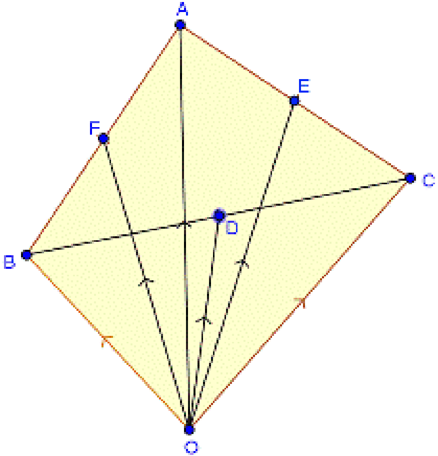
RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.4

Algebra of Vectors Ex 23.4 Q1



Here, in ΔABC, D, E, F are the mid points of the sides of BC, CA and AB respectively. And O is any point in space.

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ be the position vector of point A, B, C, D, E, F with respect to O.

So, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$

$\overrightarrow{OD} = \vec{d}$, $\overrightarrow{OE} = \vec{e}$, $\overrightarrow{OF} = \vec{f}$

$$\vec{d} = \frac{\vec{b} + \vec{c}}{2}$$

$$\vec{e} = \frac{\vec{a} + \vec{c}}{2} \qquad \text{[Using mid point formula]}$$

$$\vec{f} = \frac{\vec{a} + \vec{b}}{2}$$

$$\begin{aligned}
\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} &= \vec{d} + \vec{e} + \vec{f} \\
&= \frac{\vec{b} + \vec{c}}{2} + \frac{\vec{a} + \vec{c}}{2} + \frac{\vec{a} + \vec{b}}{2} \\
&= \frac{\vec{b} + \vec{c} + \vec{a} + \vec{c} + \vec{a} + \vec{b}}{2} \\
&= \frac{2\left(\vec{a} + \vec{b} + \vec{c}\right)}{2} \\
&= \vec{a} + \vec{b} + \vec{c} \\
&= \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}
\end{aligned}$$

So,

$$\overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$

Algebra of Vectors Ex 23.4 Q2

Here, we have to show that the sum of the three vectors ditermined by medians of a triangle directed from the vertices is zero.

Let ABC is triangle such that position vector of A, B and C are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

As AD, BE, CF are medians, D, E and F are mid points.

Position vector of $D = \dfrac{\vec{b}+\vec{c}}{2}$ [Using mid point formula]

Position vector of $E = \dfrac{\vec{c}+\vec{a}}{2}$ [Using mid point formula]

Position vector of $F = \dfrac{\vec{a}+\vec{b}}{2}$ [Using mid point formula]

Now,

$$\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$$
$$= \left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\left(\frac{\vec{c}+\vec{a}}{2}-\vec{b}\right)+\left(\frac{\vec{a}+\vec{b}}{2}-\vec{c}\right)$$
$$= \frac{\vec{b}+\vec{c}-2\vec{a}}{2}+\frac{\vec{c}+\vec{a}-2\vec{b}}{2}+\frac{\vec{a}+\vec{b}-2\vec{c}}{2}$$
$$= \frac{\vec{b}+\vec{c}-2\vec{a}+\vec{c}+\vec{a}-2\vec{b}+\vec{a}+\vec{b}-2\vec{c}}{2}$$
$$= \frac{2\vec{b}+2\vec{c}+2\vec{a}-2\vec{b}-2\vec{a}-2\vec{c}}{2}$$
$$= \frac{\vec{0}}{2}$$
$$= \vec{0}$$

$$\therefore \overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}=\vec{0}$$

Algebra of Vectors Ex 23.4 Q3

Here, it is given that $ABCD$ is a parallelogram, P is the point of intersection of diagonals and O be the point of reference.

Using triangle law in ΔAOP,

$\overrightarrow{OP} + \overrightarrow{PA} = \overrightarrow{OA}$ (i)

Using triangle law in ΔOBP,

$\overrightarrow{OP} + \overrightarrow{PB} = \overrightarrow{OB}$ (ii)

Using triangle law in ΔOPC,

$\overrightarrow{OP} + \overrightarrow{PC} = \overrightarrow{OC}$ (iii)

Using triangle law in ΔOPD,

$\overrightarrow{OP} + \overrightarrow{PD} = \overrightarrow{OD}$ (iv)

Adding equation (i), (ii), (iii), and (iv),

$$\overrightarrow{OP} + \overrightarrow{PA} + \overrightarrow{OP} + \overrightarrow{PB} + \overrightarrow{OP} + \overrightarrow{PC} + \overrightarrow{OP} + \overrightarrow{PD} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$$

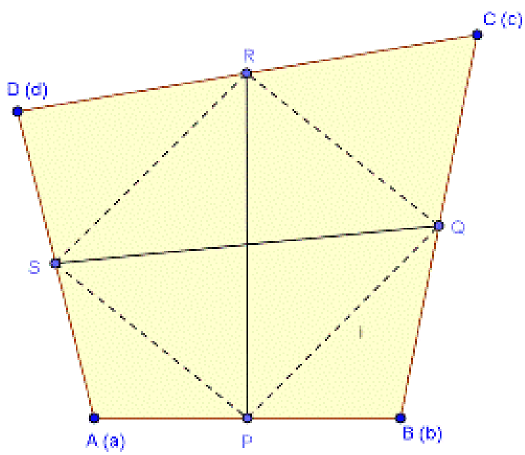
$$4\overrightarrow{OP} + \overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} + \overrightarrow{PD} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$$

$$4\overrightarrow{OP} + \overrightarrow{PA} + \overrightarrow{PB} - \overrightarrow{PA} - \overrightarrow{PB} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$$

$\left[\text{Since } \overrightarrow{PC} = -\overrightarrow{PA} \text{ and } \overrightarrow{PD} = -\overrightarrow{PB} \text{ as } P \text{ is mid point of } AC, BD\right]$

$$4\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$$

Algebra of Vectors Ex 23.4 Q4



Let $ABCD$ be a quadrilateral and P, Q, R, S be the mid points of sides AB, BC, CD and DA respectively.

Let position vector of A, B, C and D be $\vec{a}, \vec{b}, \vec{c}$, and $\vec{d}$.

So position vector of P, Q, R and S are $\left(\frac{\vec{a}+\vec{b}}{2}\right)$, $\left(\frac{\vec{b}+\vec{c}}{2}\right)$, $\left(\frac{\vec{c}+\vec{d}}{2}\right)$ and $\left(\frac{\vec{d}+\vec{a}}{2}\right)$ respectively.

Position vector of $\overrightarrow{PQ}$
= Position vector of Q − Position vector of P

$$= \left(\frac{\vec{b}+\vec{c}}{2}\right) - \left(\frac{\vec{a}+\vec{b}}{2}\right)$$
$$= \frac{\vec{b}+\vec{c}-\vec{a}-\vec{b}}{2}$$
$$= \frac{\vec{c}-\vec{a}}{2} \qquad \text{(i)}$$

Position vector of $\overrightarrow{SR}$
= Position vector of R − Position vector of S

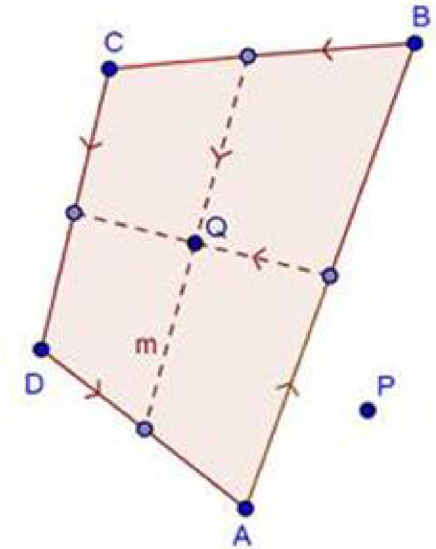
$$= \left(\frac{\vec{c}+\vec{d}}{2}\right) - \left(\frac{\vec{a}+\vec{d}}{2}\right)$$
$$= \frac{\vec{c}+\vec{d}-\vec{a}-\vec{d}}{2}$$
$$= \frac{\vec{c}-\vec{a}}{2} \qquad \text{(ii)}$$

Using (i) and (ii) ,
$\overrightarrow{PQ} = \overrightarrow{SR}$

So, $PQRS$ is a parallelogram.
Therefore, PR bisects QS [as diagonals of parallelogram]

Line segment joining the mid point of opposite sides of a quadrilateral bisects each other.

Algebra of Vectors Ex 23.4 Q5



Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of the points $A, B, C,$ and D respectively.

Then, position vector of

$$\text{mid point of } AB = \frac{\vec{a}+\vec{b}}{2}$$

$$\text{mid point of } BC = \frac{\vec{b}+\vec{c}}{2}$$

$$\text{mid point of } CD = \frac{\vec{c}+\vec{d}}{2}$$

$$\text{mid point of } DA = \frac{\vec{a}+\vec{d}}{2}$$

Q is the mid point of the line joining the mid points of AB and CD

$$\therefore \quad p.r. \text{ or } Q = \frac{\frac{\vec{a}+\vec{a}}{2}+\frac{\vec{c}+\vec{d}}{2}}{2}$$

$$= \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

Let $\vec{p}$ be the position vector of P.

Then,

$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}$$

$$= \vec{a}-\vec{p}+\vec{b}-\vec{p}+\vec{c}-\vec{p}+\vec{d}-\vec{p}$$

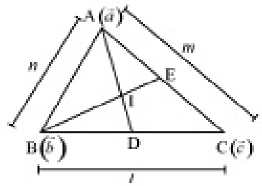
$$= \left(\vec{a}+\vec{b}+\vec{c}+\vec{d}\right)-4\vec{p}$$

$$= 4\left(\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}-\vec{p}\right)$$

$$= 4\overrightarrow{PQ}$$

Algebra of Vectors Ex 23.4 Q6

Let $A(\vec{a}), B(\vec{b})$ and $C(\vec{c})$ be the position vectors of the vertices of the triangle ΔABC and the length of the sides BC, CA and AB be l, m and n respectively.



The internal bisector of a triangle divides the opposite side in the ratio of the sides containing the angles.
Since AD is the internal bisector of the $\angle ABC$,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{n}{m} \qquad (1)$$

Therefore position vector of $D = \dfrac{n\vec{c} + m\vec{b}}{m+n}$

Let the internal bisector intersect at I.

$$\frac{ID}{AI} = \frac{BD}{AB} \qquad (2)$$

$$\frac{BD}{DC} = \frac{n}{m}$$

Therefore,

$$\frac{CD}{BD} = \frac{m}{n}$$

$$\frac{CD+BD}{BD} = \frac{m+n}{n}$$

$$\frac{BC}{BD} = \frac{m+n}{n}$$

$$BD = \frac{\text{ln}}{m+n} \qquad (3)$$

From (2) and (3), we get

$$\frac{ID}{AI} = \frac{\text{ln}}{m+n}$$

Therefore,

$$\text{Position vector of } I = \frac{\left(\frac{nc+mb}{m+n}\right)(m+n)+la}{l+m+n} = \frac{la+mb+nc}{l+m+n}$$

Similarly, we can prove that I lie on the internal bisectors of angles B and C.
Hence the three bisectors are concurrent.