frac{vx}{x}\right)}$$

$$x\frac{dv}{dx} = \frac{v}{2-\log v} - v$$

$$x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2-\log v}$$

$$\int\frac{\log v - 2}{v(\log v - 1)}dv = -\int\frac{dx}{x}$$

Let $\log v - 1 = t$

$$\frac{1}{v}dv = dt$$

$$\int\left(\frac{t-1}{t}\right)dt = -\int\frac{dx}{x}$$

$$t - \log|t| = \log\left|\frac{c}{x}\right|$$

$$\log v - 1\log(\log v - 1) = \log\left|\frac{c}{x}\right|$$

$$\log e^{\log v - 1} - \log|\log v - 1| = \log\left|\frac{c}{x}\right|$$

$$e^{\log\left(\frac{v}{e}\right)} = \frac{c}{x}|\log v - 1|$$

$$\frac{v}{e} = \frac{c}{x}|\log v - 1|$$

$$y = c_1\left\{\log\left|\frac{y}{x}\right| - 1\right\}$$

Differential Equations Ex 22.9 Q36(i)

$$\left(x^2+y^2\right)dx = 2xydy,\ y(1)=0$$

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$

It is a homogenues equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\int\frac{2v}{1-v^2} = \int\frac{dx}{x}$$

$$\log\left|1-v^2\right| = -\log|x| + \log|c|$$

$$\log\left|1-v^2\right| = \log\left|\frac{c}{x}\right|$$

$$\left|\frac{x^2-y^2}{x^2}\right| = \left|\frac{c}{x}\right|$$

$$\left|x^2-y^2\right| = |cx| \qquad \text{---(i)}$$

Put $y = 0,\ x = 1$

$$1 - 0 = c$$

$$c = 1$$

Put the value of c in equation (i),

$$\left|x^2-y^2\right| = |x|$$

$$\left(x^2-y^2\right)^2 = x^2$$

Differential Equations Ex 22.9 Q36(ii)

Here, $xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0,\ y(e) = 0$

$$\frac{dy}{dx} = \frac{y - xe^{\frac{y}{x}}}{x}$$

It is a homogeneous equation

Put $y = vx$

and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx - xe^{\frac{vx}{x}}}{x}$$

$$x\frac{dv}{dx} = v - e^{v} - v$$

$$x\frac{dv}{dx} = -e^{v}$$

$$\int -e^{-v}dv = \int\frac{dx}{x}$$

$$e^{v} = \log|xc|$$

$$v = \log(\log|xc|)$$

$$\frac{y}{x} = \log\log|x| + k$$

$$y = x\log(\log|x|) + k \qquad ---(i)$$

Put $y = 0,\ x = e$

$$0 = e\log(\log e) + k$$

$$0 = e \times 0 + k$$

$$0 = k$$

Using equation (i),

$$y = x\log(\log|x|)$$

Differential Equations Ex 22.9 Q36(iii)

$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0, y(1) = 0$

Here it is a homogeneous equation

Put $y = vx$

And

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\frac{vx}{x}$$

$$x\frac{dv}{dx} = v - \operatorname{cosec}v - v$$

$$= -\operatorname{cosec}v$$

$$\frac{dv}{\operatorname{cosec}v} = -\frac{dx}{x}$$

$$\sin v\,dv = -\frac{dx}{x}$$

$$-\cos v = -\log|x| + c$$

$$-\cos\frac{y}{x} = -\log|x| + c$$

Now putting $y = 0, x = 1$, we have

$c = -1$

Now

$$-\cos\frac{y}{x} + 1 = -\log|x|$$

$$\log|x| = \cos\frac{y}{x} - 1$$

Differential Equations Ex 22.9 Q36(iv)

$$\left(xy - y^2\right)dx - x^2dy = 0,\ \ y(1) = 1$$

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2}$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{xvx - v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = v - v^2 - v$$

$$x\frac{dv}{dx} = -v^2$$

$$-\int \frac{1}{v^2}dv = \int \frac{dx}{x}$$

$$-\left(-\frac{1}{v}\right) = \log|x| + c$$

$$\frac{x}{y} = \log|x| + c \qquad \text{---(i)}$$

Put $y = 1,\ x = 1$

$$1 = c$$

Using equation (1),

$$x = y\left[\log|x| + 1\right]$$

$$y = \frac{x}{\left[\log|x| + 1\right]}$$

Differential Equations Ex 22.9 Q36(v)

$$\frac{dy}{dx} = \frac{y(x+2y)}{x(2x+y)}, \; y(1) = 2$$

It is a homogeneous equation

Put $\quad y = vx$

and $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

So,

$$v + x\frac{dv}{dx} = \frac{vx(x+2vx)}{x(2x+vx)}$$

$$x\frac{dv}{dx} = \frac{v(1+2v)}{(2+v)} - v$$

$$x\frac{dv}{dx} = \frac{v+2v^2-2v-v^2}{2+v}$$

$$x\frac{dv}{dx} = \frac{v^2-v}{2+v}$$

$$\frac{2+v}{v^2-v}dv = \frac{dx}{x}$$

$$\int\frac{2+v}{v^2-v}dv = \int\frac{dx}{x} \qquad ---(i)$$

$$\frac{2+v}{v(v-1)} = \frac{A}{v} + \frac{B}{v-1}$$

$$\frac{2+v}{v(v-1)} = \frac{A(v-1)+Bv}{v(v-1)}$$

$$2+v = (A+B)v - A$$

Comparing the coefficients of like powers of v,

$$A = -2$$

$$A + B = 1$$

$\Rightarrow \quad -2 + B = 1$

$\Rightarrow \quad B = 3$

Using equation (i),

$$\int\frac{-2}{v}dv + 3\int\frac{1}{v-1}dv = \int\frac{dx}{x}$$

$$-2\log|v| + 3\log|v-1| = \log|cx|$$

$$|v-1|^3 = v^2cx$$

$$\frac{|y-x|^3}{x^3} = \frac{y^2}{x^2}cx$$

Differential Equations Ex 22.9 Q36(vi)

$$\left(y^4 - 2x^3y\right)dx + \left(x^4 - 2xy^3\right)dy = 0$$

$$\frac{dy}{dx} = \frac{2x^3y - y^4}{x^4 - 2xy^3}$$

It is a homogeneous equation

Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So,

$$v + x\frac{dv}{dx} = \frac{2x^3vx - x^4v^4}{x^4 - 2xv^3x^3}$$

$$x\frac{dv}{dx} = \frac{2v - v^4}{1 - 2v^3} - v$$

$$x\frac{dv}{dx} = \frac{2v - v^4 - v + 2v^4}{1 - 2v^3}$$

$$x\frac{dv}{dx} = \frac{v^4 + v}{1 - 2v^3}$$

$$\int \frac{1 - 2v^3}{v\left(v^3 + 1\right)}dv = \int \frac{dx}{x} \qquad ---(i)$$

$$\frac{1 - 2v^3}{v\left(v + 1\right)\left(v^2 - v + 1\right)} = \frac{A}{v} + \frac{B}{V + 1} + \frac{cv + D}{v^2 - v + 1}$$

$$1 - 2v^3 = A\left(v^3 + 1\right) + Bv\left(v^2 - v + 1\right) + \left(cv + D\right)\left(v^2 + v\right)$$

$$= Av^3 + A + cv^3 - Bv^2 + cv + cv^3 + cv^2 + Dv^2 + Dv$$

$$1 - 2v^3 = v^3\left(A + B + C\right) + v^2\left(-B + C + D\right) + v\left(B + D\right) + A$$

Comparing the coefficients of like powers of v

$$A = 1 \qquad ---(ii)$$

$$B + D = 0 \qquad ---(iii)$$

$$-B + C + D = 0 \qquad ---(iv)$$

$$A + B + C = -2 \qquad ---(v)$$

Solution of equation (ii), (iii), (iv), (v) gives

$$A = 1,\ b = -1,\ c = -2,\ x = 1$$

Using equation (i),

$$\int \frac{1}{v}dv - \int \frac{1}{v + 1}dv - \int \frac{2v - 1}{v^2 - v + 1}dv = \int \frac{dx}{x}$$

$$\log|v| - \log|v + 1| - \log\left|v^2 - v + 1\right| = \log|xc|$$

$$\log\left|\frac{v}{v^3 + 1}\right| = \log|xc|$$

Differential Equations Ex 22.9 Q36(vii)

Here, $x\left(x^2+3y^2\right)dx+y\left(y^2+3x^2\right)dy=0,\ y(1)=1$

$$\frac{dy}{dx}=-\frac{x\left(x^2+3y^2\right)}{y\left(y^2+3x^2\right)}$$

It is a homogeneous equation

Put $y=vx$

and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

So,

$$v+x\frac{dv}{dx}=-\frac{x\left(x^2+3v^2x^2\right)}{vx\left(v^2x^2+3x^2\right)}$$

$$x\frac{dv}{dx}=-\frac{\left(1+3v^2\right)}{v\left(v^2+3\right)}-v$$

$$x\frac{dv}{dx}=\frac{-1-3v^2-v^4-3v^2}{v\left(v^2+3\right)}$$

$$=\frac{-v^4-6v^2-1}{v\left(v^2+3\right)}$$

$$\frac{v\left(v^2+3\right)}{v^4+6v^2+1}dc=-\frac{dx}{x}$$

$$\int\frac{4v^3+12v}{v^4+6v^2+1}dv=-4\int\frac{dx}{x}$$

$$\log\left|v^4+6v^2+1\right|=\log\left|\frac{c}{x^4}\right|$$

$$\left|v^4+6v^2+1\right|=\left|\frac{c}{x^4}\right|$$

$$\left|y^4+6y^2x^2+x^4\right|=|c| \qquad \text{---(i)}$$

Put $y=1,\ x=1$

$(1+6+1)=c$

$\Rightarrow$ $c=8$

Put $c=8$ in equation (i),

$$\left(y^4+x^4+6x^2y^2\right)=8$$

Differential Equations Ex 22.9 Q36(viii)

$$\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx+xdy=0$$

$$\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx=-xdy$$

$$\sin^2\left(\frac{y}{x}\right)+\frac{y}{x}=\frac{dy}{dx}........(i)$$

Let $v=\frac{y}{x}$

$$v+x\frac{dv}{dx}=\frac{dy}{dx}$$

From eq (i)

$$\sin^2 v+v=v+x\frac{dv}{dx}$$

$$\frac{1}{\sin^2 v}dv=\frac{1}{x}dx$$

Integrating on both the sides we have,

$$\int\frac{1}{\sin^2 v}dv=\int\frac{1}{x}dx$$

$$-\cot v=\log(x)+C$$

$$-\cot\left(\frac{y}{x}\right)=\log(x)+C.........(ii)$$

Put $x=1$ $y=\frac{\pi}{4}$ in eq (ii)

$$-\cot\left(\frac{\pi}{4}\right)=\log(1)+C$$

$$C=-1$$

From eq (ii) we have

$$-\cot\left(\frac{y}{x}\right)=\log(x)-1$$

Differential Equations Ex 22.9 Q36(ix)

$$\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx+xdy=0$$

Here it is a homogeneous equation

Put $y=vx$

And

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

So,

$$v+x\frac{dv}{dx}=-\sin^2\left(\frac{vx}{x}\right)+\frac{vx}{x}$$

$$x\frac{dv}{dx}=-\sin^2 v$$

$$\frac{dv}{\sin^2 v}=-\frac{dx}{x}$$

$$\cot\left(\frac{y}{x}\right)=\log|cx|$$

$$x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0, y(2)=\pi$$

Here it is a homogeneous equation

Put $y=vx$

And

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

So,

$$v+x\frac{dv}{dx}=\frac{vx}{x}-\sin\left(\frac{vx}{x}\right)$$

$$x\frac{dv}{dx}=-\sin v$$

$$\frac{dv}{\sin v}=-\frac{dx}{x}$$

$$\text{cosec}\,v\,dv=-\frac{dx}{x}$$

$$-\log(\text{cosec}\,v+\cot v)=-\log x+c$$

Now putting $y=\pi, x=2$, we have

$c=0.301$

Now

$$-\log\left(\text{cosec}\left(\frac{y}{x}\right)+\cot\left(\frac{y}{x}\right)\right)=-\log x+0.301$$

Differential Equations Ex 22.9 Q37

Consider the given equation

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$$

This is a homogeneous differential equation.

Thus, substiuting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

in the above equation, we get,

$$x\cos\left(\frac{vx}{x}\right)\left(v+x\frac{dv}{dx}\right) = vx\cos\left(\frac{vx}{x}\right)+x$$

$$\Rightarrow \cos v\left(v+x\frac{dv}{dx}\right)=v\cos\left(\frac{vx}{x}\right)+1$$

$$\Rightarrow v\cos v+x\cos v\frac{dv}{dx}=v\cos v+1$$

$$\Rightarrow x\cos v\frac{dv}{dx}=1$$

$$\Rightarrow \cos v\,dv=\frac{dx}{x}$$

Integrating both the sides,

$$\Rightarrow \int\cos v\,dv=\int\frac{dx}{x}$$

$$\Rightarrow \sin v = \log x + C$$

$$\Rightarrow \sin\left(\frac{y}{x}\right) = \log x + C \ldots(1)$$

Given that when $x = 1$, $y = \frac{\pi}{4}$

Substituting the values, x = 1 and y = $\frac{\pi}{4}$

in equation (1), we get,

$$\Rightarrow \sin\left(\frac{\frac{\pi}{4}}{1}\right) = \log 1 + C$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\right) = 0 + C$$

$$\Rightarrow \frac{1}{\sqrt{2}} = C$$

Subsituting the value of C, in equation (1) we get,

$$\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{\sqrt{2}}$$

Differential Equations Ex 22.9 Q38

consider the given equation

$$(x - y)\frac{dy}{dx} = x + 2y$$

This is a homogeneous equation.

Substiuting y=vx and $\frac{dy}{dx} = \left(v + x\frac{dv}{dx}\right)$ in

the above equation, we have,

$$(x - vx)\left(v + x\frac{dv}{dx}\right) = x + 2vx$$

$$\Rightarrow (1 - v)\left(v + x\frac{dv}{dx}\right) = 1 + 2v$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + 2v}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + 2v}{1 - v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + 2v - v(1 - v)}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + 2v - v + v^2}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v + v^2}{1 - v}$$

$$\Rightarrow \frac{(1 - v)dv}{(1 + v + v^2)} = \frac{dx}{x}$$

Integrating on both the sides, we have,

Integrating on both the sides, we have,

$$\Rightarrow \int \frac{(1-v)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{(1+v+v^2)} - \int \frac{1}{2}\frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{v^2+\frac{1}{4}+v+\frac{3}{4}} - \frac{1}{2}\int \frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2}\int \frac{(2v+1)dv}{(1+v+v^2)} = \int \frac{dx}{x}$$

$$\Rightarrow \frac{3}{2}\times\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}} - \frac{1}{2}\log(1+v+v^2) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2v+1}{\sqrt{3}} - \frac{1}{2}\log(1+v+v^2) = \log x + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}} - \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right) = \log x + C \ldots(1)$$

Given that when x = 1, y = 0

Substituting the values, in the above equation, we get,

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2\times 0+1}{\sqrt{3}} - \frac{1}{2}\log(1+0+0^2) = \log 1 + C$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}} - \frac{1}{2}\times 0 = 0 + C$$

$$\Rightarrow C = \sqrt{3}\times\frac{\pi}{6}$$

$$\Rightarrow C = \frac{\pi}{2\sqrt{3}}$$

Thus, equation (1) becomes,

$$\sqrt{3}\tan^{-1}\frac{2\left(\frac{y}{x}\right)+1}{\sqrt{3}} - \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right) = \log x + \frac{\pi}{2\sqrt{3}}$$

$$\Rightarrow \sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}} - \frac{\pi}{2\sqrt{3}} = \log x + \frac{1}{2}\log\left(1+\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}} - \frac{\pi}{\sqrt{3}} = \log x^2 + \log\left(\frac{x^2+xy+y^2}{x^2}\right)$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\frac{2y+x}{x\sqrt{3}} - \frac{\pi}{\sqrt{3}} = \log(x^2+xy+y^2)$$

Differential Equations Ex 22.9 Q39

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

$$\frac{dy}{dx} = \left(\frac{1}{\frac{x}{y} + \frac{y}{x}} \right) \ldots\ldots.(i)$$

Let $v = \frac{y}{x}$

$$x\frac{dv}{dx} + v = \frac{dy}{dx}$$

From (i) we have,

$$x\frac{dv}{dx} + v = \left(\frac{1}{\frac{1}{v} + v} \right)$$

$$\left(-\frac{1}{v^3} - \frac{1}{v} \right) dv = \frac{1}{x} dx$$

Integrating on both the sides we have

$$\frac{1}{2v^2} - \log v = \log x + C$$

$$\Rightarrow \frac{x^2}{2y^2} = \log\left(\frac{y}{x} \times x \right) + C \ldots\ldots\ldots(ii)$$

Put $x = 0$, $y = 1$

$$0 = \log(1) + C$$

$$C = 0$$

From eq (ii) we have

$$\frac{x^2}{2y^2} = \log(y)$$