RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.5

Here, $\vec{a} = \sqrt{3}\hat{i} + \hat{j}$

Let $\vec{b}$ is any vector parallel to $\vec{a}$

So, $\vec{b} = \lambda\vec{a}$ (where λ is any scalar)

$$= \lambda\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

$$\vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$

$$\left|\vec{b}\right| = \sqrt{\left(\lambda\sqrt{3}\right)^2 + (\lambda)^2}$$

$$= \sqrt{3\lambda^2 + \lambda^2}$$

$$= \sqrt{4\lambda^2}$$

$$\left|\vec{b}\right| = 2\lambda$$

$$4 = 2\lambda$$

$$\lambda = \frac{4}{2}$$

$$\lambda = 2$$

$$\therefore \quad \vec{b} = \lambda\sqrt{3}\hat{i} + \lambda\hat{j}$$

$$\vec{b} = 2\sqrt{3}\hat{i} + 2\hat{j}$$

Algebra of Vectors Ex 23.5 Q4

(i) Here, $A = (4, -1)$
$B = (1, 3)$

Position vector of $A = 4\hat{i} - \hat{j}$
Position vector of $B = \hat{i} + 3\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= (\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})$$

$$= \hat{i} + 3\hat{j} - 4\hat{i} + \hat{j}$$

$$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$$

$$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$|\overrightarrow{AB}| = 5$$

$$\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$$

(ii) Here, $A = (-6, 3)$
$B = (-2, -5)$

Position vector of $A = -6\hat{i} + 3\hat{j}$
Position vector of $B = -2\hat{i} - 5\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= (-2\hat{i} - 5\hat{j}) - (-6\hat{i} + 3\hat{j})$$

$$= -2\hat{i} - 5\hat{j} + 6\hat{i} - 3\hat{j}$$

$$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$$

$$|\overrightarrow{AB}| = \sqrt{(4)^2 + (-8)^2}$$

$$= \sqrt{16 + 64}$$

$$= \sqrt{80}$$

$$= \sqrt{16 \times 5}$$

$$= 4\sqrt{5}$$

$$|\overrightarrow{AB}| = 4\sqrt{5}$$

$$\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$$

Algebra of Vectors Ex 23.5 Q5

Here, $A = (-1, 3)$

$B = (-2, 1)$

Position vector of $A = -\hat{i} + 3\hat{j}$

Position vector of $B = -2\hat{i} + 1\hat{j}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (-2\hat{i} + \hat{j}) - (-\hat{i} + 3\hat{j})$

$= -2\hat{i} + \hat{j} + \hat{i} - 3\hat{j}$

$= -\hat{i} - 2\hat{j}$

So,

Coordinate of the position vector equivalent to $\overrightarrow{AB} = (-1, -2)$

Algebra of Vectors Ex 23.5 Q6

Here, $A = (-2, -1)$

$B = (3, 0)$

$C = (1, -2)$

Let $D = (x, y)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= (3\hat{i} - 0 \times \hat{j}) - (-2\hat{i} - \hat{j})$

$= 3\hat{i} - 0 \times \hat{j} + 2\hat{i} + \hat{j}$

$\overrightarrow{AB} = 5\hat{i} + \hat{j}$

$\overrightarrow{DC}$ = Position vector of C – Position vector of D

$= (\hat{i} - 2\hat{j}) - (x\hat{i} + y\hat{j})$

$= \hat{i} - 2\hat{j} - x\hat{i} - y\hat{j}$

$\overrightarrow{DC} = (1 - x)\hat{i} + (-2 - y)\hat{j}$

Since $ABCD$ is a parallelogram, which have equal and parallel opposite sides.

So, $\overrightarrow{AB} = \overrightarrow{DC}$

$5\hat{i} + \hat{j} = (1 - x)\hat{i} + (-2 - y)\hat{j}$

Comparing components of LHS and RHS

$5 = 1 - x$

$x = 1 - 5$

$x = -4$

$1 = -2 - y$

$y = -2 - 1$

$y = -3$

So, coordinate of D is $(-4, -3)$

Algebra of Vectors Ex 23.5 Q7

Here, $A(3,4), B(5,-6), C(4,-1)$

$\vec{a} = 3\hat{i} + 4\hat{j}$
$\vec{b} = 5\hat{i} - 6\hat{j}$
$\vec{c} = 4\hat{i} - \hat{j}$

Now,

$$\vec{a} + 2\vec{b} - 3\vec{c} = \left(3\hat{i} + 4\hat{j}\right) + 2\left(5\hat{i} - 6\hat{j}\right) - 3\left(4\hat{i} - \hat{j}\right)$$
$$= 3\hat{i} + 4\hat{j} + 10\hat{i} - 12\hat{j} - 12\hat{i} + 3\hat{j}$$
$$= \hat{i} - 5\hat{j}$$

$\therefore \vec{a} + 2\vec{b} - 3\vec{c} = \hat{i} - 5\hat{j}$

Algebra of Vectors Ex 23.5 Q9

$\left|\overrightarrow{AB}\right| = 5$ units

$\left|\overrightarrow{BC}\right| = \sqrt{(8)^2}$

$\left|\overrightarrow{BC}\right| = 8$ units

$\left|\overrightarrow{AC}\right| = \sqrt{(-3)^2 + (8)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$\left|\overrightarrow{AC}\right| = 5$ units

Here, $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{AC}\right|$

$5 = 5$

So, there are two sides AB, and BC of ΔABC have same length.

ΔABC is an isosceles triangle.

Algebra of Vectors Ex 23.5 Q10

Let $\vec{a} = \hat{i} + \sqrt{3}\hat{j}$

Suppose $\vec{b}$ is any vector parallel to $\vec{a}$

$\vec{b} = \lambda\vec{a}$ where λ is a scalar

$= \lambda\left(\hat{i} + \sqrt{3}\hat{j}\right)$

$\vec{b} = \lambda\hat{i} + \sqrt{3}\lambda\hat{j}$

$|\vec{b}| = \sqrt{(\lambda)^2 + \left(\sqrt{3}\lambda\right)^2}$

$= \sqrt{\lambda^2 + 3\lambda^2}$

$= \sqrt{4\lambda^2}$

$= 2\lambda$

Unit vector of $\vec{b} = \dfrac{\vec{b}}{|\vec{b}|}$

$$\hat{b} = \frac{\lambda\hat{i} + \sqrt{3}\lambda\hat{j}}{2\lambda}$$

$$\hat{b} = \frac{\left(\hat{i} + \sqrt{3}\ \hat{j}\right)}{2}$$

$$\hat{b} = \frac{1}{2}\left(\hat{i} + \sqrt{3}\ \hat{j}\right)$$

Algebra of Vectors Ex 23.5 Q11

(i) Here, $P = (3, 2)$

Position vector of $P = 3\hat{i} + 2\hat{j}$

Component of P along x-axis $= 3\hat{i}$

Component of P along y-axis $= 2\hat{j}$

(ii) Here, $Q = (-5, 1)$

Position vector of $Q = -5\hat{i} + \hat{j}$

Component of Q along x-axis $= -5\hat{i}$

Component of Q along y-axis $= \hat{j}$

(ii) Here, $R = (-11, -9)$

Position vector of $R = -11\hat{i} - 9\hat{j}$

Component of R along x-axis $= -11\hat{i}$

Component of R along y-axis $= -9\hat{j}$

(iv) Here, $S = (4, -3)$

Position vector of $S = 4\hat{i} - 3\hat{j}$

Component of S along x-axis $= 4\hat{i}$

Component of S along y-axis $= -3\hat{j}$